Simulated Performance of Diffractive Optical Elements using a Helmholtz Equation Solver

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Outline

1. Electron-Beam Fabrication of Diffractive Optics at JPL
2. Iterative Finite-Difference Helmholtz Equation Solver
3. Grating Simulation
4. Computer Generated Hologram Simulation
5. Cylindrical Lens Simulation

Electron-Beam Fabrication of Diffractive Optics at JPL

- Surface contouring of thin film poly-methyl methacrylate (PMMA, Plexiglas) using JEOL JBX5DII 50 kV electron-beam lithography system
- Can deliver 64 different exposure doses in each pattern (~50 depth levels)
- Proximity effect (backscattered e-beam dose) measured for different substrates and deconvolved out of the desired pattern
- Development in acetone yields exponential depth vs. dose response curve
- Patterns composed of square pixels (typically 0.5 to 2 μm) writing rate $\sim 10^5$ pixels/minute
- Quantitative characterization using atomic force microscopy (AFM)
- Fresnel lenses, gratings, computer-generated holograms (collaborators A. Gmitro - Univ. of Arizona, Peter Guilfoyle - Opticomp)
- Wavelength-size pixels and fabrication errors (side-wall etching) require modeling beyond the Fourier optics approximations (flat plate, perfect phase delay over entire pixel)

Iterative Finite-Difference Helmholtz Equation Solver

- Alternating-direction-ire plicit (AD I) technique [G. R. Hadley, Opt. Lett. 19, 84 (1994)]
- Surface relief boundary conditions are properly treated
- Physical structures easily specified as polygons
- Requires minimal storage
- Currently running on Pentium PC, HP workstation, and JPL Cray Y-MP
- Technique is applicable to numerous problems

Helmholtz equation

$$\nabla^2 U + k_0^2 \varepsilon(x,z) U = 0$$

Alternating-direction-implicit (ADI) equations for *n+1st* iteration

$$(\omega_n + \delta_z^2) V = (\omega_n - \delta_x^2) U^n$$
$$(\omega_n + \delta_x^2) U^{n+1} = (\omega_n - \delta_z^2) V$$

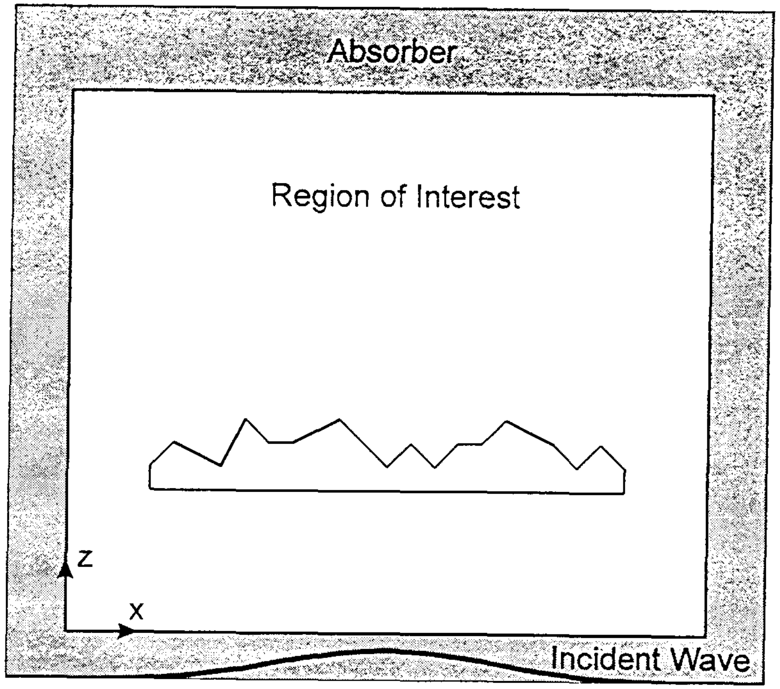
$$\delta_x^2 U_{i,j} = \frac{U_{i-1,j} - 2U_{i,j} + U_{i+1,j}}{(k_0 \Delta x)^2} + \frac{\varepsilon_{i,j}}{2} U_{i,j}$$
$$\delta_z^2 V_{i,j} = \frac{V_{i,j-1} - 2V_{i,j} + V_{i,j+1}}{(k_0 \Delta z)^2} + \frac{\varepsilon_{i,j}}{2} V_{i,j}$$

- ω_n are complex acceleration parameters (may use a cyclic sequence)
- Hadley's two-parameter sequence

$$\omega_1 = -200 + i0$$
$$\omega_2 = (0.05 + i1.1)\mathrm{Re}\{\varepsilon/2\}$$

- Proper node numbering results in tridiagonal matrices

Computational Domain



- Incident field complex amplitude is specified along z = O boundary
- Field is forced to zero on all other sides
- Absorbers surround region of interest
  - Index-matched at ROI edges
  - Imaginary part increases as d^p with distance d into the absorber
  - Absorber along lower boundary absorbs waves reflected back toward source

Convergence Critera

1. Field converges to *n* significant digits everwhere in region of interest (*n* typically 4).

2. Finite difference form of Helmholtz equation converges to some error tolerance (typically **1x1** O^-IO^).

Execution Times

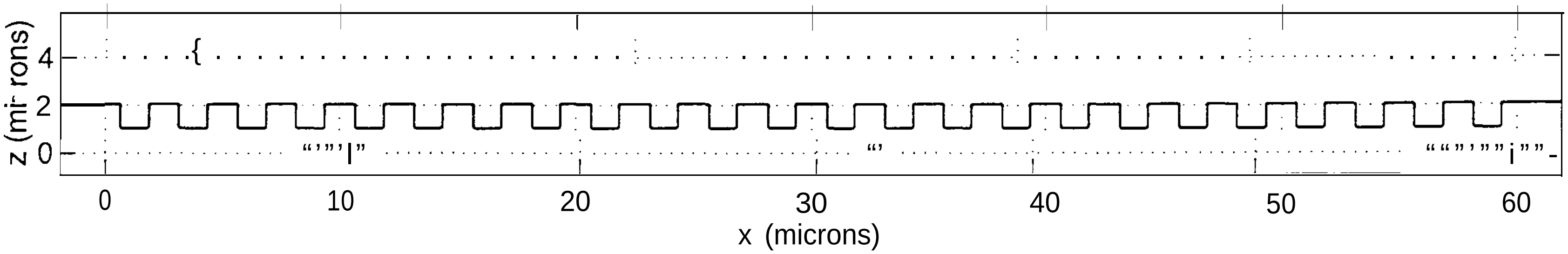
- Scale linearly with number of mesh points
- Typical problem with 100,000 mesh points
  - 90 MHz Pentium PC: 1000 sec
  - HP 715/75 workstation:
  - Cray YMP2E/232:

Grating Simulation

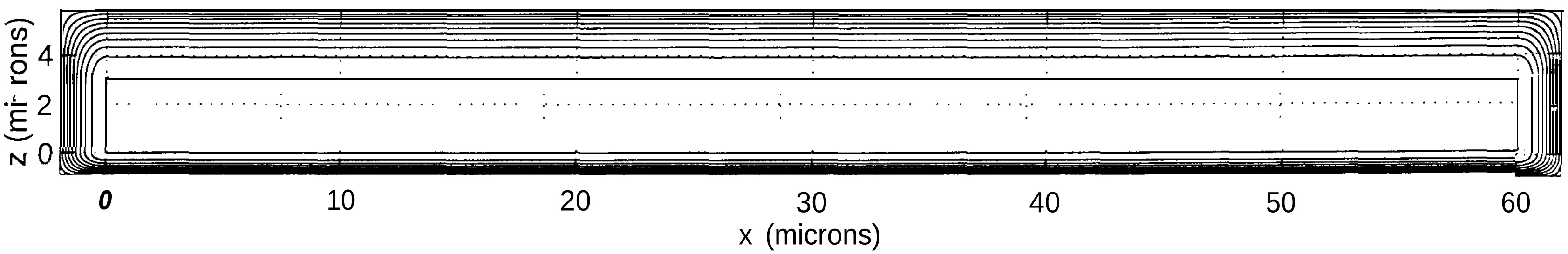
- Rectangular groove surface relief grating: period = 2.5 μm, groove depth = 1.0 μm, duty cycle = 50%, refractive index = 1.5, freespace wavelength = 1.0 μm, angle of incidence = 0°, TE polarization
- Simulated using ADI technique and rigorous coupled-wave analysis (RCWA)
- Input wave for ADI technique had unity amplitude (center 30 pm) with raised cosine edges (1 O μm on each side)
- Calculated angular spectrum of plane waves from field slices though ADI results
  - In front of grating: $E_{tot} = E_{inc} + E_{refl}$
  - in back of grating: $E_{tot} = E_{trans}$

