LOW FREQUENCY GUIDED PLATE WAVES PROPAGATION IN FIBER REINFORCED COMPOSITES

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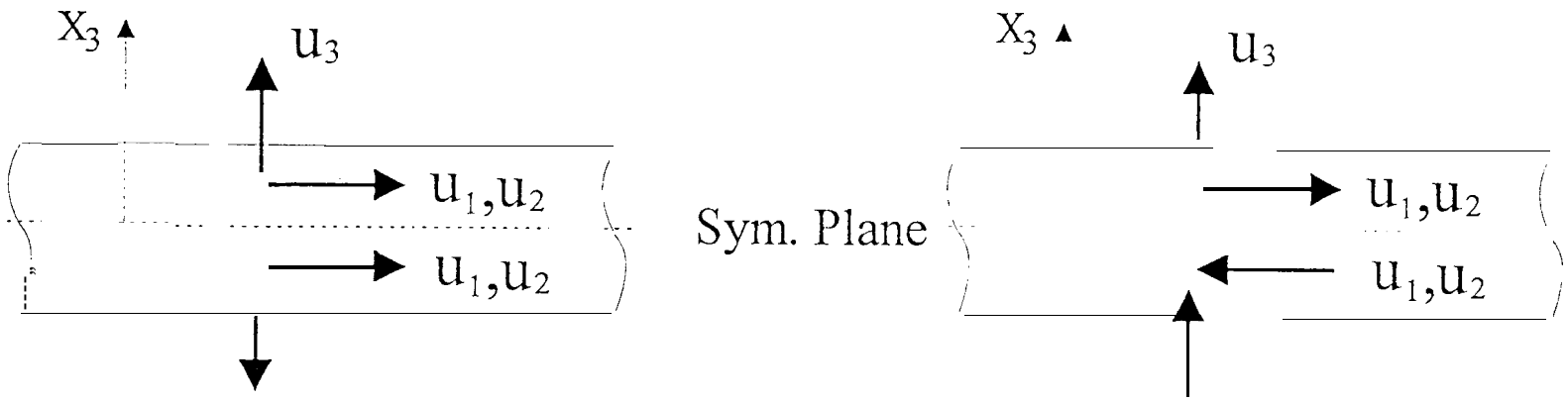
INTRODUCTION

- Application of conventional destructive techniques for the determination of the elastic stiffness constants of composite materials can be costly and often inaccurate.

- Reliable nondestructive evaluation (NDE) methods for the determination of the integrity of composite materials and structures are needed.

- Guided. wave propagation in isotropic plate have been studied theoretically and experimentally by many authors (e.g. Medick, German).

- In contact coupling ultrasonic experiments, the lowest symmetric (extensional) and antisymmetric (flexural) modes are easy to measure.



