# GRAVITATION AND CELESTIAL MECHANICS INVESTIGATIONS WITH GALILEO

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**Abstract.** The gravitation and celestial mechanics investigations during the cruise phase aad Orbiter phase of the Galileo mission depend on Doppler and ranging measurements generated by the Deep Space Network (DSN) at its three spacecraft tracking sites in California, Australia, and Spain. Other investigations which also rely on DSN data, and which like ours fall under the general discipline of spacecraft radio science, arc described in a companion paper by Howard *et al.* (1992). We group our investigations into four broad categories as follows: (1) the determination of the gravity fields of Jupiter and its four major satellites during the orbital tour, (2) a search for gravitational radiation as evidenced by perturbations to the coherent Doppler link between the spacecraft and Earth, (3) the mathematical modeling, and by implication tests, of general relativistic effects on the Doppler and ranging data during both cruise and orbiter phases, and (4) an improvement in the ephemeris of Jupiter by means of spacecraft ranging during the Orbiter phase. The gravity fields arc accessible because of their effects on the spacecraft motion, determined primarily from the Doppler data. For the Galilean satellites we will determine second degree and order gravity harmonics that will yield new information on the central condensation and likely composition of material within these giant satellites (Hubbard aad Anderson, 1978). The search for gravitational radiation is being conducted in cruise for periods of 40 days centered around solar opposition. During these times the radio link is least affected by scintillations introduced by solar plasma. Our sensitivity to the amplitude of sinusoidal signals approaches $10^{-15}$ in a band of gravitational frequencies between $10^{-4}$ and $10^{-3}$ Hz, by far the best sensitivity obtained in this band to date, In addition to the primary objectives of our investigations, we discuss two secondary objectives: the determination of a range fix oa Venus during the flyby 0010 February, 1990, and the determination of the Earth's mass (GM) from the two Earth gravity assists, EGA1 in December 1990 and EGA2 in December 1992.

## 1. introduction

The gravitational investigations discussed in this review arc a subset of radio science investigations that use the Galileo telecommunication subsystem and Earth-station radio systems of the Deep Space Network (DSN). Unlike the Voyager mission to the outer planets, where there was one radio science team representing both gravitational science and propagation science, NASA has followed the examples of earlier missions and has selected for the Galileo mission two experiment teams representing respectively the two scientific disciplines (see the companion paper by Howard *et al.* (1992), for details and for a description of the overall radio-science system). Because the two teams have instrumentation in common, there is an inevitable overlap in the overall planning of radio science activities, both in the spacecraft sequencing and in the DSN scheduling. Nevertheless, we review here only the gravitational science which is organized as follows. Anderson is the Team Leader for Gravity and Celestial Mechanics; he proposed investigations in the area of celestial mechanics, including relativistic time delay and relativistic red shift. Estabrook and Armstrong arc Team Members; they proposed a

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search for gravitational radiation. Campbell is Science Coordinator for the team, and in addition is involved in the analysis of data for the celestial mechanics investigations. Krisher is participating in the relativistic celestial mechanics, including the relativistic time delay and red shift experiments. Lau is participating in applications of the JPL ephemeris system to the celestial mechanics experiments, in particular the tests of general relativity and the improvement of the Jupiter ephemeris.

At this early stage in the mission, we have received data from the Venus encounter, the first Earth Gravity Assist (EGA1), and the cruise phase. We are actively engaged in the analysis of these data and in planning our experiments for the cruise gravitational-wave opportunities and the next Earth flyby, EGA2. As with any planning activity on space missions, the optimization of spacecraft and DSN radio systems, spacecraft trajectories, and mission operations for our team must be accomplished in competition with other mission and science requirements, hence the ideal conditions for gravitational science will be achieved only rarely. Yet even under less than ideal conditions, we are satisfied with the current status of mission planning, not only for the cruise experiments prior to Jupiter arrival, but also for the high-priority measurements of the gravitational fields of the Galilean satellites during the satellite tour. The Galileo mission offers major new opportunities for gravitational science and celestial mechanics.

Radio data for our team consist of ranging and Doppler measurements generated by the DSN at its three sites at Goldstone California, near Madrid Spain, and near Canberra Australia. The DSN uses low-noise, highly phase-stable receivers, and a distribution system for frequency and timing based on hydrogen masers. These systems, continually under improvement, make use of the latest technologies in digital electronics and fiber optics, for example. Though these improvements are primarily motivated by the requirements for tracking and communicating with spacecraft in deep space, for example Pioneer 10/11 and Voyager 1/2 at distances beyond the orbit of Neptune, we recognize that new or improved radio science measurements are often enabled as well.

The importance of frequency stability in the end-to-end Doppler System will be discussed for each of the investigations in the following sections. The general idea is that gravitational fields, whether produced by asteroids, planets, satellites or even gravitational waves (GW), will affect the path of the Galileo spacecraft and hence the frequency of the radio link between DSN stations and the spacecraft. The lower the noise in the radio link over the frequency band of interest, the smaller the gravitational signal that can be detected and measured in that band. Our task is to identify and subsequently measure gravitational signals that yield information on masses, densities, and the internal structure of planets and satellites. In some cases the sensitivity is good enough to test the foundations of gravitational theory at the Einstein level, or to search for GW produced by extreme events at the galactic center or in external active galactic nuclei, or even in the early universe, including the Big Bang.

Until May 1991, all Doppler and ranging data are being generated with the spacecraft's low-gain antenna, thus the Earth-spacecraft uplink is limited to 2215 MHz (14.17 cm wavelength) in the radio S-band. With the unfurling of the spacecraft's high-gain antenna in May 1991, the uplink frequency can be either $\sim 2115$ MHz or, by using selected 34-m radio antennas at each of the three DSN sites, the uplink can be transmitted at $\simeq$ 7167 MHz in the $X$-band. Upon receiving either the S-band or X-band transmission from the ground, but not both simultaneously, the spacecraft's transponder and radio subsystem will generate the phase coherent, simultaneous downlink signals with an $S/X$ carrier coherency ratio of 11/3. The transponded signals will be transmitted by the spacecraft with up to 20 W of power beamed to the Earth through a 5-m parabolic dish, the high-gain antenna. If the high-gain antenna is not fully deployed as planned, all our investigations will be carried out at S-hand using the low-gain antenna.