    Separate simulation with uniform index was used to find E_{inc}
- Calculated the power density flowing in the z direction for each order
- Compared ADI efficiencies to RCWA efficiencies

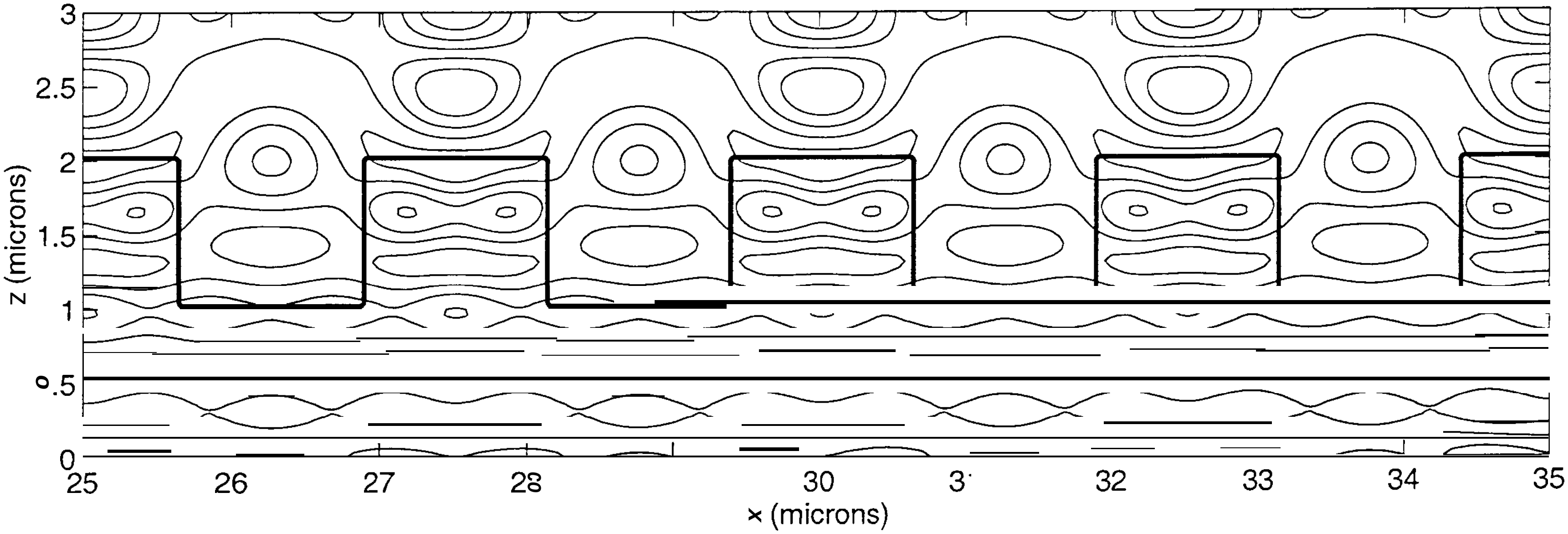
Grating: Real Part of Refractive Index

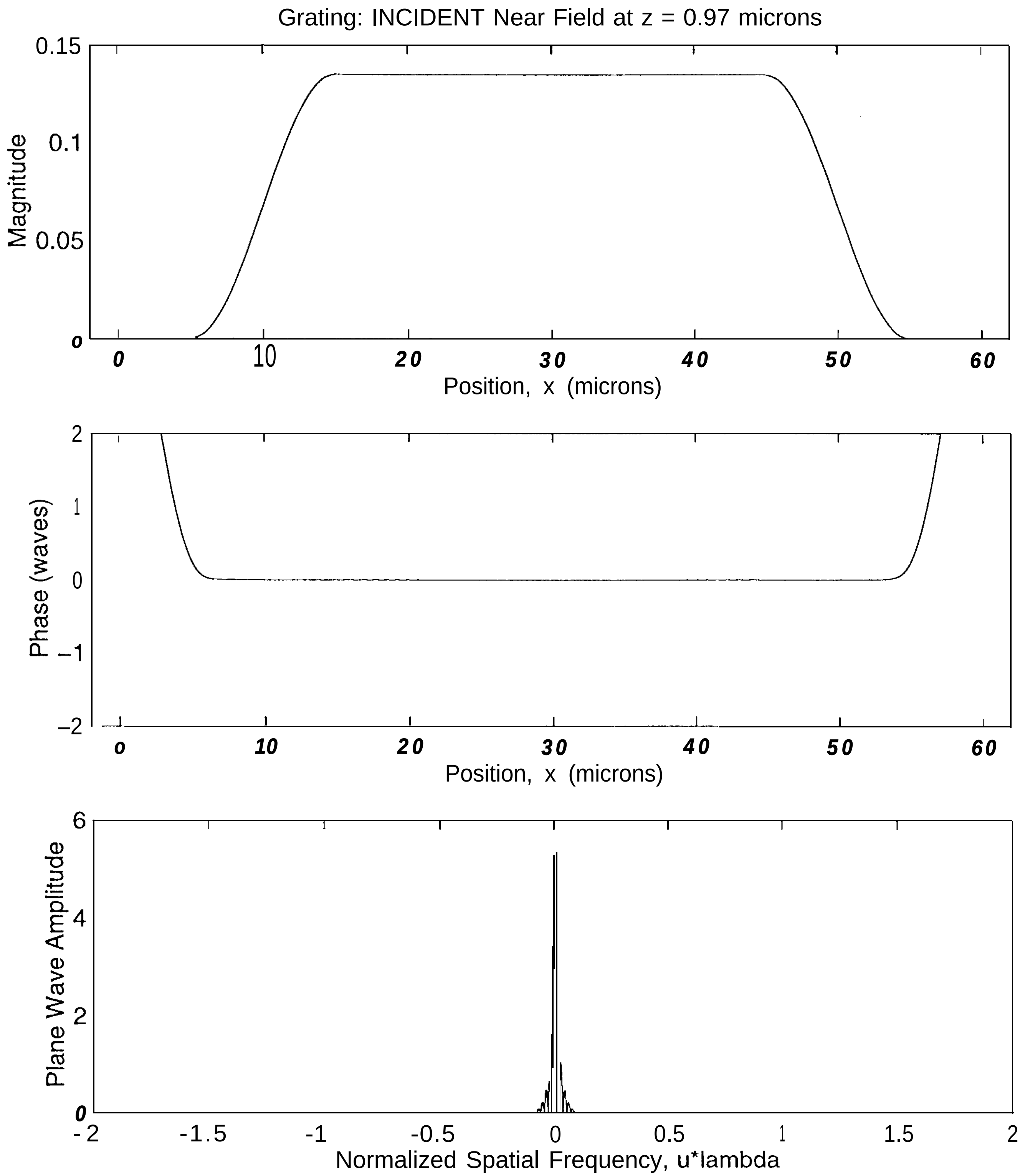


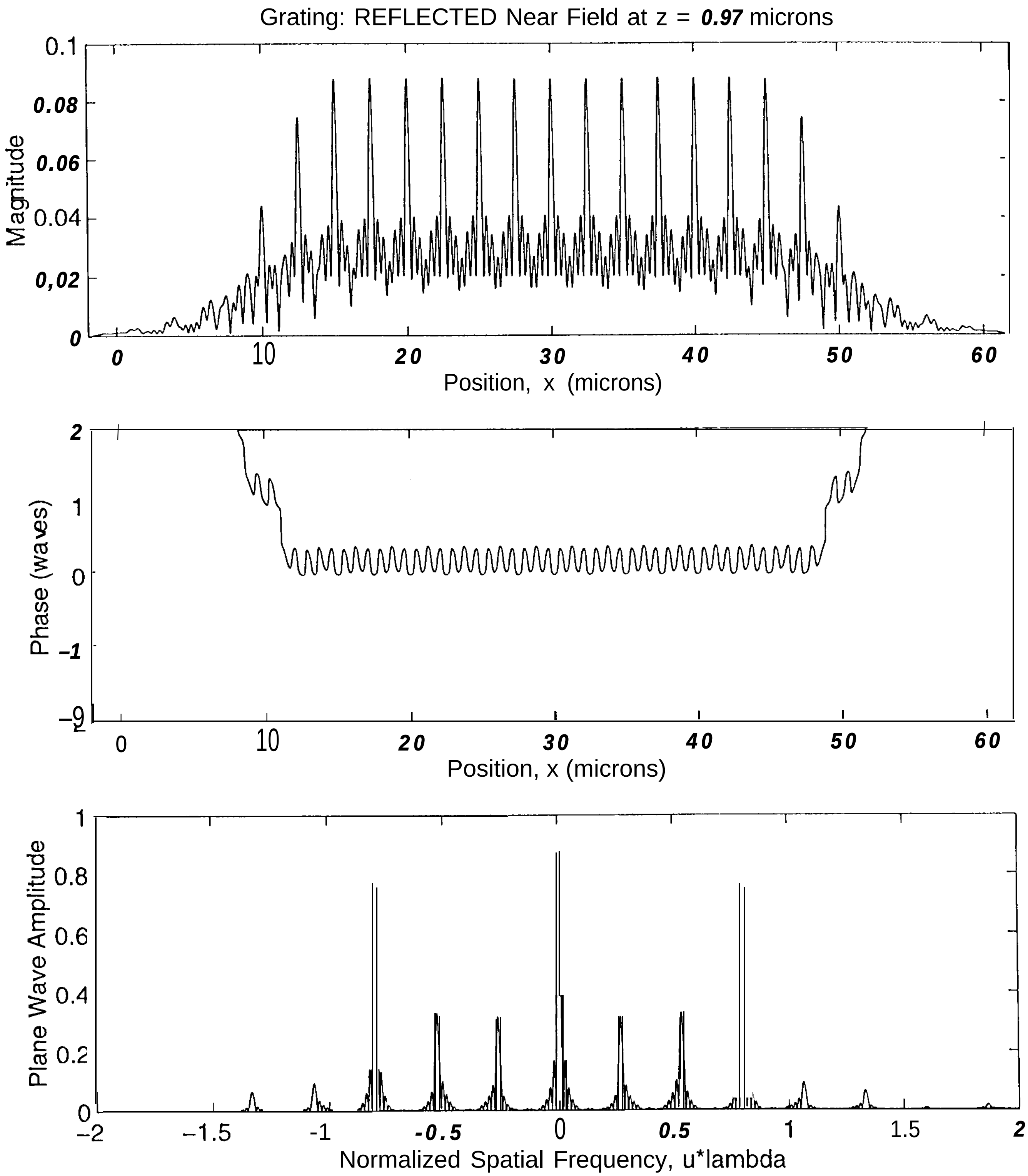
Grating: Imaginary Part of Refractive Index



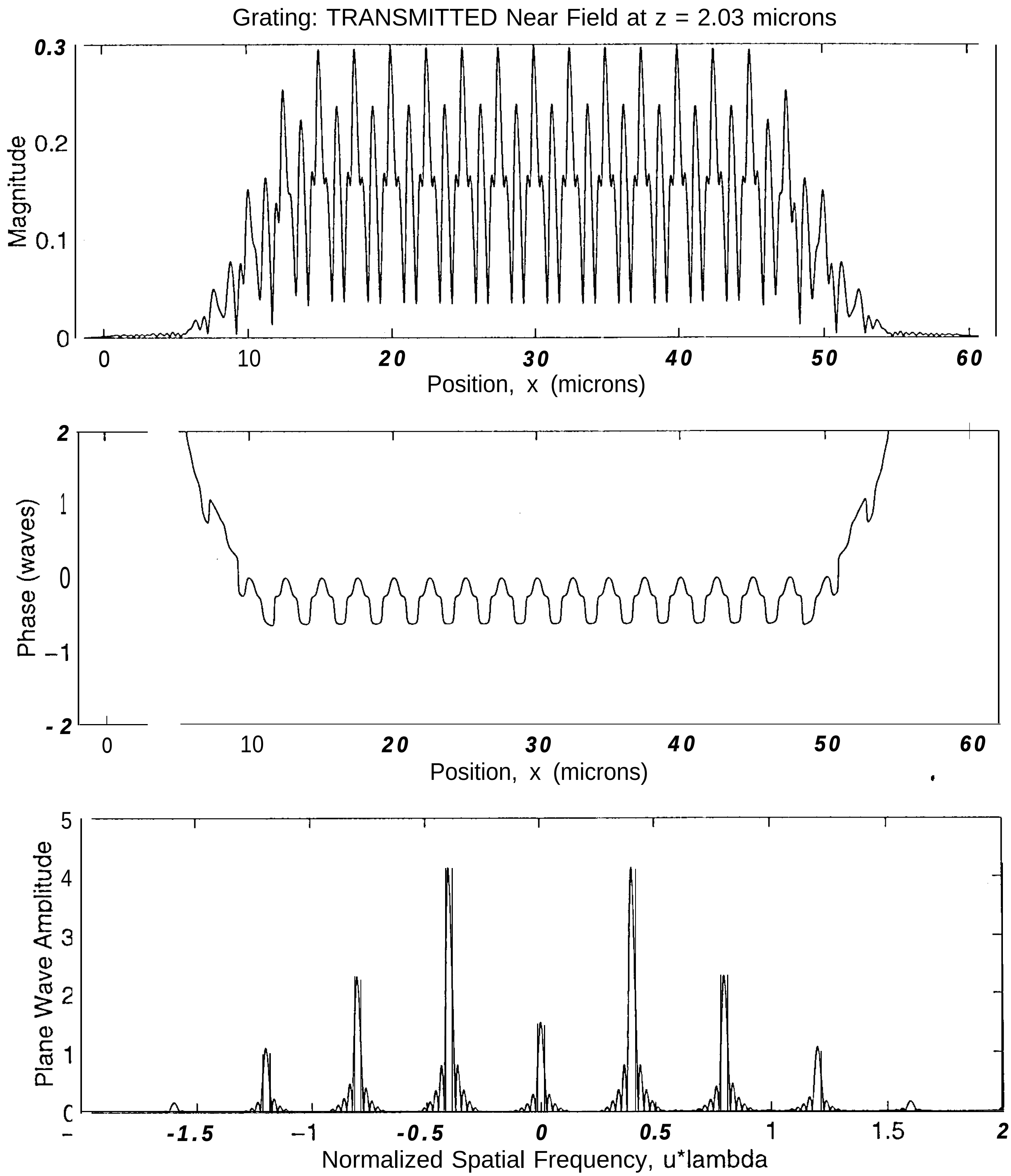
Grating: Instantaneous Electric Field




Grating: INCIDENT Near Field at z = 0.97 microns
Magnitude
0.15
0.1
0.05
0
0
10
20
30
40
50
60
Position, x (microns)
Phase (waves)
2
1
0
−1
−2
0
10
20
30
40
50
60
Position, x (microns)
Plane Wave Amplitude
6
4
2
0
-2
-1.5
-1
-0.5
0
0.5
1
1.5
2
Normalized Spatial Frequency, u*lambda



Grating: REFLECTED Near Field at z = 0.97 microns
Magnitude
0.1
0.08
0.06
0.04
0,02
0
0
10
20
30
40
50
60
Position, x (microns)
Phase (waves)
2
1
0
-1
-2
0
10
20
30
40
50
60
Position, x (microns)
Plane Wave Amplitude
1
0.8
0.6
0.4
0.2
0
-2
-1.5
-1
-0.5
0
0.5
1
1.5
2
Normalized Spatial Frequency, u*lambda



Grating: TRANSMITTED Near Field at z = 2.03 microns
Magnitude
0.3
0.2
0.1
0
0
10
20
30
40
50
60
Position, x (microns)
Phase (waves)
2
1
0
−1
- 2
0
10
20
30
40
50
60
Position, x (microns)
Plane Wave Amplitude
5
4
3
2
1
0
-1.5
−1
-0.5
0
0.5
1
1.5
2
Normalized Spatial Frequency, u*lambda