(a) Symmetric Mode

(b) Antisymmetric Mode

Symmetric and Antisymetric Mode of Guided Waves

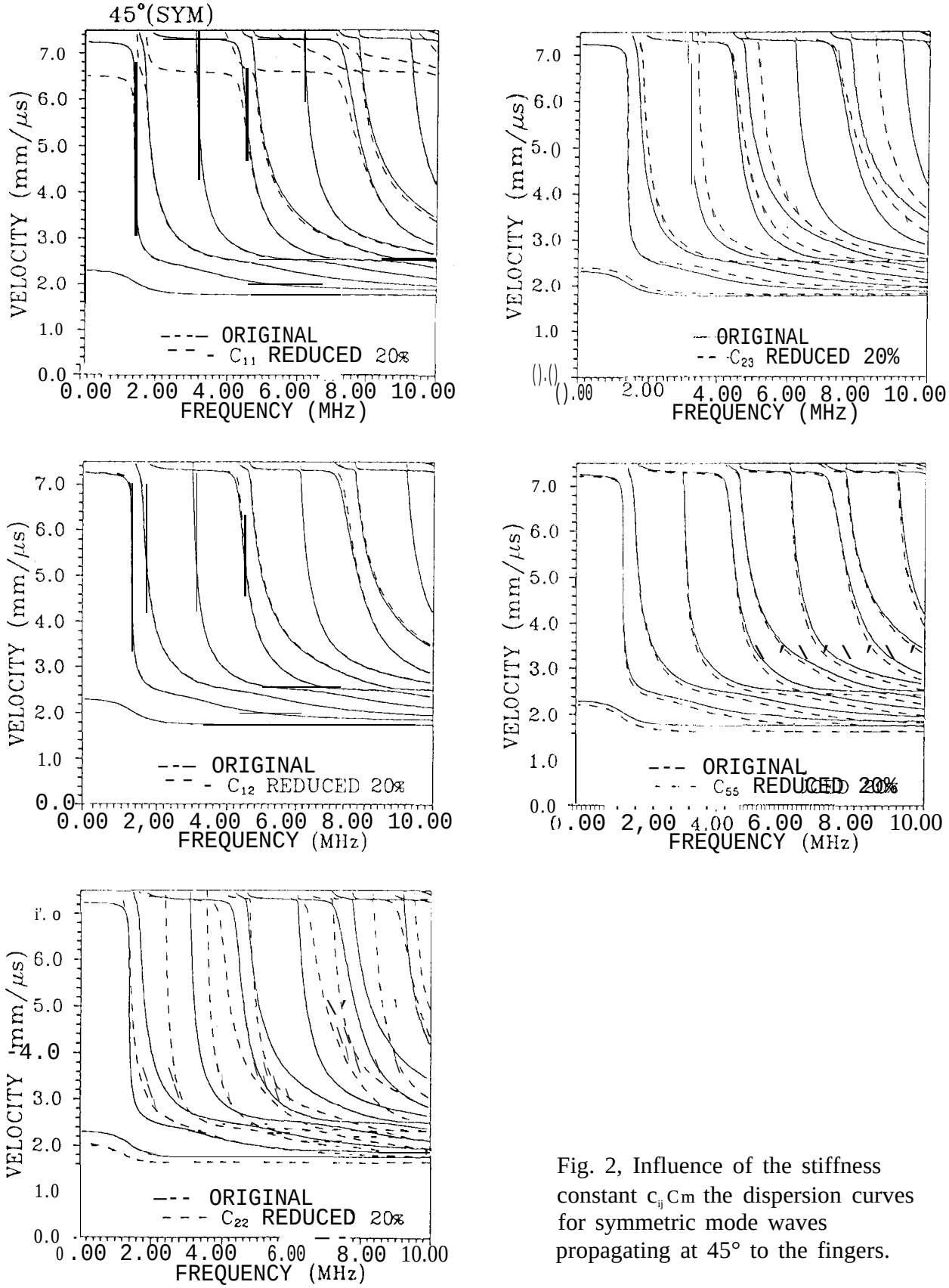
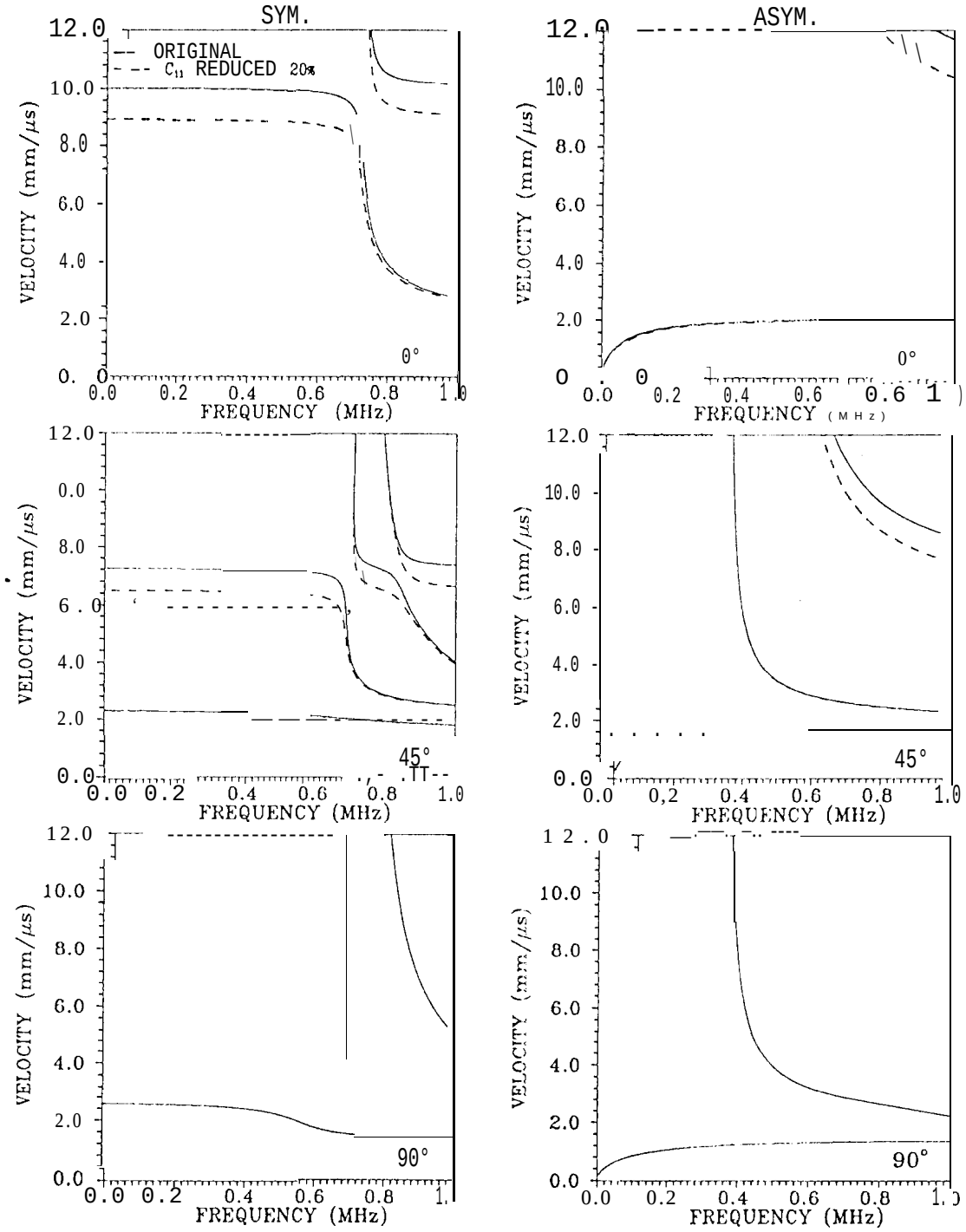