The advantage of generating Doppler and ranging data with $X$-band cm both the uplink and downlink lies in the reduction of plasma effects in radio-frequency measurements at higher frequencies. Because the refractive index of cold plasma is inversely proportional to the square of the frequency of the link, the perturbations of the radio phase arc inversely proportional to the first power of the frequency. in terms of a figure of merit, defined by frequency fluctuations divided by the center frequency for the link, the improvement goes inversely as the radio frequency squared. The scintillation noise in the beamed radiation is reduced by a factor (3/1 1)2 for X-band Doppler with respect to S-band. Similarly, the uncertainty in the group velocity of the ranging modulation (pseudo-random code) is reduced by a factor of (3/1 1)' for X-band ranging, Another advantage of the Galileo spacecraft is that the larger high-gain antenna, compared to the 3.66-m dish used on Voyager, will yield about a 38 dB antenna gain at S-band and about a 50 dB gain at X-band. Once the high-gain antenna is unfurled, the gravitational investigations will no longer be limited by a poor signal to noise ratio at the DSN receivers, whether located at 34 m or 70 m stations. Although in principle it is possible to integrate any coherent radio signal for a long enough time that the phase can be measured to a fraction of a cycle, in practice there arc limits to what can be done with a weak signal buried in noise. Besides, for some of our measurements, in particular the GW search and the determination of mass signals rich in high-frequency harmonics, wc arc interested in relatively short Doppler integration times in the range of 1to 10 s. For comparison, the G W search with Pioneer 10/ 11, limited by a weak signal from the spacecraft's 8 W transmitter and 1 m dish, requires Doppler integration times of 100 s or longer (Anderson *et al.*, 1990a). Of course at some point integration times can become so long that systematic effects from sources such as interplanetary plasma arc important, not poor signal to noise, but for Galileo this occurs for periods longer than 1000 s at $X$-band. Fortunately, even at Jupiter distance, there will be plenty of signal at both x- and S-bands. Signal to noise is not a concern for any of our experiments.

A primary aim of the celestial mechanics experiments is to measure the shapes of the gravitational fields of Ganymede, Io, and Europa. The results will allow us to make a better selection of models for the interior of the satellites. The experiment provides data on the masses and moments of inertia, and these data constrain the central density, differentiation of materials within the satellites, and chemical composition and physical states of the interiors. This is possible because Galileo will approach the satellites much closer than did any earlier spacecraft, hence gravitational forces will be larger and easier to observe.

## 2. Gravitational Experiments During Cruise

Ongoing investigations and data analysis at the time of the writing of this review concern (1) an liar[h-Venus range fix from the Venus encounter on 10 February, 1990, and (2) a test of the redshift of the onboard Ultra Stable Oscillator (USO) in the gravitational field of the Sun.

Galileo is only the second U.S. spacecraft to provide ranging data at Venus. The first transponded ranging at Venus was generated with Mariner 5, but an accurate range fix to the planet was not obtained. Instead, the emphasis was upon combining the spacecraft radio data with simultaneous radar ranging to Venus to determine the radius of the planet (Anderson *et* al., 1968). Galileo's ranging data during encounter will provide data on the Venus ephemeris. Unfortunately, the two U. S. orbiters of Venus do not have a ranging capability: Pioneer 12 (IWO) because of a spacecraft radio system dating from the early days of the Pioneer missions (no ranging transponder), and Magellan because NASA rejected all relativity proposals that would have placed a requirement on the M agellan mission for ranging. With no requirement for ranging, the Magellan Project used the ranging port of their transponder for a telemetry channel, thus gaining a much-needed increased bit rate. nut even in the absence of spacecraft ranging, data from radar altimeters on PVO and Magellan will result in improvements to the ephemeris. The limiting error in ground-based radar ranging to Venus is km-sized topographic variations on the planet. By calibrating the radar ranging for distances between the center of mass of Venus and the sub-radar points on the planet's surface, as determined by the altimeter data (Pettengill *et* al., 1980), a much improved set of Venus radar ranging will result. The accuracy will not be as good as what could be obtained by ranging to an orbiter, but the r.m. s. radar ranging residuals will be improved from about 10 $\mu$s to about 1 $\mu$s, comparable to the accuracy of the Galileo range fix.

Wc arc in the process of analyzing the Galileo ranging data generated during the Venis encounter. Ranging residuals referenced to a best-fit flyby orbit arc shown in Figure 1. Wc estimate that the reduced range fix between the centers of mass of Earth and Venus will be accurate to about $\pm$ 1.0 $\mu$s ( $\pm$ 150 m in distance). This information, in combination with radar ranging to Mercury and Venus dating back to 1966, as well as the Viking Lander ranging to Mars between 1976 and 1982 and the Mariner 10 range fixes to Mercury, will be used to improve the ephemerides of the inner planets and to check for agreement with current gravitational theory. For example the current determination of the excess relativistic precession of Mercury's perihelion, 42.96 arc scc pcr century in excess of the inertial 530 arc scc pcr century from planetary perturbations, is accurate to 0.2 arc scc pcr century and is in agreement with General Relativity (Shapiro *et al.*, 1976; Anderson *et* al., 1987, 1990b). There is a potential for improving this result by about a factor of 2 by analyzing all available ranging data for the inner planets, including the Venus radar ranging calibrated for topography and the Galileo range fix. Improved accuracy for the general relativistic parameter $\alpha_1$ maybe obtained as well (see Will, 1981, for a definition of $\alpha_1$ as well as other parameters that characterize general relativistic orbital corrections).

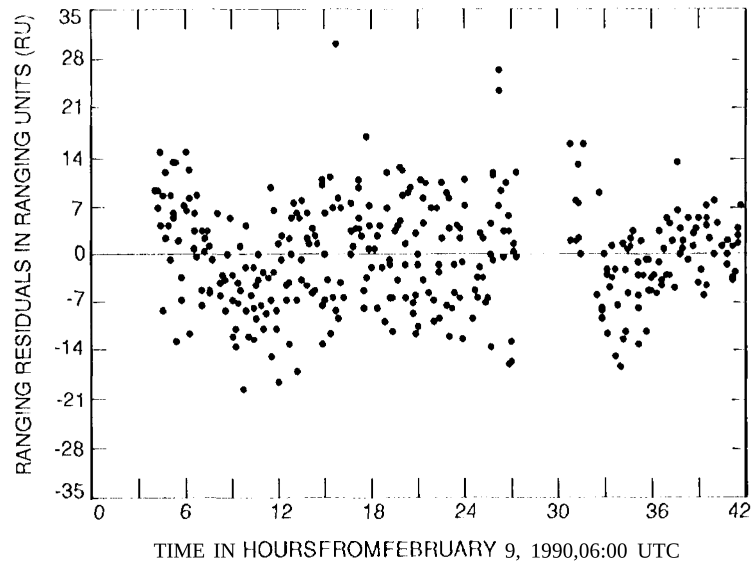


Fig. I, Ranging residuals (observed range minus computed range) for the encounter of the Galileo spacecraft with Venus on 10 February, 1990. The residuals correspond to ranging data generated at three 70-m DSN stations (DSS14, 11 SS43, DSS63). The units for the residuals (RU) are expressed in term.s of the output from the ranging hardware aod are a function of the exciter frequency. For the Venus encounter, 1 RU = 1.056 ns of round-trip time delay, or 0.158 m in distance. The r.m.s. ranging residual is aboat 7 ns (1,1 m). Any unknown bias in the data is less than 80 ns.