RCWA-ADI Comparison

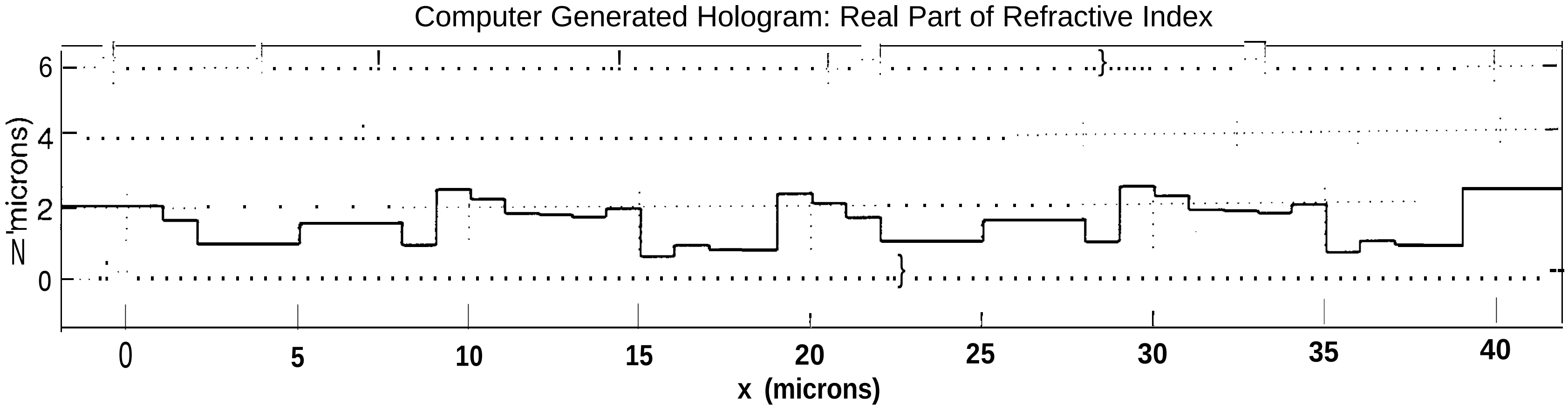
Rectangular groove grating: period = 2.5 pm, depth = 1.0pm
grating index = 1.5, cover index = 1.0

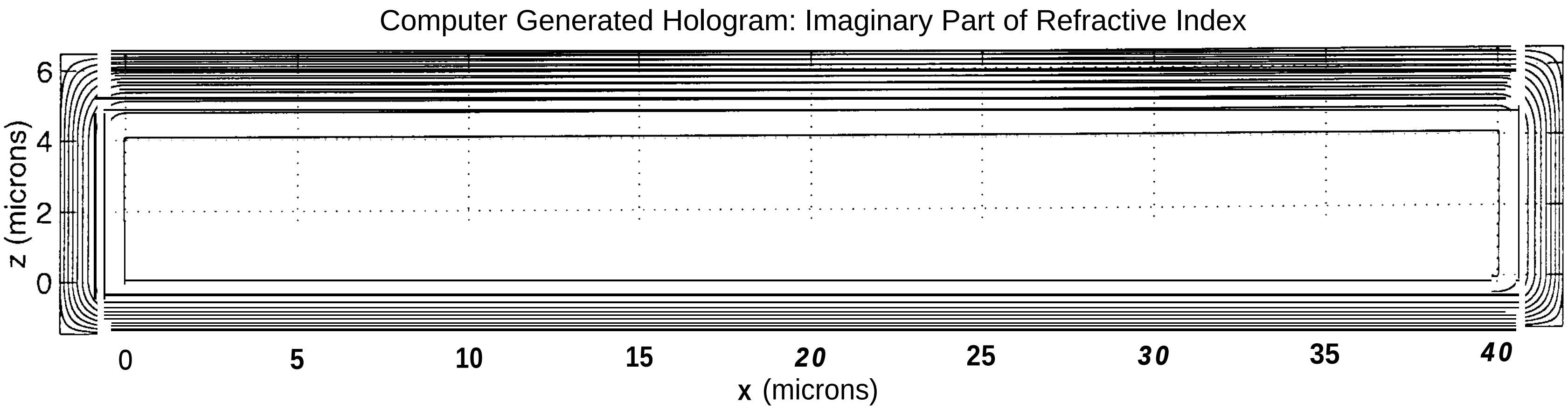
Order	RCWA Backward DE	ADI Backward DE	Error	RCWA Backward Rel. DE	ADI Backward Rel. DE	Error
-3	0.009809	**0.0104**	6.1%	**0.16476**	**0.1699**	**3.1%**
-2	0.003349	**0.00352**	5.1%	**0.05626**	**0.0575**	**2.2%**
-1	0.003403	**0.00329**	-3.2%	**0.05715**	**0.0538**	**-5.9%**
0	0.026413	**0.02679**	1.4%	**0.44365**	**0.4376**	-1.4%
1	0.003403	**0.00329**	-3.2%	**0.05715**	**0.0538**	**-5.8%**
2	0.003349	**0.00352**	5.0%	**0.05626**	**0.0575**	**2.1%**
3	0.009809	**0.0104**	6.1%	**0.16476**	**0.1699**	3.1%
	Total	Total		Total	Total	
	0.059535	0.06123	2.8%	1	1	

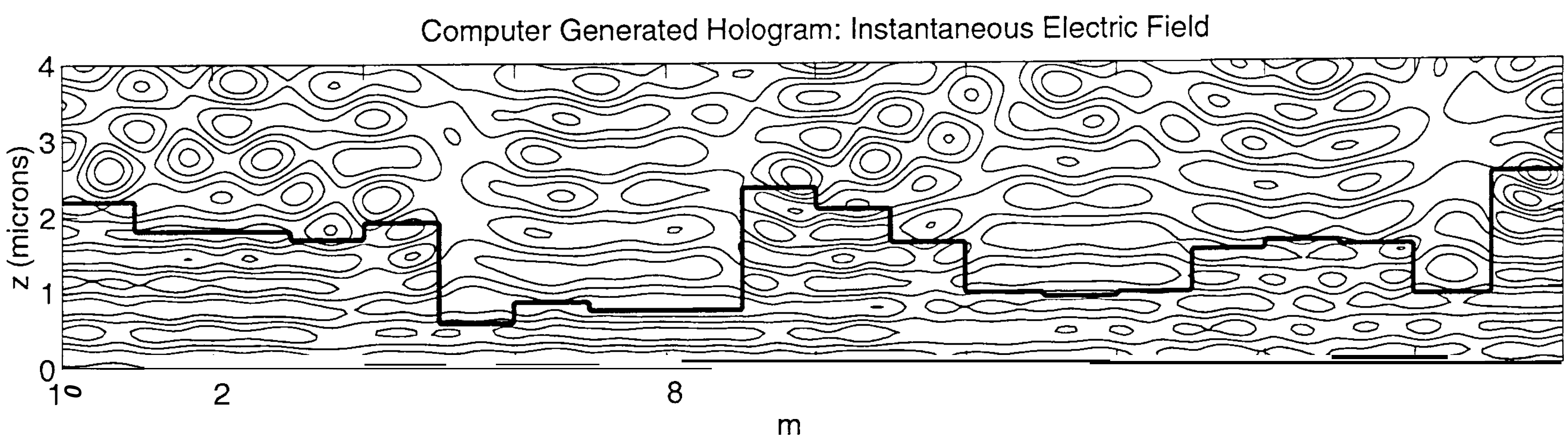
Order	RCWA Forward DE	ADI Forward DE	Error	RCWA Forward Rel. DE	ADI Forward Rel. DE	Error
-2	**0.075094**	0.07335	**-2.3%**	**0.07985**	**0.0795**	-0.5%
-1	**0.367745**	0.36174	-1.6%	**0.39102**	**0.3918**	**0.2%**
0	**0.054788**	0.05299	**-3.3%**	**0.05826**	**0.0574**	-1.5%
1	**0.367745**	0.36174	-1.6%	**0.39102**	**0.3918**	**0.2%**
2	**0.075094**	0.07335	**-2.3%**	**0.07985**	**0.0795**	-0.5%
	Total	**Total**		**Total**	**Total**	
	0.940465	**0.92319**	-1.8%	1	1	
	Grand Total	**Grand Total**				
	1	**0.98441**	**-1.670**			