Fig. 2, Influence of the stiffness constant c_{ij} Cm the dispersion curves for symmetric mode waves propagating at 45° to the fingers.


SYM.
ASYM.
ORIGINAL
C11 REDUCED 20%
VELOCITY (mm/μs)
FREQUENCY (MHz)
0°
45°
90°

SYMMETRIC PLATE WAVE IN A FIBER-REINFORCED COMPOSITE LAMINATE

a) Exact Linear Elastic Solution

$$\Delta_1 \cot(\zeta_1 \omega h) + \Delta_2 \cot(\zeta_2 \omega h) + \Delta_3 \cot(\zeta_3 \omega h) = 0$$

$$\Delta_1 = \zeta_2[(\xi_2^2 + \zeta_3^2)q_{22} - (\xi_2^2 - \zeta_3^2)q_{12}][(a_5 - a_3)\xi_1^2 q_{11} - (a_1 - 2a_4)\xi_2^2 q_{21} - a_1\zeta_1^2 q_{21}]$$

$$\Delta_2 = -\zeta_1[(\xi_2^2 + \zeta_3^2)q_{21} - (\xi_2^2 - \zeta_3^2)q_{11}][(a_5 - a_3)\xi_1^2 q_{12} - (a_1 - 2a_4)\xi_2^2 q_{22} - a_1\zeta_1^2 q_{22}]$$

$$\Delta_3 = 4a_4\xi_2^2\zeta_1\zeta_2\zeta_3(q_{11}q_{22} - q_{12}q_{21})$$

when frequency times thickness tends to zero, i e $\omega H \to 0$

$$\frac{\Delta_1}{\zeta_1} + \frac{\Delta_2}{\zeta_2} + \frac{\Delta_3}{\zeta_3} = 0$$

or

$$
\begin{aligned}
& a_1 a_2^2 b_3 n_1^4 n_2^2 \quad a_2 a_3^2 b_3 n_1^4 n_2^2 \quad 2a_2^2 a_4 b_3 n_1^4 n_2^2 + a_2 a_3 a_5 b_3 n_1^4 n_2^2 - 2a_2 a_3 a_4 b_3 n_1^2 n_2^4 \\
& + \quad 2a\, a_2 a_3 b_1 b_2 \quad 2a_3^3 b_1 b_2 \qquad\qquad b_2 + 2a_3^2 a_5 b_1 b_2 \quad 2a_1 a_2 b_3 \\
& + a_3^2 b_3 + 4a_2 a_4 b_3 - a_3 a_5 b_3 + 2a_2 a_3 a_4 b_1 b_3 + a_1 a_2 a_5 b_1 b_3 \quad 2a_2 a_4 a_5 b_1 b_3 \\
& + 2a_2 a_3 a_4 b_2 b_3 + a_1 a_2 a_5 b_2 b_3 - 2a_2 a_4 a_5 b_2 b_3) n_1^2 n_2^2 \\
& + (2a_1 a_3 a_5 b_1 b_2 \quad 4a_3 a_4 a_5 b_1 b_2 \quad 2a_3 a_4 b_3) n_2^4]\rho V^2 \\
& + \qquad\qquad + a_3^3 b_2 b_3 \quad 2a_3^2 a_5 b_1 b_2 b_3 + a_3 a_5^2 b_1 b_2 b_3) n_1^2 \\
& + (-2a_1 a_3 b_1 b_2 + 4a_3 a_4 b_1 b_2 \quad a_1 b_3 \quad 2a_4 b_3 \\
& - 2a_3 a_4 b_1 b_3 - a_1 a_5 b_1 b_3 + 2a_4 a_5 b_1 b_3 - 2a_3 a_4 b_2 b_3 - \\
& + 2a_4 a_5 b_2 b_3 - a_1 a_3 a_5 b_1 b_2 b_3 \\
& + 2a_3 a_4 a_5 b_1 b_2 b_3 + a_1 a_5^2 b_1 b_2 b_3 \quad 2a_4 a_5^2 b_1 b_2 b_3) n_2^2](\rho V^2)^2 \\
& + a_1 a_3 b_1 b_2 b_3 \, \rho V^{2\ 3}
\end{aligned}
$$