Another application of more accurate ephemerides for the inner planets is to determine or limit a possible time variation in the gravitational constant $G$ as measured in atomic units. The ephemerides are based on astronomical ephemeris time as independent variable, but the measurements of range are made in atcmic-time units, hence it is operationally possible to detect a systematic difference between ephemeris time and atomic time that could be attributable to a time-varyiag G.

The expected effect is small. For a Hubble constant $H_0$ of 75 km $s^{-1}$ $Mpc^{-1}$, the cosmic expansion rate is 7.67 x $10^{-11}$ $yr^{-1}$. The orbital motion of the binary pulser PSR 1913+ 16 yields a determination of $\dot{G}/G$ of (1.0 $\pm$ 2.3) x $10^{-11}$ $yr^{-1}$, a result that is consistent with zero (Damour *et al.*, 1988). Lunar laser ranging to corner reflectors on the Moon, and Viking Lander DSN ranging, can be used to limit $|\dot{G}/G|$ (Williams *et al.*, 1978; Hellings *et al.*, 1983; Reasenberg, 1983), however both the Moon and Mars are affected by dynamical systematic noise (geophysical effects for the Moon and unknown asteroid masses for Mars), and it is generally agreed that a limit of $|\dot{G}/G| < 3 \times 10^{-11}$ $yr^{-1}$ is the best that can be achieved with either of these bodies at present, although the work of Hellings *et al.* (19.S3) would suggest that a limit somewhat smaller than this could ultimately be achieved with confidence by means of a careful modelling of the asteroid belt between Mars and Jupiter. The motions of Mercury, Venus and Earth are not so affected by the asteroids, hence by turning our attention to ranging data for Mercury and Venus, and by using the Viking Lander data to determine the mbit of the Earth, it should be possible to determine $\dot{G}/G$ with a 10 accuracy of $\pm$ 0.2 x $10^{-11}$ $yr^{-1}$ (Anderson *et al.*, 1990 b).

Another prediction of General Relativity that is being tested with the Galileo spacecraft during cruise is the gravitational redshift in the field of the Sun (Will, 1981 ; Krisher, 1990). Only one method has been used previously to test the solar redshift. This has involved determining the shift in the positions of spectral lines of elements in the Sun. This type of measurement is difficult to perform accurately, however, resulting so far in only a 5% test of the redshift (Snider, 1972, 1974). The measurement with Galileo depends on observing the gravitational shift in the frequency of an oscillator deep within the gravitational potential of the Sun. The location of the spacecraft in the Sun's field is determined by the phase-coherent Doppler data based on the transponded radio signal. Then the measurement of frequency at the spacecraft is accomplished by breaking the phase lock with the uplink. This noncoherent transmission is referenced to an oven-controlted crystal oscillator, the Ultra Stable Oscillator (USO), a spare Voyager USO with similar frequency stability to those flown on Voyager 1 and Voyager 2. The one-sided power spectral density of the Voyager 2 11S0 is shown in Figure 2 over a range of Fourier frequency from 2X $10^{-5}$ Hz (13-hour period) to 0.5 Hz (2-s period). The spectral density represents the noise in fractional frequency $\Delta v/v$ for Voyager 2 noncoherent transmissions during its cruise. At low Fourier frequencies the noise is characterized by a combination of a flicker-frequency component ($f^{-1}$) and a random walk component ($f^{-2}$). At the Saturn flyby in 1980 we were able to determine the redshift in the Voyager 1 transmissions to an accuracy of 1% even in the presence of low-frequency noise characterized by Figure 2 (Krisher *et al.*, 1990). A 1% test in the solar field is a possibility with Galileo.

From 28 November, 1989 to the present, the command sequence for the spacecraft has included a switch from coherent to noncoherent (USO referenced) tracking on roughly a weekly schedule. Two hours of one-way USO Doppler data have been extracted and recorded by the DSN each week during these noncoherent periods. We intend to follow the frequency shift in the USO data as the spacecraft proceeds from Earth to Venus, back to the Earth at EGA1 and then to EGA2 (see Figure 3). The gravitational shift in the S-band transmission (2295 MHz) will be roughly 10117, (minimum to maximum frequency shift) over a period of 100 days, for a fractional frequency shift $\Delta v/v = 4.4 \times 10^{-9}$. By Figure 2, the worst-case estimate of the inherent USO random walk over 100 days is 0.5117, (one sigma) or 2 x $10^{-10}$ in fractional frequency, which would imply a potential measurement accuracy of 5%. However, by using properties of the predicted gravitational frequency shift, it might be possible to achieve at least another factor of 5 improvement in accuracy, provided that possible systematic errors do not corrupt the determination.

The USO stability characterized by Figure 2 is specified in terms of the frequency deviation from a straight-line fit to the oscillator drift. Hence, during the data analysis for the solar redshift, we will remove a bias and a linear trend in the USO frequency data. The residual frequency shift over the total interval of data will consist of the

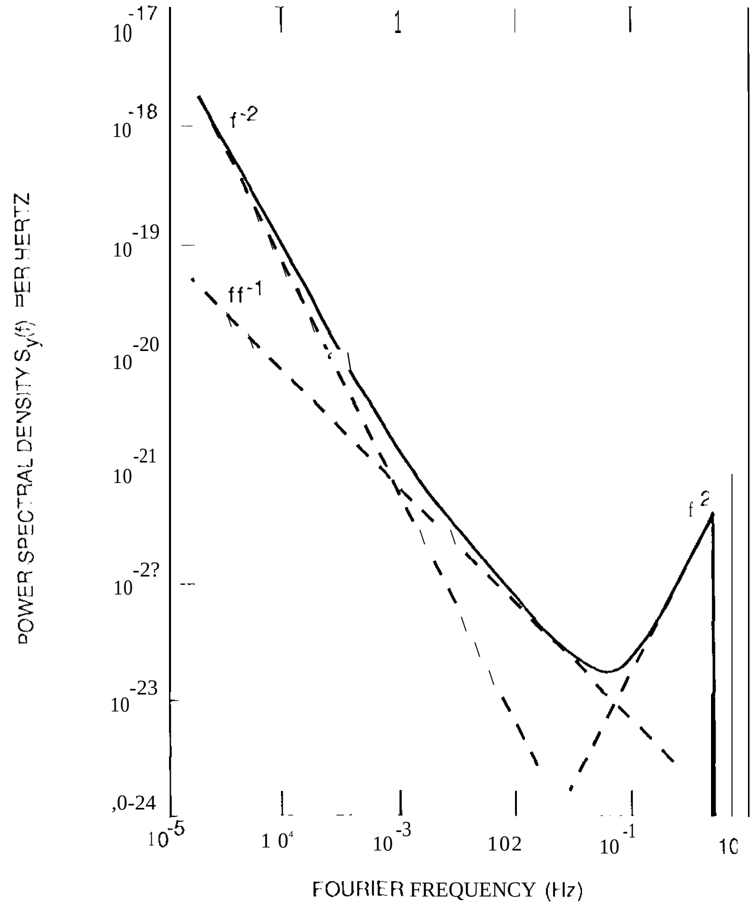


Fig. 2. One-sided power spectral density for measurements of fractional frequency for the Ultra Stable Oscillator (USO) flown on Voyager 2. The Doppler shifts in USO-referenced transmissions from Voyager 2 during cruise were referenced to ground-based hydrogen masers. The raw data were detrended by first removing effects of spacecraft-station relative motion, and then by fitting a bias and linear drift to the resulting Doppler residuals. The USO in the Galileo Orbiter's radio system is a Voyager spare, hence its noise properties should be similar. The spectral density can be represented by three power laws in Fourier frequency $f$ as shown.

redshift signal and red noise. With a reasonable model for the noise at periods longer than the 13 hours of Figure 2, the signal can be extracted by standard statistical techniques (Wiener filter or minimum-variance regression analysis).