Computer Generated Hologram Simulations

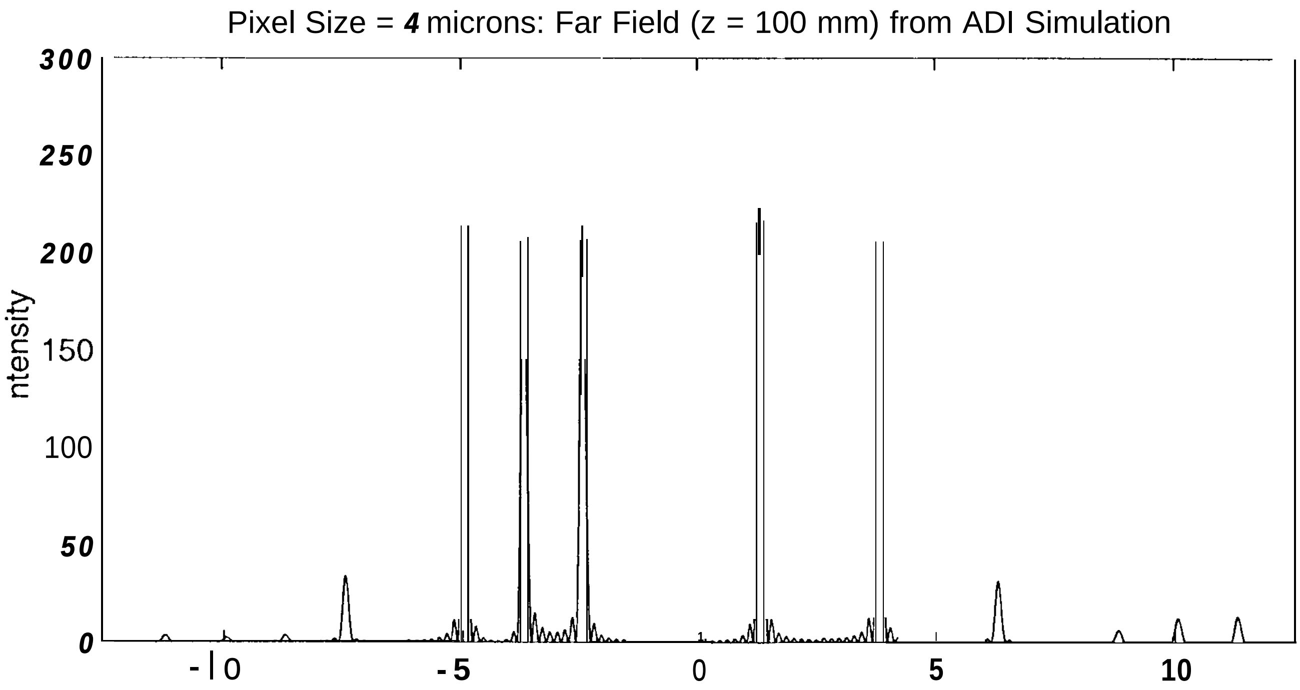
- Used Gerchberg-Saxton iterative algorithm to design one-dimensional CGHS with pixel widths of 0.5, 1, 2, 3, and 4 μm
- Calculated near field using ADI technique
- Calculated Fourier optics near field from pixel depths (quantized to match ADI grid)
- Extracted single-cell CGH near fields and arrayed them 8 times (isolates far field spots)
- Propagated both solutions to the far-field using Fraunhofer approximation and compared intensity patterns


Computer Generated Hologram: Real Part of Refractive Index
z (microns)
x (microns)
0
2
4
6
0
5
10
15
20
25
30
35
40

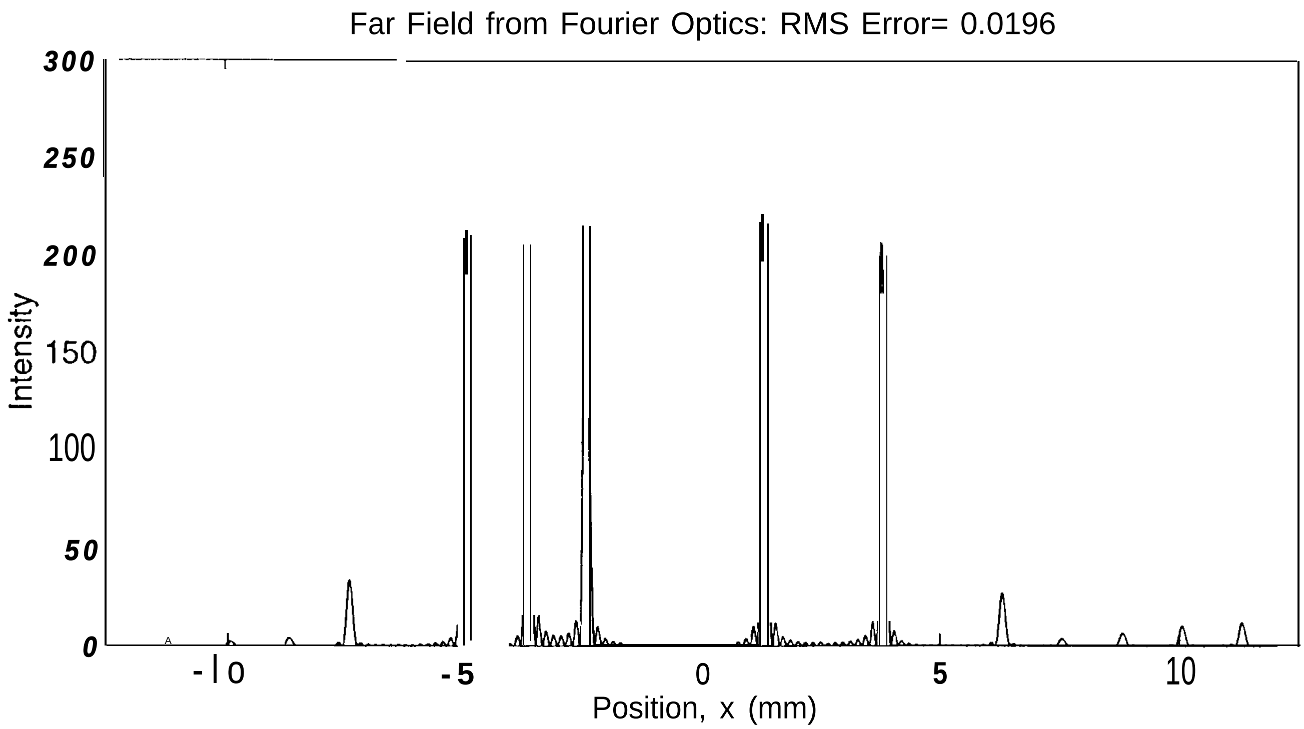

Computer Generated Hologram: Imaginary Part of Refractive Index
z (microns)
x (microns)
0
2
4
6
0
5
10
15
20
25
30
35
40



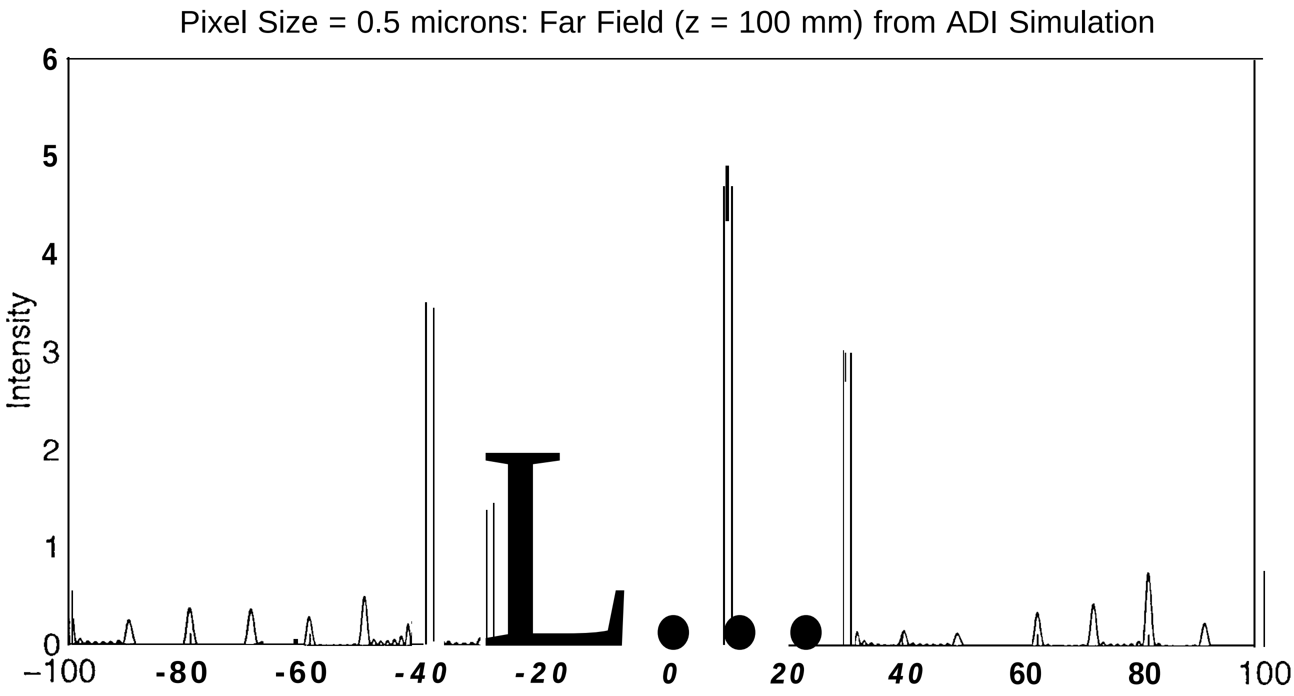
Computer Generated Hologram: Instantaneous Electric Field
z (microns)
4
3
2
1
0
1
0
2
8
m