$$(b_1 - b_2)(\rho V^2 - c_{55}n_1^2)(\rho V^2 - c_{11}n_1^2)\Omega(c)ij,\ n_1,\ n_2) = 0$$

$$\Omega(c_{ij},\ n_1,\ n_2) = (-c_{12}^2 c_{55} + c_{11}c_{22}c_{55})n_1^4$$

$$(-2c_{12}^2 c_{22} + c_{11}c_{22}^2 + 2c_{12}^2 c_{23} - c_{11}c_{23}^2 - 2c_{12}c_{22}c_{55} + 2c_{12}c_{23}c_{55})n_1^2 n_2^2$$

$$+(c_{22}^2 c_{55} - c_{23}^2 c_{55})n_2^4$$

$$+[(c_{12}^2 - c_{11}c_{22} - c_{22}c_{55})n_1^2 + (-c_{22}^2 + c_{23}^2 - c_{22}c_{55})n_2^2]\rho V^2 + c_{22}\rho V^4$$

$\Omega(c_{ij}, n_1, n_2) = 0$ represents the dispersion equation of the limit **of** the lowest symmetric mode.

Group velocity V_e

$$V_e = \frac{\partial\Omega/\partial n}{\partial\Omega/\partial V}$$

Wave propagates along 0°

$$(\rho V^2 - c_{55})(c_{22}\rho V^2 + c_{12}^2 - c_{11}c_{22}) = 0$$

$$V_{1,2} = \sqrt{\frac{c_{55}}{\rho}}, \quad \sqrt{\frac{c_{11} - c_{12}^2/c_{22}}{\rho}}$$

Wave propagates along 0°

$$(\rho v^2 - c_{55})(c_{22}\rho v^2 - c_{22}^2 + c_{23}^2) = 0$$

$$V_{1,2} = \sqrt{\frac{c_{55}}{\rho}}, \quad \sqrt{\frac{c_{22} - c_{23}^2/c_{22}}{\rho}}$$

For isotropic material

$$(\rho V^2 - c_{55})(c_{11}\rho V^2 - 4c_{11}c_{55} + 4c_{55}^2) = 0$$

$$V_{1,2} = \sqrt{\frac{c_{55}}{\rho}}, \quad 2\sqrt{\frac{c_{55}}{\rho}}\sqrt{1 - \frac{c_{55}}{c_{11}}}$$

APPROXIMATE PLATE THEORIES

Classical plate theory

$$\begin{vmatrix} \hat{c}_{11}n_1^2 + \hat{c}_{55}n_2^2 - \rho v^2 & (\hat{c}_{12} + \hat{c}_{55})n_1 n_2 \\ (\hat{c}_{12} + \hat{c}_{55})n_1 n_2 & (\hat{c}_{55}n_1^2 + \hat{c}_{22}n_2^2) - \rho v^2 \end{vmatrix} = 0$$