## 2.1. Determination of the Mass of the Earth

The masses of all the planets in the solar system, with the exception of Earth and Pluto, have been determined by radio tracking of spacecraft during flybys. Now with EGA1 and EGA2, the Earth will be added to the list of planetary flybys. The DSN will generate both transponded range and Doppler data with the Galileo spacecraft for two intervals,

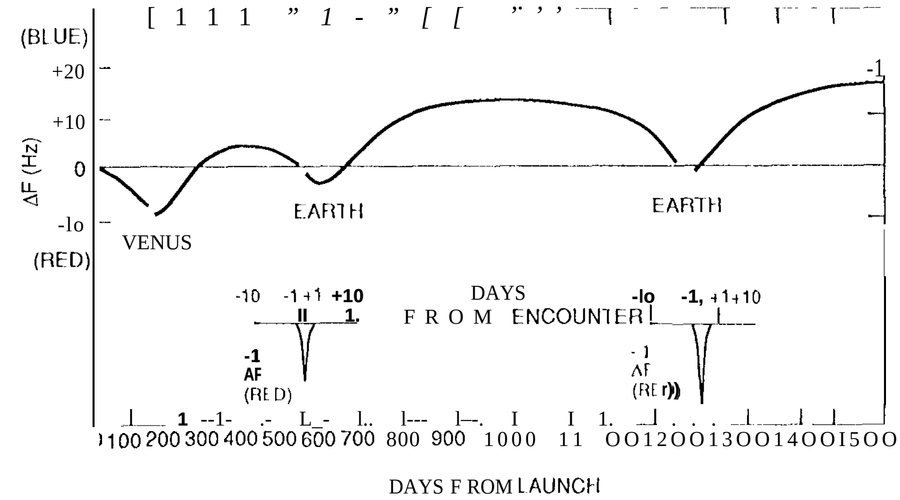


Fig. 3. Expected gravitational redshift caused by solar gravity as evidenced in radio transmissions from the Galileo spacecraft to Earth at 13 cm wavelength. The arrows indicate the times of the Venus flyby and the two Earth gravity assists, EGA1 and EGA2. The two insets in the figure show the expected redshifts caused by Earth gravity for each gravity assist. Figure courtesy of J. P. Brenkle.

defined by three days prior to encounter to three days after encounter at EGA1 and EGA2. Because the DSN's frequency and timing system is based on hydmgcn-maser frequency standards, the instrumental error in measuring the Doppler shift will be better than $2 \times 10^{-14}$ in fractional frequency $\Delta\nu/\nu$ for sufficiently long Doppler Integration times. The corresponding measurement of velocity will be accurate to about $3 \times 10^{-6}$ m $s^{-1}$, with a seeming possibility of measuring *GM* of the Earth to an accuracy of about one part per billion. But in fact the actual error in the measurement could be as much as a factor of 100 larger than that. Several considerations contribute to this possibly larger error. Most importantly, the instrumental error, referenced as it is to the hydrogen masers, is a fraction of the total error budget for the determination of *GM*. Anion.gothcrimportant error sources are (1) the correlations between *GM* and the determination of the spacecraft's trajectory, (2) noise introduced into the S-band (*2. 3 GHz) radio signal by the Earth's troposphere and ionosphere, (3) uncertainties in the absolute location of the tracking stations at Goldstone California, near Canberra Australia, and near Madrid Spain, and (4) buffeting of the spacecraft by drag and by react ion forces generated by subsystems on the spacecraft itself. These error sources will be most important for the more distant flyby EGA1, where the error in *GM* could pessimistically be as high as 0.3 ppm. However they are less of a concern for EGA2, so by combining the Doppler data from the two flybys into a single determination of *GM*, along with a determination of the two assumed independent flyby trajectories, the realistic error (1 $\sigma$) in *GM* should be less than 0.07 ppm.

One would think that *GM* for our own planet would be so well known that another determination by the Galileo flybys would be at best superfluous. But in fact there are subtleties with previous determinations that make a relatively straightforward and independent determination by the flyby technique worthwhile. One recognizes that the external gravitational potential function $\phi$ for the Earth is best determined by observing secular and long-period changes in the orbits of as many artificial satellites as possible. Yet the complexity of $\phi$, both on a local and on a global scale, necessitates a determination of a wide range of spherical harmonics, with the result that there is little room left for a good determination of the zero degree harmonic, the total mass $GM$.

In fact, it was not until 1978 or 1979 that good results from satellites first became available. Recently, Ries *et al.* (1989) have derived a value of $GM$ from Earth-orbiting satellites, including LAGEOS. Their value is so accurate that in order to assure consistency with General Relativity, one must specify whether the result is given for geocentric coordinates or solar-system barycentric coordinates (Huang *et al.*, 1990). In geocentric coordinates, the natural coordinate system for Earth-satellite work, the result is $GM = 398600.4405 \pm 0.0010$ km$^3$ s$^{-2}$, where $c = 299792.458$ km s$^{-1}$. In solar-system barycentric coordinates, the system used by JPL in orbit-determination software, the result would be about 0.015 ppm smaller, thus the $GM$ result from the Galileo flybys should be consistent with a value of $GM = 398600.435 \pm 0.001$ km$^3$ s$^{-2}$. We must wait until after EGA2 in December 1992 before we can verify that there is this agreement between the Doppler tracking of the Galileo spacecraft during Earth flyby and laser ranging to the LAGEOS satellite. A good agreement will provide increased confidence that flyby determinations for other planets are sound.

### 2.2. Search for Gravitational Radiation

There is strong theoretical support for the idea that matter undergoing asymmetrical motion will radiate gravitationally, and that the radiation will propagate at the characteristic speed c in the form of gravitational waves (GW) in the space-time metric, or tensor potential, of General Relativity (Thorne, 1987). Though GW have never been detected with certainty, there is observational evidence that the theory is correct; the predicted effect of gravitational radiation reaction is seen in the acceleration of the mean orbital motion of the binary pulsar PSR 1913 +16 (Taylor and Weisberg, 1989).