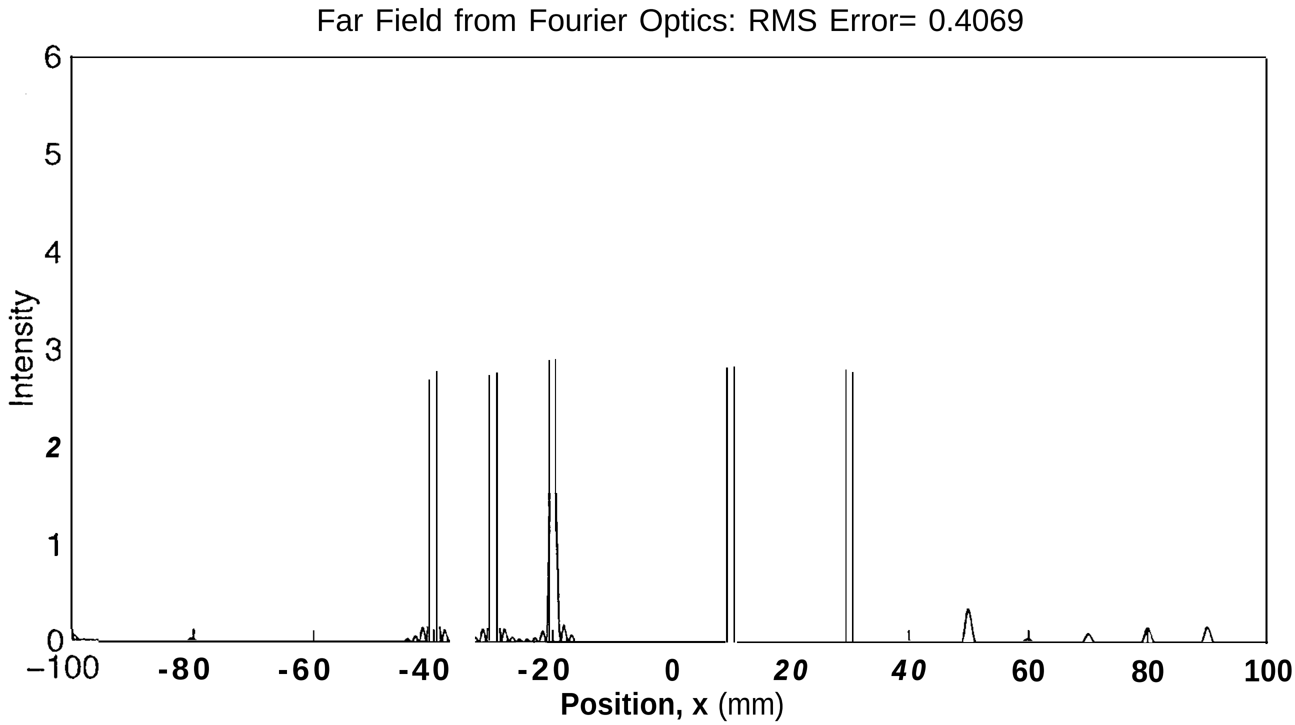


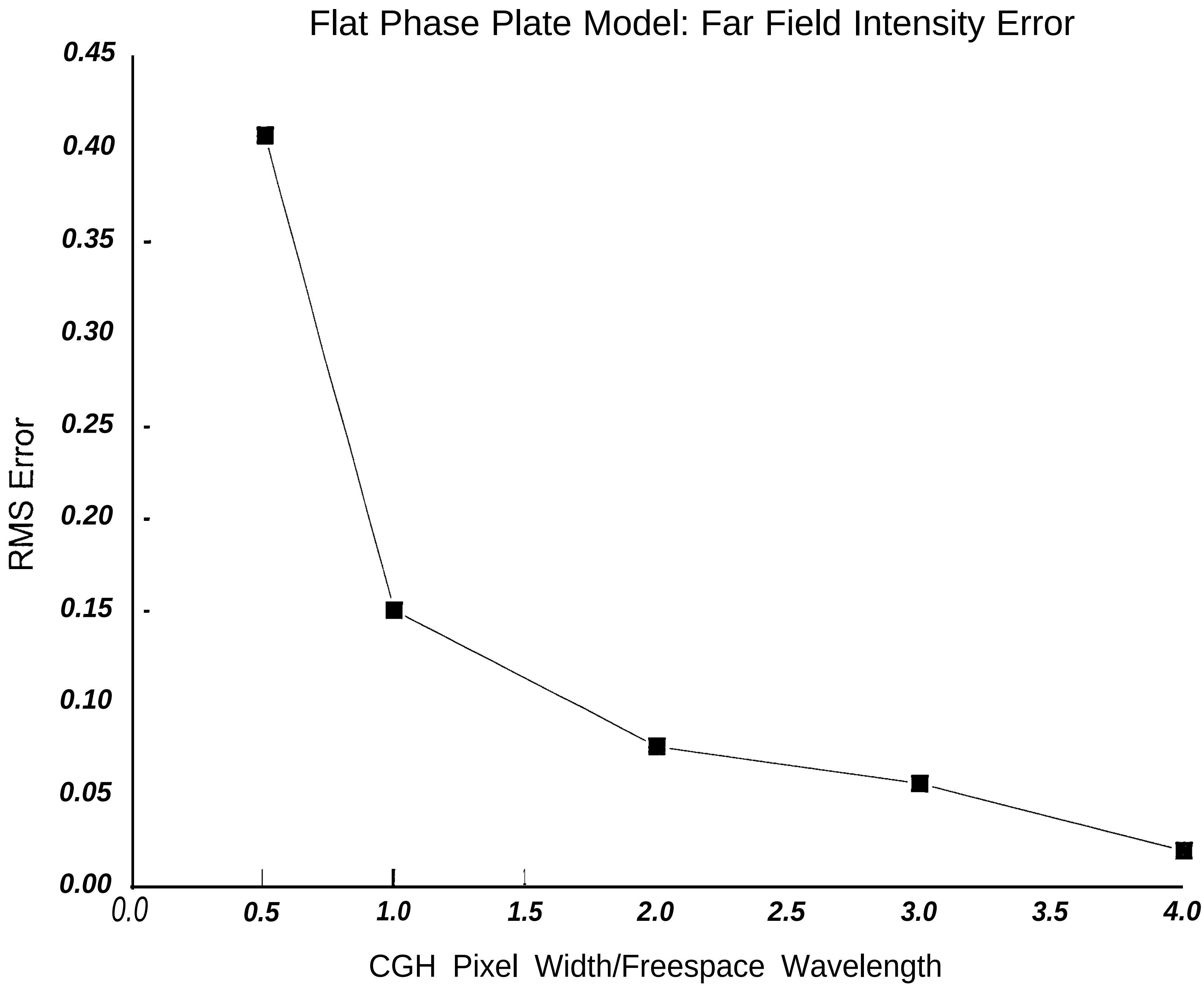
Pixel Size = 4 microns: Far Field (z = 100 mm) from ADI Simulation
ntensity
300
250
200
150
100
50
0
-10
-5
0
5
10



Far Field from Fourier Optics: RMS Error= 0.0196
Intensity
300
250
200
150
100
50
0
-10
-5
0
5
10
Position, x (mm)


Pixel Size = 0.5 microns: Far Field (z = 100 mm) from ADI Simulation
Intensity
6
5
4
3
2
1
0
−100
-80
-60
-40
-20
0
20
40
60
80
100
L...