where

$$\hat{c}_{11} = c_{11} - c_{12}^2/c_{22}$$

$$\hat{c}_{22} = c_{22} - c_{23}^2/c_{22}$$

$$\hat{c}_{12} = c_{12} - c_{12}c_{23}/c_{22}$$

$$\hat{c}_{55} = c_{55}$$

Shear Deformation Plate Theory

$$u_1 = u_1^0(x_1, x_2, t)$$

$$u_2 = u_2^0(x_1, x_2, t)$$

$$u_3 = x_3\psi_3(x_1, x_2, t)$$

Assume plane wave solutions of (21) in the form

$$u_1^0 = U_1^0 e^{i(k_1x_1 + k_2x_2 - \omega t)}$$

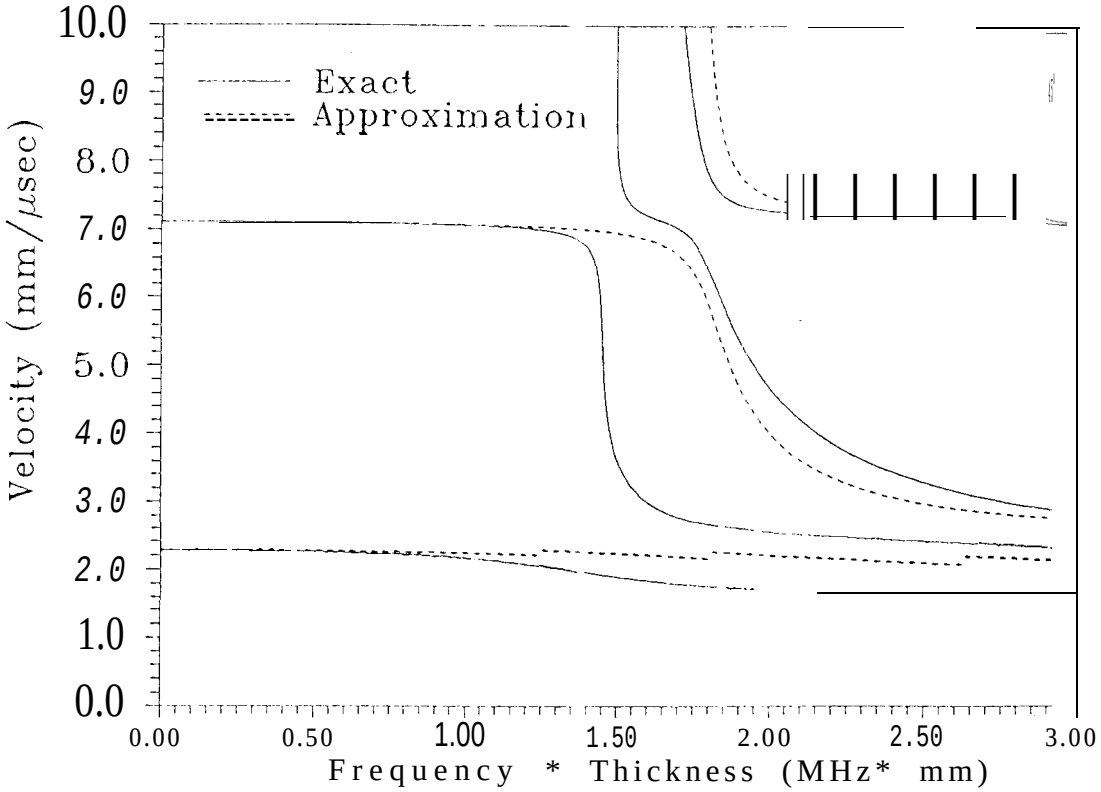
$$u_2^0 = U_2^0 e^{i(k_1x_1 + k_2x_2 - \omega t)}$$

$$\psi_3 = \Psi_3 e^{i(k_1x_1 + k_2x_2 - \omega t)}$$