For several selected periods during Galileo's cruise to Jupiter, its $X$-band microwave tracking link will be continuously monitored to detect any Doppler frequency fluctuations that could be caused by passing, cosmically generated, long period gravitational waves. These observations will be conducted when other spacecraft experimental activity is at a minimum and when the spacecraft is in the anti-solar direction (so that solar plasma interference with the microwave link is minimized), and will utilize the high precision H-maser time-keeping standards of the Deep Space Network. The resultant overall strain (Doppler) measurement precision will be $h \sim \Delta v/v \sim 5 \times 10^{-15}$, which is at the threshold theoretically calculated for pulses of millihertz frequency waves from several classes of extragalactic sources (for a comprehensive review see Thorne, 1987; Schutz, 1989). By obtaining up to 40 days of continuous $X$-band Doppler, as requested by our team, it may be possible to search for periodic waves to a threshold of $\sim 10^{-16}$.

Gravitational waves are propagating gravitational fields, 'ripples' in the curvature of space-time that carry energy ant] momentum and move at finite speed. All relativistic theories of gravity agree on the existence of these waves, although the theories may differ in number of polarization slates, propagation speed, efficiency of wave generation, etc. in General Relativity, gravitational waves arc transverse, have two polarization states, and propagate at the speed of light. As a wave passes through space, it changes the geometric curvature of the space-time, in directions transverse to the propagation direction. Thus the effect may be described as a strain in space, a dimensionless fractional change in the distances between any massive objects present, coupled with a similar fractional change in the rates at which separated clocks keep time. The amplitude of a gravitational wave is characterized by the dimensionless 'strain amplitude' $h = \Delta l/l$.

While gravitational waves can in principle be produced in the laboratory (e.g., by a steel bar spinning about an axis perpendicular to its length), the resulting signal would be far too weak to be detected with any foreseeable technology. Even the waves from the binary pulsar PSR 1913 +16 arc at far too low an amplitude to be detected directly. Extreme astrophysical events involving at least solar mass objects undergoing high acceleration arc required.

Several experimental groups worldwide arc building and operating resonant bar and laser-interferometer gravitational wave antennas, sensitive to kilohertz waves expected from various (solar-mass) sources in our Galaxy. For the millihertz frequencies expected from larger extra-galactic sources, for example from extreme events in the nuclei of active galaxies, massive binary black holes, or possibly the Big Bang, we require 'antennas' on a large scale of 1 AU or more such as the Earth–Galileo microwave link (for gravitational waves of period P, the antenna response falls rapidly if $P$ is larger than the round-trip-light-time of the link, Estabrook and Wahlquist, 1975; Estabrook, 1985). Searches for millihertz gravitational waves can only be done in space. Previous searches have been performed with the Viking spacecraft, Voyager 1, Pioneer 10, Pioneer 11, and they have been scheduled with Ulysses (Armstrong *et al.*, 1979, 1987; Hellings *et al.*, 1981; Anderson *et al.*, 1984, 1990a; Anderson and Mashhoon, 1985; Estabrook, 1988; Armstrong, 1989).

Implementation of up-down coherent $X$-band microwave links on Galileo allows for the first time a spacecraft Doppler search at the lower strain thresholds for which plausible astrophysical sources may exist. The DSN has recently installed $X$-band transmitters, and implemented careful improvements to the phase stability of the timing and frequency stabilization electronics of selected 34-m radio antennas at all three sites in California, Australia, and Spain (Peng, 1988).

The Galileo high-gain antenna is scheduled to be unfurled in May 1991, after the first Earth encounter, and just before the first solar opposition (see Figure 1 in the companion paper by Howard *et al.* (1 992) for a description of events important to Radio Science in general, and the GW search in particular). After necessary spacecraft control and pointing procedure have been tested, a week of continuous $X$-band Doppler data will be requested for purposes of categorizing thresholds for system phase noise and for demonstrating ground operations at the 34-m high precision tracking sites. This will also be the first significant gravitational wave search opportunity, inasmuch as the range on the Earth-Earth leg of the cruise trajectory will have opened rapidly to $73 \times 10^{6}$ km - or **487-s** round-trip-light-time. We calculate the burst sensitivity for 10 mHz waves ($P = 100$ s) to be $\sim 8 \times 10^{-15}$. Similar searches will be done at the second opposition. November 1992 and third opposition, February, 1993.

Forty-day gravitational wave searches are scheduled for Opposition 4 and 5, enroute to Jupiter. The longer round-trip-light-times allow searches for millihertz waves ($P \sim 100$ s) at limits set by the $X$-band capability. At Opposition 4, May, 1994, Galileo will be $5 \times 10^{8}$ km from Earth, 3367 seconds round-trip-ligth-time; at Opposition 5, June, 1995, $6.3 \times 10^{8}$ km, or 4233 s round-trip-light-time. The expected sensitivities for these 40-day searches are: for bursts, $5 \times 10^{-15}$; for periodic (sine) waves 1 to $3 \times 10^{-16}$, depending on frequency in the millihertz band; and for a possible gravitational wave background, $2 \times 10^{-15}$ (r.m.s. in 118 $\mu$Hz band, limited by DSN station stability). If the high-gain antenna is not successfully deployed, these opportunities will still exist at $S$-band, but with decreased sensitivity.

Finally an opportunity may occur in April 1993 for a joint spacecraft Doppler tracking experiment. If present plans go through, the Mars Observer spacecraft will also be flying enroute to Mars. It also will have $X$-band capability, both receive and transmit. Both it and Galileo will be in roughly anti-solar directions, though not at strict opposition. In fact they will be sufficiently separated in the sky that they can be simultaneously tracked from DSN stations at different longitudes. A continuous ~-week joint tracking experiment is being proposed. Joint experiments would provide much more convincing evidence for any putative gravitational wave detection near the system sensitivity threshold.

## 2.3. Mass Determinations for Small Bodies

There is the possibility that the Galileo spacecraft may fly close enough to one or more small bodies that a meaningful mass determination can be obtained. There are no plans to fly near small satellites of Jupiter or any known comets, but during two passes through the asteroid belt between Mars and Jupiter, once near aphelion in December 1991 at a distance of about 2.3 AU, and again in 1993 on the transfer trajectory between EGA2 and Jupiter arrival (see Figure 1 of Howard *et al.*, 1992), close encounters with one or two asteroids are likely. The accuracy in the determination of their masses depends on the distance of closest approach and on the relative spacecraft-asteroid velocity. The fractional error in mass is given by the expression,

$$\frac{\Delta m}{m} = kc \frac{bV}{(Gm)} \frac{\Delta v}{v},$$

where $m$ is the mass of the asteroid, $b$ is the impact parameter of the asteroid-spacecraft hyperbolic flyby, $V$ is the flyby velocity on the hyperbolic asymptote, $G$ and $c$ are the usual physical constants, $\Delta v/v$ is the accuracy of fractional-frequency Doppler shift over a characteristic time for the flyby ($\sim 10\, b/V$), and $k$ is a dimensionless factor that depends on both the geometry of the flyby as observed from Earth, and the sample interval for the Doppler data. If $b$ can be determined from optical navigation data to an accuracy of a few pixels, rather than from a simultaneous Doppler solution for $b$ and $Gm$, the value of $k$ will lie in the range 0.4 to 1.0, thus it can be taken equal to unity for a conservative estimate of the mass error.