Far Field from Fourier Optics: RMS Error= 0.4069
Intensity
6
5
4
3
2
1
0
−100
-80
-60
-40
-20
0
20
40
60
80
100
Position, x (mm)

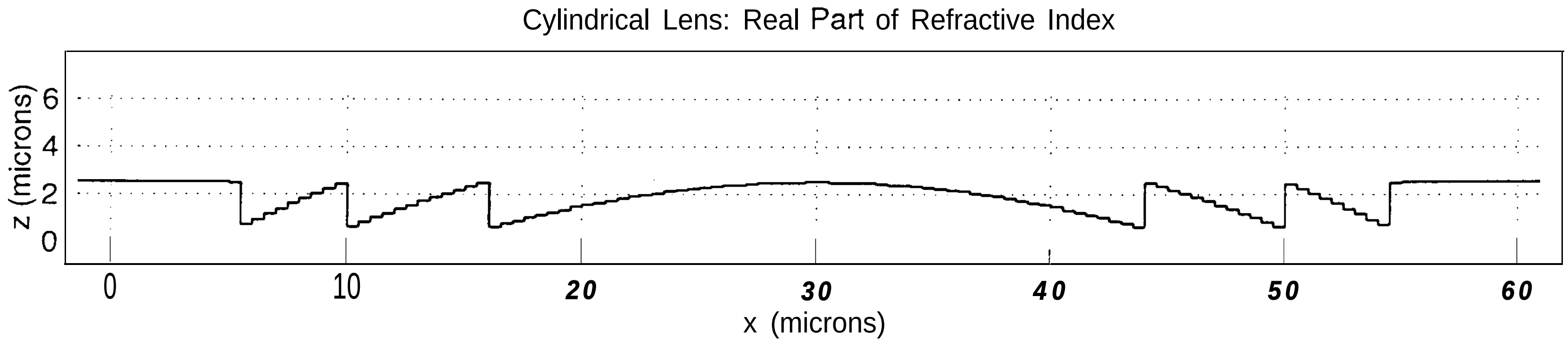

Flat Phase Plate Model: Far Field Intensity Error
RMS Error
0.45
0.40
0.35
0.30
0.25
0.20
0.15
0.10
0.05
0.00
0.0
0.5
1.0
1.5
2.0
2.5
3.0
3.5
4.0
CGH Pixel Width/Freespace Wavelength

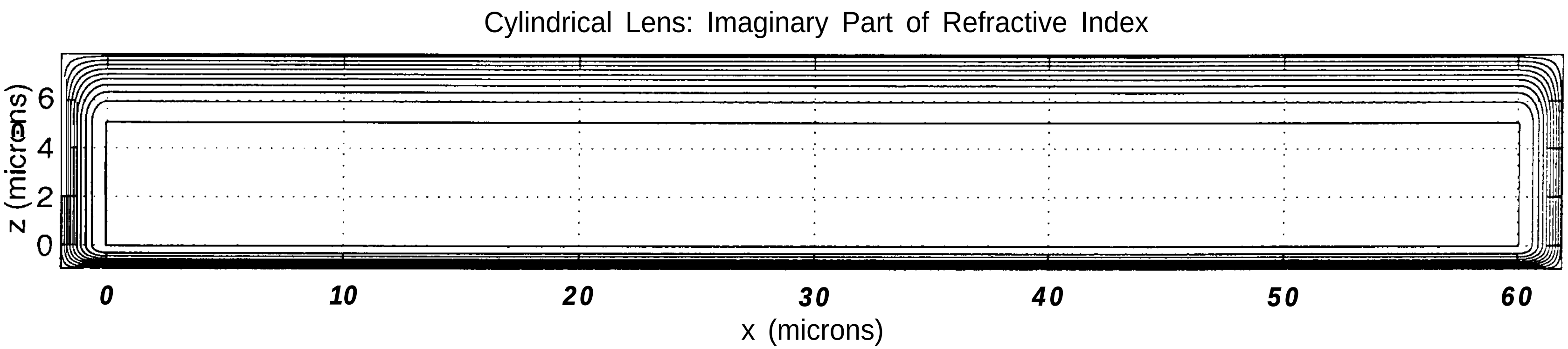
Cylindrical Lens Simulation

- Foca ength 00 μm, width 50 μm, lens index 1 5, focal medium index 1.0
- Calculated near field using ADI technique
- Propagate field by convolving with impulse response of free space

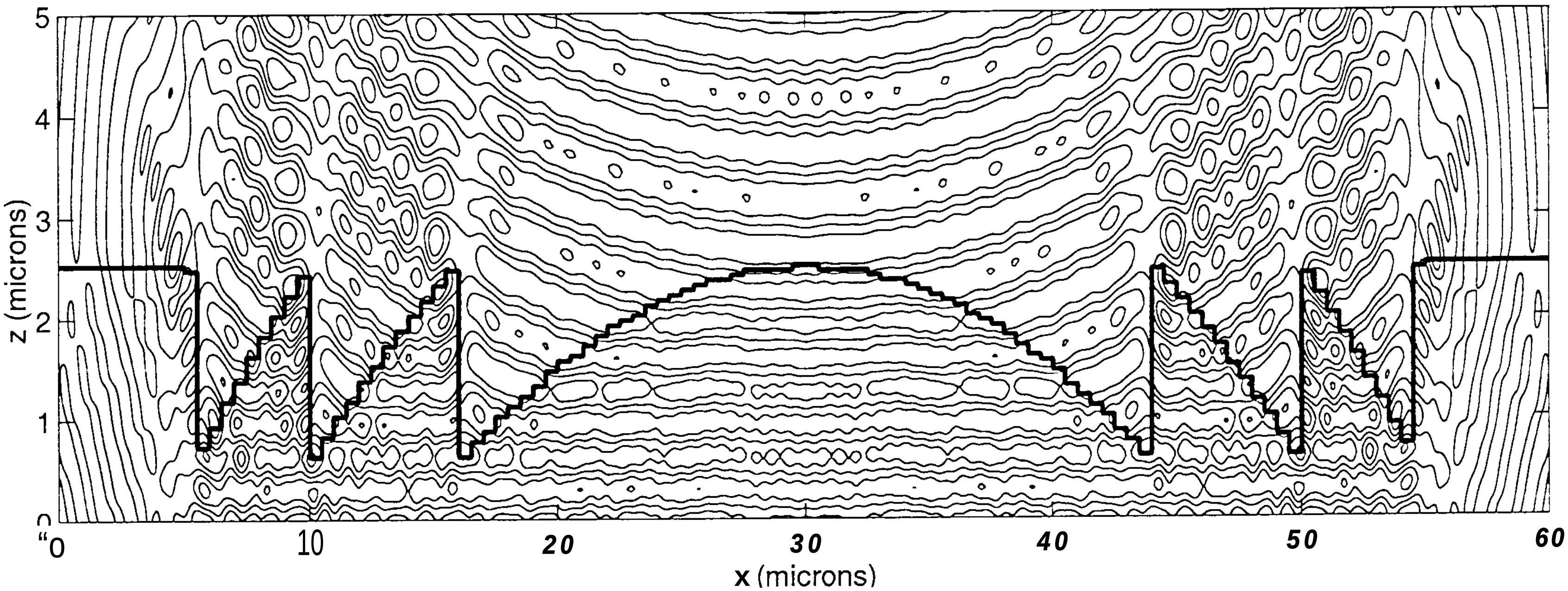
$$U(x, z - z_0) = U(x, z_0) * h(z - z_0)$$
$$= F.T.^{-1}\left(F.T.\{U(x, z_0)\} F.T.\{h(z - z_0)\}\right)$$