where k_1, k_2 and k_3 represent the wavenumbers along the x_1, x_2 and x_3 directions, respectively, and ω is the circular frequency.

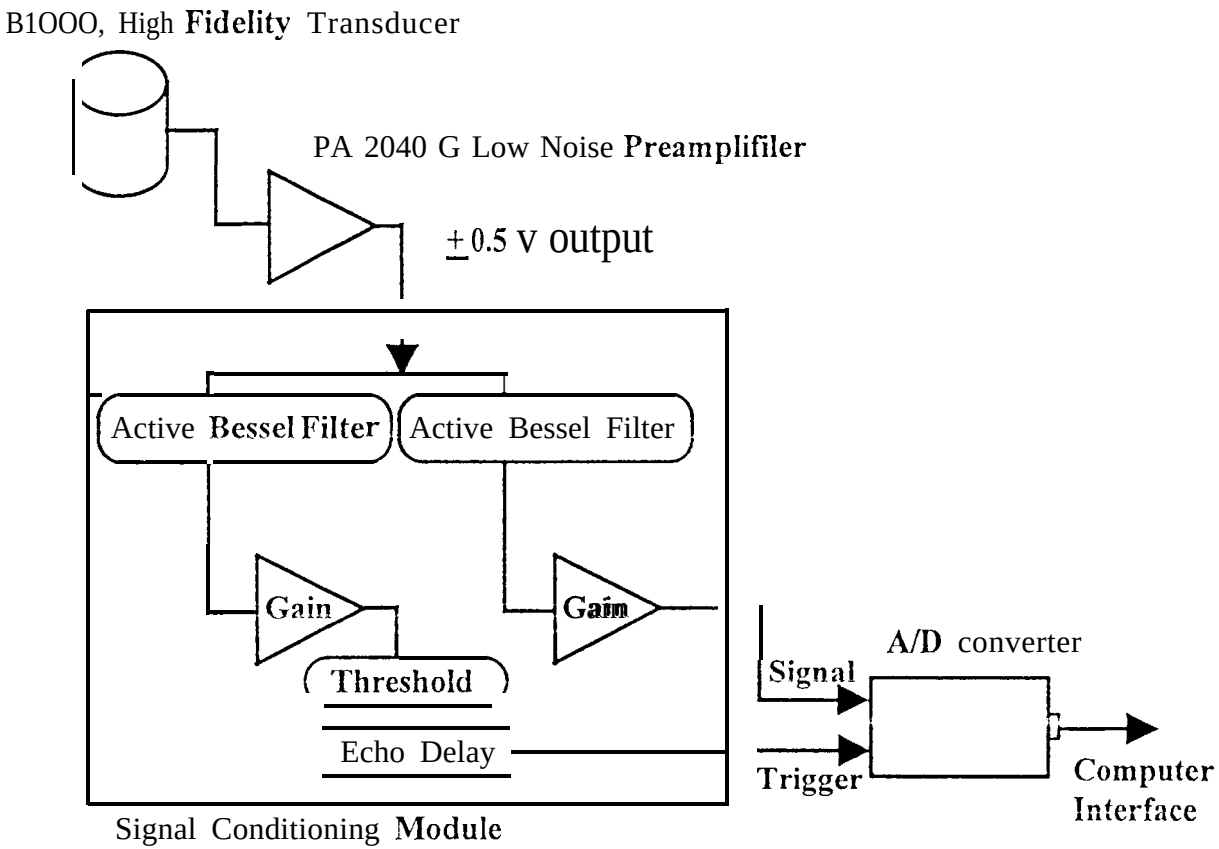
$$\begin{bmatrix} -(A_{11}k_1^2 + A_{55}k_2^2) + I_1\omega^2 & -(A_{12} + A_{55})k_1k_2 & iA_{12}k_1 \\ -(A_{12} + A_{55})k_1k_2 & -(A_{55}k_1^2 + A_{22}k_2^2) + I_1\omega^2 & iA_{23}k_2 \\ iA_{12}k_1 & iA_{23}k_2 & D_{55}k_1^2 + D_{44}k_2^2 + A_{22} + I_3\omega^2 \end{bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \end{Bmatrix}$$