The mean density $\rho$ of the small body is of fundamental interest, so both the mass and volume must be determined. For close approaches to asteroid-sized bodies, it is the mass that limits the determination of density, hence $\Delta\rho/\rho$ is given by the same expression as $\Delta m/m$. Volume is determined by imaging data to smaller fractional error. In order to minimize the error $\Delta\rho/\rho$, it is necessary to minimize the product $bV(\Delta v/v)$. We have no significant control over the flyby velocity, though it is small (of order 8 km s$^{-1}$ for the first pass through the asteroid belt between EGA 1 and EGA2), but the impact parameter is fairly easy to control by small midcourse corrections. The fractional Doppler error $\Delta v/v$ can be minimized for Galileo by tracking the spacecraft with the $X$-band uplink instead of S-band.

The Galileo Project has targeted the asteroid 951 Gaspra for a flyby on October 29, 1991. Unfortunately, the expected radius of the asteroid is only 8 km, hence the expected $Gm$ is small, about $4 \times 10^{-4}$ km$^3$ s$^{-2}$. The planned flyby distance is 1600 km, and with a flyby velocity of 8 km s$^{-1}$ and a Doppler error of $5 \times 10^{-14}$, the formula for $\Delta m/m$ yields an uncertainly of 50%. Detailed numerical simulations with Navigation software confirm this result. Furthermore, we have shown that coherent $X$-band tracking during the flyby is an absolute requirement, even for a 50% determination. The only possibility for improving on this result is to fly closer. Because the error decreases linearly with the distance, a flyby at 800 km would yield a density determination to about 25%. With regard to distinguishing between a predominantly icy or predominantly rocky composition, 25% is better than 50%, but the improvement is probably not significant enough to change the Project's decision for a 1600 km flyby.

## 3. Experimental Celestial Mechanics During the orbital Tour

For the first time we have an opportunity to conduct gravitational investigations with an orbiter of a giant planet. Not only that, but we expect a high level of performance from the Orbiter's radio subsystem, at least when compared to previous missions to the outer solar system, or even to the Viking Orbiter of Mars. Furthermore the DSN, with substantial improvements to its worldwide network of tracking stations, will be in a position to take full advantage of Galileo's improved capabilities. Our investigations will rely primarily on transponded Doppler and ranging data, supplemented by ground-based astrometric data on Jupiter and its satellites, and by star-satellite imaging data from the orbiter itself. All these various types of data will be available for analysis by means of archiving and software systems used by the Galileo Navigation Team. The Doppler and ranging data generated by the DSN specifically for the gravitational investigations will be transferred to Archival Tracking Data Files (ATDF) by the same procedures and in the same format as for the Navigation Team, thus navigational data and gravitational data can be merged for subsequent data reduction and analysis.

Whenever possible, Doppler data will be generated in the $X$-band ($\simeq$ 7167 MHz uplink and 84'22 h4117 downlink) for purposes of minimizing scintillation noise from the interplanetary medium. Similarly, for purposes of minimizing the effects of the interplanetary medium on the ranging data, modulation on the $X$-band uplink will be available on both the $X$-band downlink and the S-band downlink at 2296 MHz. The DSN's ranging assembly (SRA) at the 34 m stations, equipped with X-band uplink capability, will autocorrelate the pseudo-random code modulated on the uplink with the received modulation cm the downlink, and simultaneously will determine the difference ($S$-$X$) in the group delay of the signals at the two downlink frequencies.

The end-to-end ranging system consists not only of DSN hardware but also software used by the Navigation Team. The final output of this system is S-band and $X$-band ranging residuals in units of microseconds, where the residuals are referenced to predicted ranges from a spacecraft trajectory computed by the Orbit Determination Program (ODP). For details regarding the computation of these range residuals at any given time of observation, as well as the computation of Doppler residuals, see Moyer (1971).

### 3.1. EXPECTED ACCURACY OF RANGING DATA

The instrumental accuracy (1 $\sigma$) of the ranging hardware at a DSN station is about $\pm$7 ns ($\pm$1 m) for Galileo. I'here is also an unknown ranging bias, perhaps as large as 80 ns (12 m) because of group delays through the transmission media, through the spacecraft radio system, and through the transmission path at the station. This unknown bias can be minimized by calibration of the signal path. The delay through the spacecraft radio system, as measured prior to launch on a prototype system, is 713 ns at X-band and 715 ns at S-band, with unknown variations possibly as large as 30 ns because of environmental conditions on the spacecraft. The delay through the station signal path, on the order of 12000 ns, is calibrated at least once for each ranging pass, with perhaps a few exceptions when there is no time in the schedule for station calibrations. However, it is usually a straightforward matter to interpolate the station calibrations when there are gaps.

A significant portion of the unknown group delay is caused by propagation of the radio signal through interplanetary plasma, but most of this contribution can be calibrated by means of the dual-hand downlink. For an electrostatic plasma, the observed range residuals at X-band and S-band arc. respectively,

$$\Delta l_x = \Delta l_g + P_u / v_u^2 + P_d / v_{xd}^2 \,,$$

$$\Delta l_s = \Delta l_g + P_u / v_u^2 + P_d / v_{sd}^2 \,,$$

where $\Delta l_g$ is the non-dispersive group delay independent of the plasma, 1' is proportional to the columnar electron content of the beam, v is the carrier frequency, and the subscripts $u$ and $d$ refer to the uplink and downlink transmissions. For Galileo the common uplink frequency can be either $v_u$ = 2115 MHz or $v_u$ = 7167 MHz. The dual-hand downlink transmissions arc at frequencies $v_{xd}$ = 841S MHz and $v_{sd}$ = 2296 MHz.

To the first order, the downlink contribution to the $X$- and S-band residuals is

$$P_d = \frac{v_{sd}^2 \, v_{xd}^2}{(v_{xd}^2 - v_{sd}^2)} (\Delta l_s - \Delta l_x)$$

and with this measured value of $P_d$, an estimate of $P_u$ can be obtained by interpolation (Muhleman and Anderson, 1981). We have made no attempts to predict the nature of the power spectral density for the calibrated ranging residuals during the orbital phase, but instead will compute periodograms empirically from the actual calibrated residuals after the unfurling of the high-gain antenna.

### 3.2. Expected Accuracy of Doppler Data

The characteristics of the Doppler noise were discussed in Section 2.2 for favorable geometries with Sun-Earth-Probe (SEP) or solar elongation angles greater than 160 arc deg. We expect an Allan variance with $X$-band uplink of $\sigma_y = 2 \times 10^{-14}$, and about $\sigma_y = 10^{-13}$ with S-band uplink, at periods typical of the celestial mechanics investigations ( > 600 s). For SEP angles less than 90 arc deg, $\sigma_y$ could be a factor of 10 to 100 times larger for the respective uplinks, and for SEP angles less than 10 arc deg, we have asked that no critical measurements be scheduled at all. The measurement of phase, or equivalently range change, is much more accurate than the measurement of absolute range, by a factor of about $10^3$ for $X$-band, hence the calibration of Doppler by means of the dual-band downlink is far less effective than for range.