where $n_1 = \cos\phi$ and $n_2 = \sin\phi$; ϕ is the wave propagating angle, and $k_1 = \omega/V\, n_1$, $k_2 = \omega/V\, n_2$, V is the phase velocity.

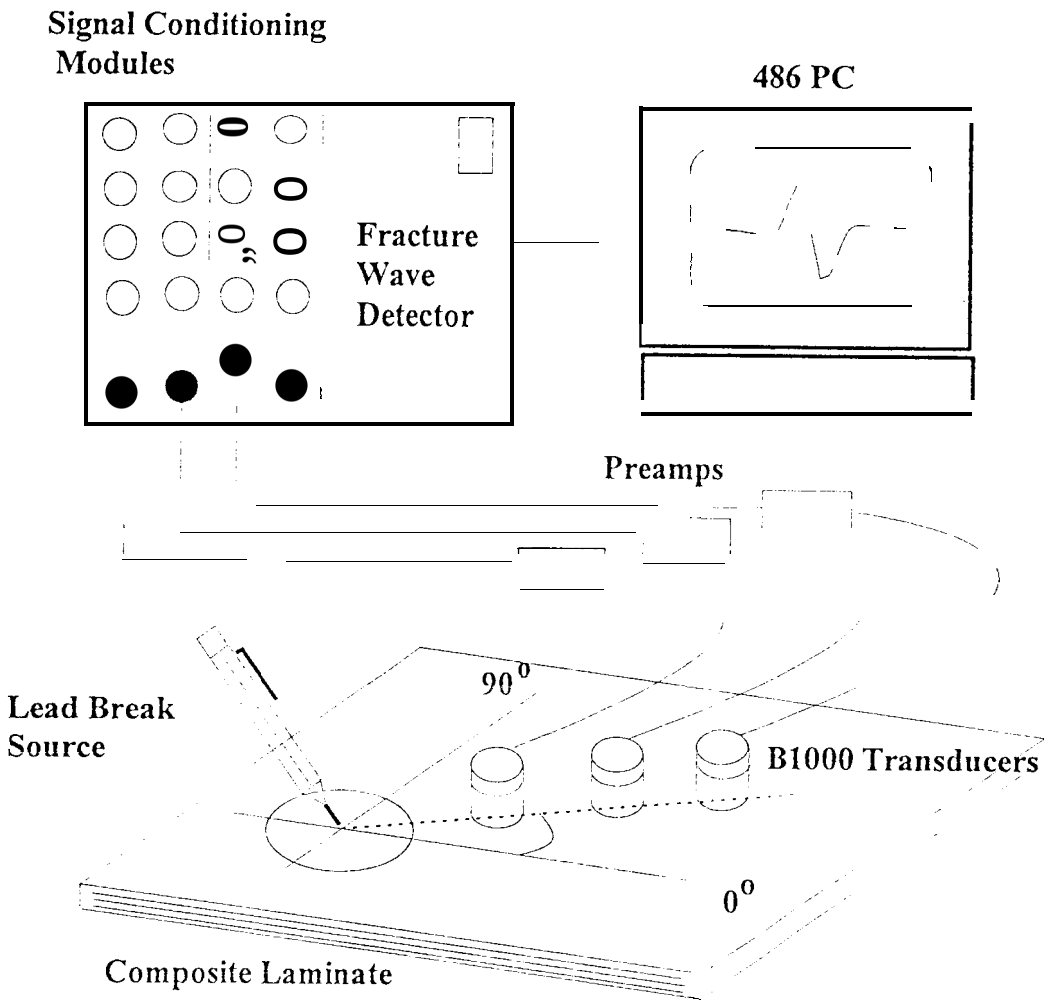

Exact
Approximation
Velocity (mm/μsec)
10.0
9.0
8.0
7.0
6.0
5.0
4.0
3.0
2.0
1.0
0.0
0.00
0.50
1.00
1.50
2.00
2.50
3.00
Frequency * Thickness (MHz* mm)

EXPERIMENT

A $[0]_{16}$ 12 x 12 cm^2 unidirectional graphite/epoxy plate was used.



F4000 Fracture Wave Detector Block Diagram

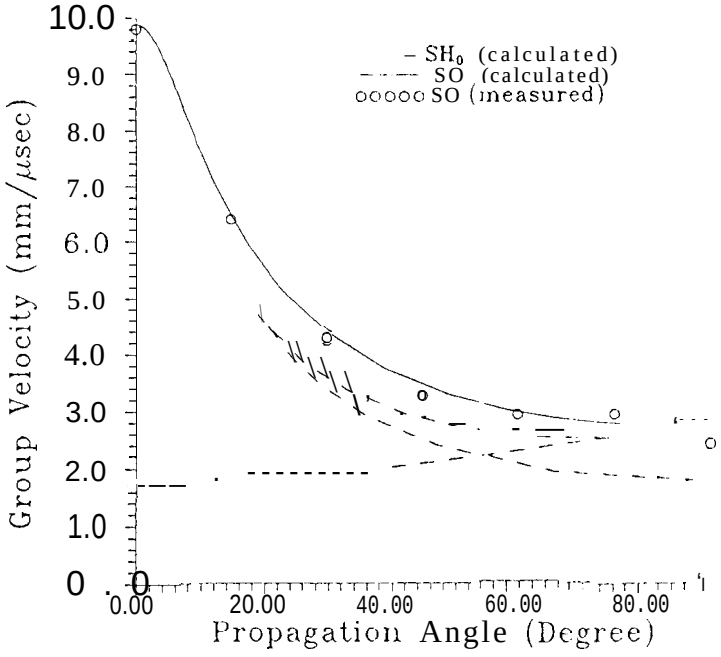


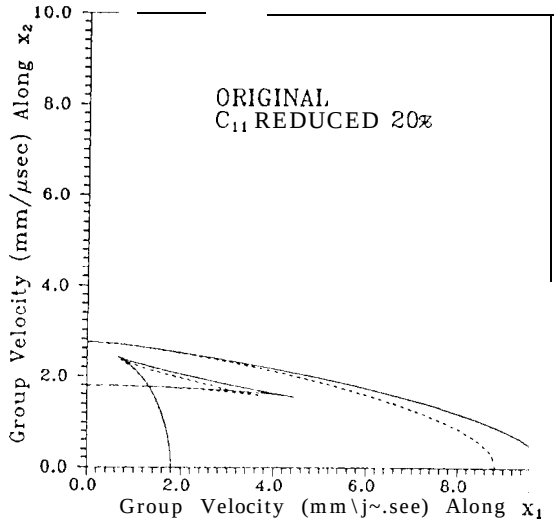
The Experimen al Setup.

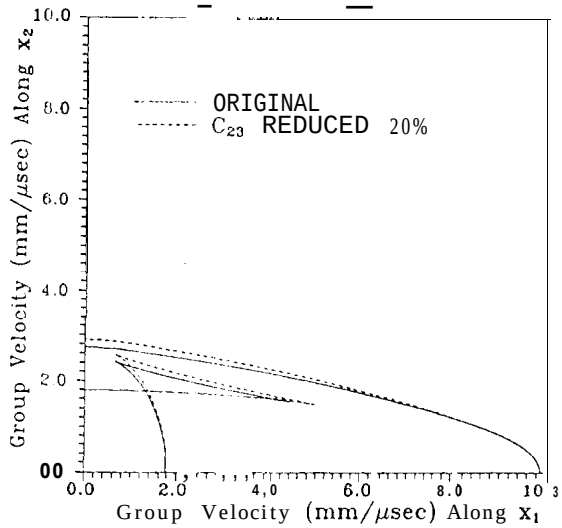
RESULT

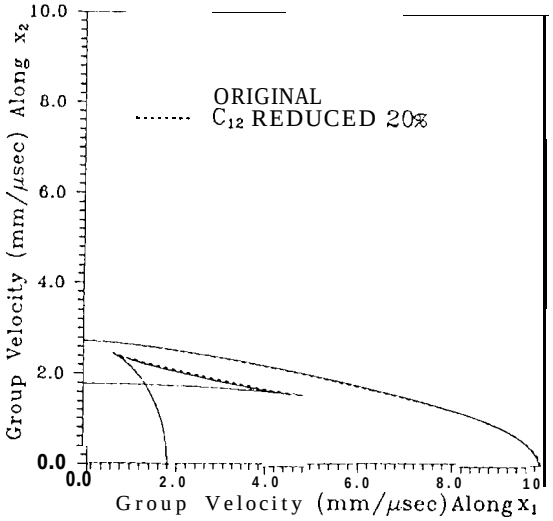
The elastic properties of the test Graphite/Epoxy laminate were extract from the measured group velocity as follows:

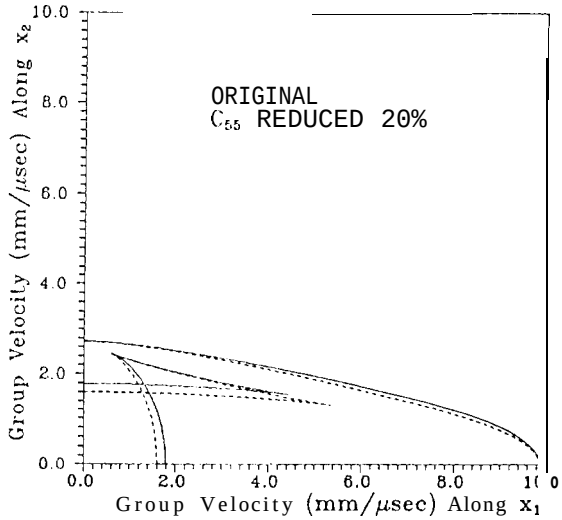
$$c_{11} = 155.01,\ c_{12} = 6.44,\ c_{22} = 15.6,\ c_{23} = 7.89,\ c_{55} = 5.00\ \text{(Gpa)}.$$

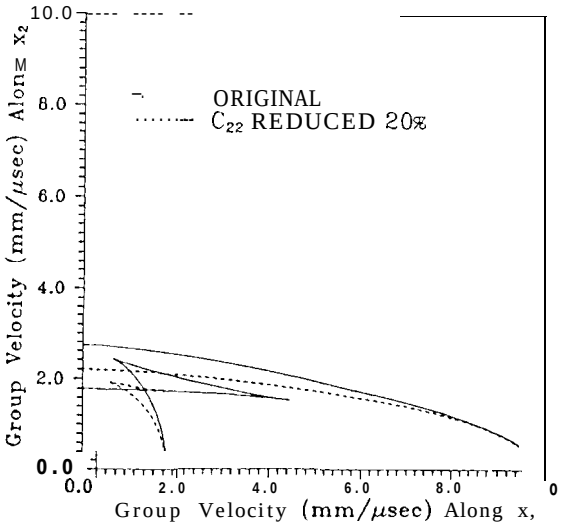

10.0
9.0
8.0
7.0
6.0
5.0
4.0
3.0
2.0
1.0
0
0.00
20.00
40.00
60.00
80.00
Group Velocity (mm/μsec)
Propagation Angle (Degree)
SH0 (calculated)
SO (calculated)
SO (measured)


ORIGINAL
C11 REDUCED 20%
Group Velocity (mm/μsec) Along x2
Group Velocity (mm\j~.see) Along x1


ORIGINAL
C23 REDUCED 20%
Group Velocity (mm/μsec) Along x2
Group Velocity (mm/μsec) Along x1


ORIGINAL
C12 REDUCED 20%
Group Velocity (mm/μsec) Along x2
Group Velocity (mm/μsec) Along x1


ORIGINAL
C55 REDUCED 20%
Group Velocity (mm/μsec) Along x2
Group Velocity (mm/μsec) Along x1


ORIGINAL
C22 REDUCED 20%
Group Velocity (mm/μsec) Along x2
Group Velocity (mm/μsec) Along x,

CONCLUDING REMARKS

- The phase and group velocities of symmetric plate waves were studied.
- A comparison between measured and calculated group velocity for a unidirectional graphite epoxy was presented.
- The agreement of the results indicates the viability of the proposed technique using the lowest extensional mode to determine the elastic constants.
- A parametric study was conducted to determine the influence of the stiffness constants on the dispersion curve of the symmetric plate waves. AH but c_{12} were found to have an influence on the dispersion curves at the low frequency range.