### 3.3. Non-Gravitational Accelerations

Another noise source of concern for the celestial mechanics investigations is buffeting of the spacecraft in the range of periods from 1000 to 100000 s. During the design phase of the mission around 1980, we imposed two requirements on the spacecraft as follows:

(1) During a satellite encounter the orbiter shall not average non-mean-zero accelerations greater than $10^{-11}$ km s$^{-2}$ on any axis when measured over a time period of 1000 s.

(2) The uncertainty in the unmodelled portion of any such non-gravitational accelerations shall not exceed $10^{-12}$ km s$^{-2}$ when measured over any time period exceeding 1000 s.

It has yet to be demonstrated during the orbital phase that these requirements can be met, and until we gain experience with real data, the spectral distribution of the spacecraft-generated forces is unknown. During critical periods for the celestial-mechanics measurements, reaction forces from spacecraft subsystems could be minimized by keeping spacecraft activity to a minimum, but here again we need experience with real data during the Orbiter phase before we can be definitive on the level of acceptable spacecraft activity and before we can identify particularly troublesome spacecraft systems.

We anticipate that the spectral density of acceleration noise from spacecraft systems will increase with increasing frequency. However, because the corresponding Doppler spectrum (first integral of acceleration) is proportional to the acceleration spectrum divided by $f^2$, the Doppler spectrum will at best be white, and it could instead increase at lower frequencies. In either case we expect that it will be considerably larger than the buffeting noise expected from fluctuations in the solar wind, and it should be larger than the $f^{-3}$ spectrum from fluctuating solar radiation pressure for periods shorter than $10^5$ s (Woodward and Hudson, 1983). The Doppler error budget is probably dominated by the approximately $f^{-1}$ plasma scintillation spectrum unless orbital fits over several days are required, in which case fluctuating solar radiation pressure could be a problem. It will probably be necessary to fit long arcs of Doppler data with a batch-sequential filter, where the duration of each batch is on the order of 12 to 24 hr. Of course our current evaluation of the spacecraft reaction forces could be too optimistic if it turns out that there are unexpected low-frequency components in the acceleration spectrum, as in the case of Voyager.

## 4. Gravitational Experiments During the Orbital Tour

During the orbital tour, the fundamental problem for a number of gravitational experiments is to determine the orbit of the Galileo spacecraft as accurately as possible with respect to the center of mass of the Jupiter system. With this fundamental problem solved, we can determine parameters of scientific interest by observing deviations in the barycentric orbit caused by various sources of gravitation, for example the rotational and tidal distortions of the Galilean satellites. Because the orbital tour will require a number of midcourse maneuvers in order to achieve the desired encounter conditions with the satellites, our best-determined orbit may in fact consist of segments between maneuvers, rather than a continuous spacecraft ephemeris for the duration of the tour. Even so, the orbit determination accuracy is expected to be consistently good over more than 99% of the orbital path. For the first time we are ranging to an orbiter of another planet with X-band uplink and dual-band downlink, also the Doppler accuracy is improved over previous missions to Jupiter, and we have the advantage of a spacecraft with reduced disturbances from the attitude control system, at least compared to Voyager.

Before discussing our primary scientific objective of the satellite gravitational fields, and our secondary objective of tests of general relativity, we should point out that an accurate orbit is needed by other investigation groups on Galileo. The required satellite-spacecraft geometries will be determined by the spacecraft ephemeris in combination with accurate ephemerides for the satellites. Although this information will be provided to the Galileo Project by the Navigation Team in agreed upon formats, it will be generated with the cooperation of our team and with merged files of navigation and radio science data. In practice the reconstructed trajectory determined by the Navigation Team for each satellite encounter probably will be identical to the reference trajectory used by our team to determine the values and $1\sigma$ errors for the gravitational parameters. By making use of results generated by the Navigation Team, as well as relying on their facilities, we can proceed with our investigation in the most cost-effective manner. As a byproduct, Galileo investigators will receive the most accurate Jupiter-centered trajectory possible, for example for the analysis of wind profiles in Jupiter's atmosphere with the Pmbc-Orbiter Doppler link (Pollack *et al.*, 1992).

### 4.1. Gravitational fields for Jupiter and its satellite

Four spacecraft have visited Jupiter on flyby trajectories. The Pioneer 10 and 11 spacecraft came much closer to Jupiter than Voyager 1 and 2, and in fact the minimal improvement expected from the Voyager flybys caused the Voyager Project to scrub the Celestial Mechanics experiment at Jupiter. No radio science data were scheduled for celestial mechanics, although the Voyager Navigation Team scheduled enough data to allow a determination of the flyby mbits for the two Voyager encounters. Subsequently, Campbell and Synnott (1 985) combined the Voyager navigation data with Pioneer 10/11 data archived for the Pioneer Celestial Mechanics Experiment, and with data from the four flybys they were able to improve substantially over the final results published by Null (1976) for the Pioneer 10/11 investigation. The accuracies for the masses of tbc Galilean satellites were improved by factors of 3 to 12, but no satellite was ever approached close enough to provide even a detection of second degree and order gravitational harmonics. A summary of our current knowledge of gravitational parameters for the Jovian system is given in Table I.

It is unlikely that Doppler data generated by the DSN with the Galileo Orbiter, even with $X$-band up and down, will yield improvements of any significance to the gravi-

TABLE 1

Combined Pioneer 10/11 and Voyager 1/2 gravity results

| Parameter[a] | Value | Realistic uncertainty[b] |
|---|---|---|
| GM (System) | 126712767 | 100 |
| GM (10) | 5961 | 10 |
| GM (Europa) | 3200 | 10 |
| GM (Ganymede) | 9887 | 3 |
| GM (Callisto) | 7181 | 3 |
| $J_2$ | 14 736 | 1 |
| $J_3$ | 1.4 | 5 |
| $J_4$ $\times 10^6$ | -587 | 5 |
| $J_6$ | 31 | 20 |
| $C_{22}$ | -0.03 | 0.05 |
| $S_{22}$ | 0.007 | 0.0s |
| $\alpha$ (pole) | 268.001 | 0,005 |
| $\delta$ (pole) | 64.504 | 0.001 |

[a] GM units arc $km^3 s^{-2}$, $\alpha$ and $\delta$ in arc deg arc for the rotation pole in 1950.0 coordinates, the gravitational harmonics for Jupiter arc rcl'erred to an equatorial radius of 71 398 km.

[b] 1-$\sigma$ realistic errors as opposed to much smaller formal errors from a covariance analysis

tational field of Jupiter. However, flybys of the Galilean satellites will definitely yield new results on second-degree gravity harmonics for these satellites. For purposes of mission planning of the orbital tour, we have imposed two requirements on the satellite flybys:

(1) At least one encounter with Io, Europa, and Callisto with uninterrupted tracking at less than 1400 km altitude and a solar elongation (SEP) angle greater than 10 arc deg.

(2) At least two encounters with Ganymede at varying latitudes with uninterrupted tracking at less than 1400 km altitude and a solar elongation (SEP) angle greater than 10 arc deg.

In addition we have requested 20 hours of Doppler data and 10 hours of ranging data, centered on the time of closest approach, for all encounters that satisfy one of the two mission requirements. The Io encounter will of necessity satisfy the conditions for a successful gravitational flyby, although the expected Doppler data will be of limited value because of the scheduled JOI maneuver. Nevertheless we expect to measure differences in the principal moments of inertia for Io to an accuracy of one percent or better. It should be possible to discriminate between plausible interior models for this satellite. Perhaps in combination with imaging data on its shape, a good model can be developed for the chemical composition and physical size of Io's core and envelope respectively (for a discussion of these measurements of Galilean satellite interior structure see Hubbard and Anderson, 1978).

In requirement (2) on the flyby conditions for Ganymede, we have recognized that the satellite is likely to be in hydrostatic equilibrium, and that an independent determination of the rotational and tidal response can be achieved by two flybys, one in a near equatorial orbit and the other in a near polar orbit. So far the mission planning for the orbital tour has been able to provide these two flybys. The constants that we will measure directly are $G[C - (A+B)/2]$ and $G(B - A)$, and with $G$ known to four or five significant figures from laboratory Cavendish experiments, the differences in the moments will easily be accurate to ten percent. One percent or better is likely with $X$-band up and down. The two Ganymede flybys scheduled for gravity harmonics should give reasonably definitive information about interior structure for Ganymede.

Useful gravity data is also expected for Europa, although there is a possibility that departures from hydrostatic equilibrium may confuse the issue for this satellite (see Hubbard and Anderson, 1978). The experiment is most marginal for the outermost satellite Callisto with its relatively weak response to rotation and tides. Even so, the second-degree gravity harmonics will be determined for Callisto, and perhaps some useful information will be forthcoming.

## 4.2. TESTS OF GENERAL RELATIVITY

Subsequent to some of the early radar and spacecraft ranging experiments in the mid to late 1960's, there has been a continuing interest in testing general relativity by means of planetary orbiters or landers (Will, 1981). The idea is to use the Doppler and ranging data to a spacecraft anchored to a planet for purposes of ranging to that planet to better accuracy than can be achieved with radar bounce. In the case of Jupiter, spacecraft ranging is the only technique presently available for measuring the distance between the center of the Earth and the center of Jupiter. Pioneer 1(1 provided a range fix cm Jupiter for 4 December, 1973 to an accuracy of ±6 km, and Pioneer 11 provided a similar measurement a year later on 3 December, 1974 to an accuracy of ± 1.5 km (Standish, 1990). The more sophisticated radio system on Voyager, including its ranging transponder and dual-band (S and $X$) downlink, provided even more accurate ephemeris data on S March, 1979 for Voyager 1 and 9 July, 1979 for Voyager 2, although the analysis of these Voyager data is still in progress at JPL.

Two or more years of ranging to the Galileo Orbiter will provide numerous range fixes on Jupiter to an accuracy on the order of 150 m, where the limiting accuracy is set by the orbit determination error along the Earth–Jupiter line, not by the instrumental error of the ranging system. In combination with the earlier radio measurements with Pioneer and Voyager, as well as radio and optical data from liarlh-based observatories that provide angular positions on the sky, a much improved ephemeris for the planet will bc available by early 1998. The most immediate application of an improved Jupiter ephemeris will bc to improve our knowledge of the perturbations caused by Jupiter on the orbits of the inner planets, particularly Mars, with the result that existing data, for example ranging measurements to the Viking landers, will bc more sensitive to small general relativistic orbital corrections, and to perturbations by asteroids.

It will bc possible to measure the excess time delay in the Galileo ranging modulation caused by solar gravity. The detailed nature of this relativistic effect was first published by Shapiro (1964), and it has since been tested several times by radar and spacecraft ranging. The most accurate experiment with the Viking Orbiter and Landers agrees with the prediction of General Relativitiy to ±0.1 % (Reasenberg *et al.*, 1979). For ray paths that pass near the Sun, the extra round-trip delay is given by (Will, 1981),

$$\Delta t = \frac{2R_G}{c} \ln\left(\frac{4r_e r_p}{r_i^2}\right),$$

where $r_e$ is the Sun–Earth distance, $r_p$ is the Sun-spacecraft distance, and $r_i$ is the impact parameter, the closest approach distance of the ray to the center of the Sun. The gravitational radius is $R_G = 2GM/c^2$, where $M$ is the solar mass and $R_G = 2953{,}25$ m.

The excess relativistic time delay is maximum for ray paths that graze the Sun, and , at the distance of Jupiter it amounts to 271 $\mu$s. Wc expect to determine the Earth-spacecraft distance to 150 m, so a measurement of the excess time delay to about 0.5 % seems feasible. Although this is about a factor of five less accurate than the Viking result, it is important to remember that only onc solar conjunction for Viking yielded ±0.1 % accuracy. Other published tests from radar and spacecraft ranging arc accurate to at best ±2% (for a compilation of results see Will, 1981). With Galileo, the excess delay $\Delta t$ can bc measured annually at each solar conjunction for the duration of the mission. There is justification for determining $\Delta t$ to the sub-1% level more than once, simply from the standpoint of good experimental practice, but in addition the predicted effect can bc tested for a fairly wide range of directions in inertial space by means of the single Viking measurement plus two or more Galileo measurements.

Another experiment of possible interest is the measurement of the redshift in the spacecraft's oscillator (USO) caused by the gravitational field of Jupiter. After radiation hardening of the crystal by Jupiter's charged-particle environment, particularly during the close approach at the orbital radius of 10 for JOI, the redshift will be measured for each orbital revolution to an accuracy of about $\pm 1\%$.

Finally, we should mention the possibility of measuring general relativistic effects on the orbits of the Galilean satellites and the spacecraft as discussed by Hiscock and Lindblom (1979). There is no doubt but that the ephemerides for the Galilean satellites will be improved by star-satellite imaging data and by spacecraft ranging data during a close satellite encounter. However, **it is** unknown whether the relativistic components of orbital precession for the satellites can be isolated from the far larger Newtonian processions. All we can do is to perform enough data analysis to find out. Similarly with X-band Doppler up and down, the analysis of Hiscock and Lindblom suggest that the relativistic parameter $\beta$ (see Will, 1981, for a definition) can be determined to better than $\pm 20\%$ from the spacecraft's motion, and given fractional frequency stability of $\Delta \nu/\nu = 5 \times 10^{-15}$ at solar opposition, the error on $\beta$ could be as small as $\pm 3\%$.

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