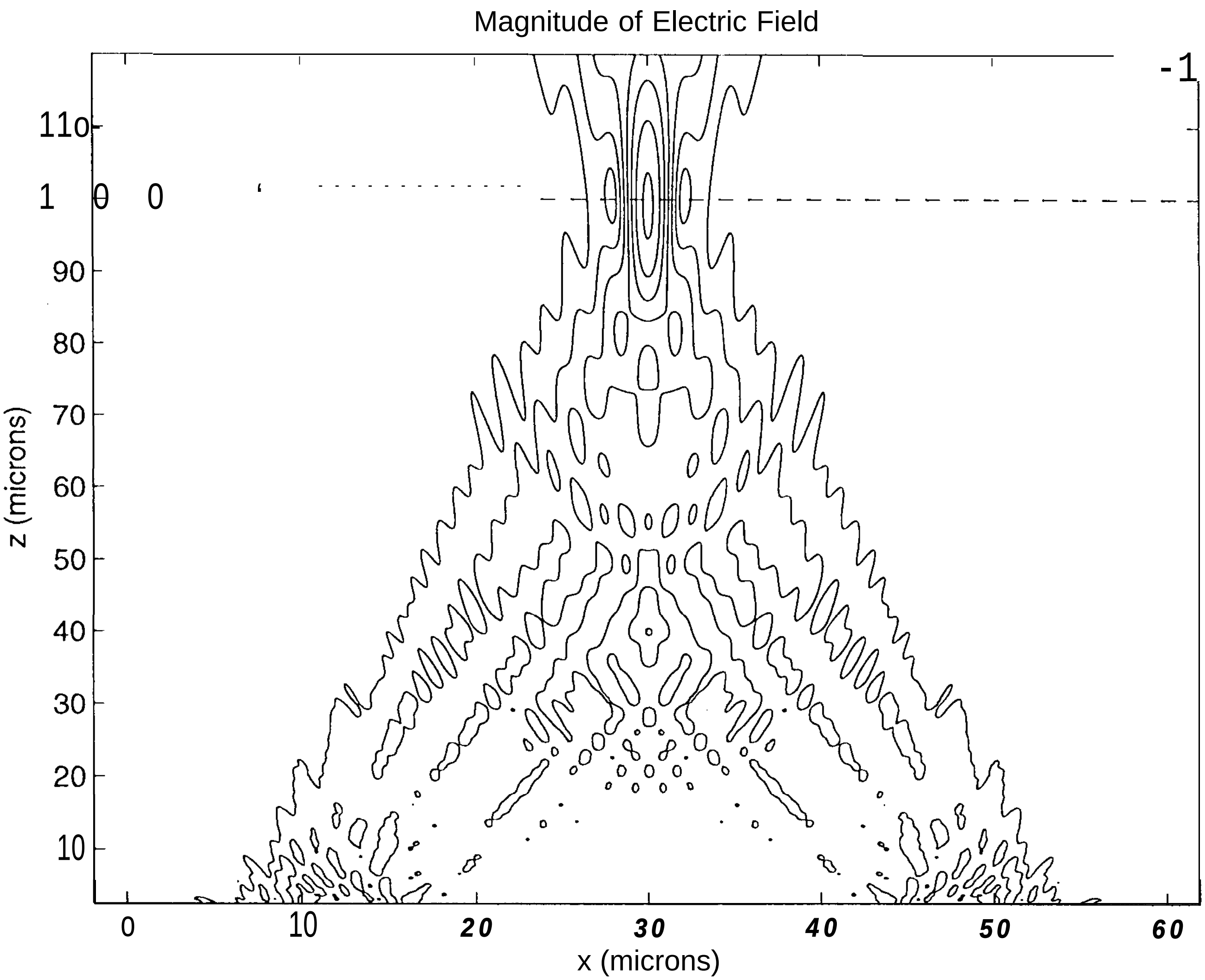
$$F.T.\ h(z - z_0) = \begin{cases} \exp\left(i(z - z_0)\frac{2\pi}{\lambda}\sqrt{1 - (u\lambda)^2}\right), & (u\lambda)^2 \le 1 \\ \exp\left(-(z - z_0)\frac{2\pi}{\lambda}\sqrt{(u\lambda)^2 - 1}\right), & (u\lambda)^2 > 1 \end{cases}$$


Cylindrical Lens: Real Part of Refractive Index
z (microns)
0
2
4
6
0
10
20
30
40
50
60
x (microns)

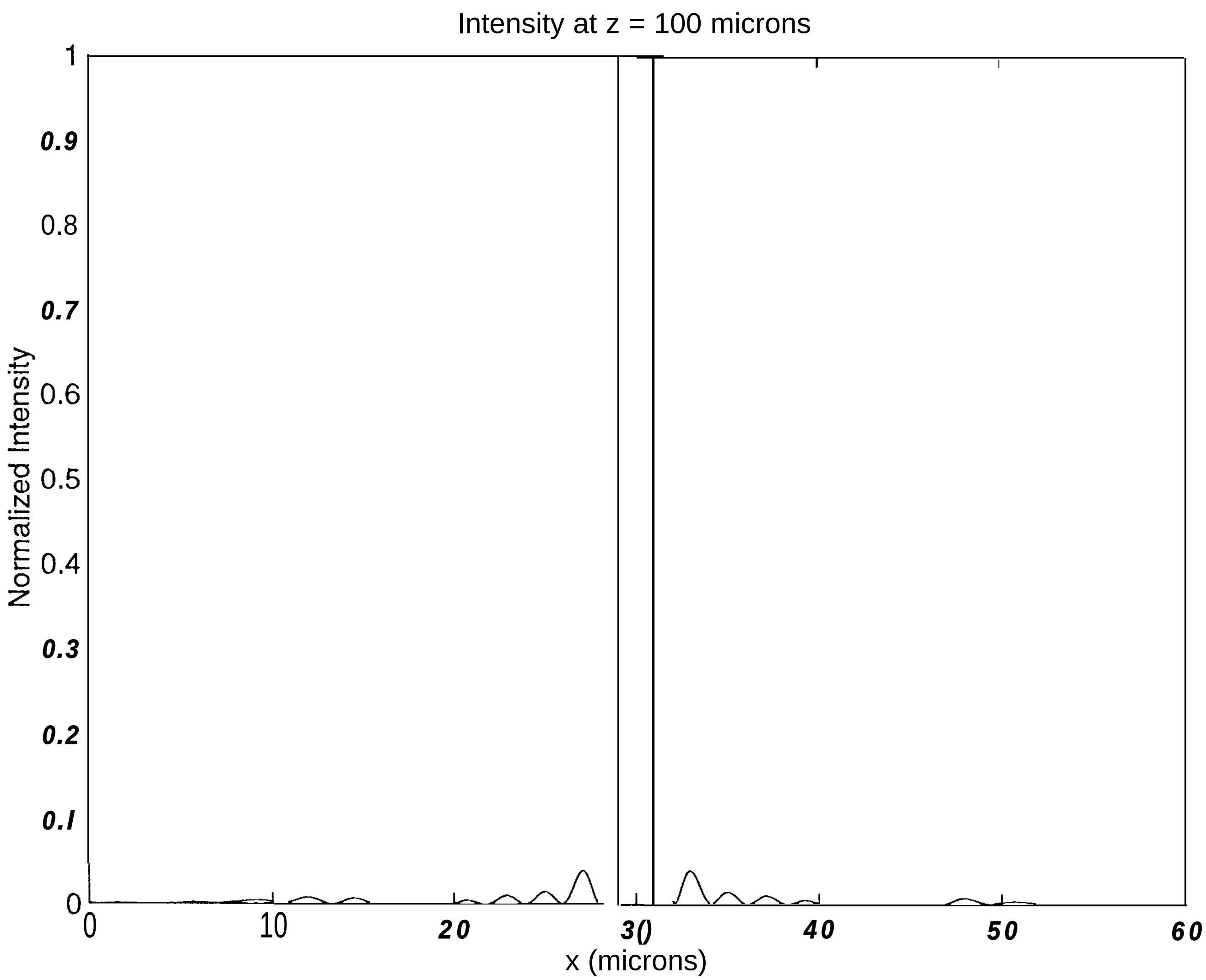

Cylindrical Lens: Imaginary Part of Refractive Index
z (microns)
0
2
4
6
0
10
20
30
40
50
60
x (microns)

Cylindrical Lens: Instantaneous Electric Field





Magnitude of Electric Field
-1
110
1 0 0
90
80
70
60
50
40
30
20
10
z (microns)
0
10
20
30
40
50
60
x (microns)


Intensity at z = 100 microns
Normalized Intensity
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
0
10
20
3()
40
50
60
x (microns)

Summary

- Extended Hadley's ADI Helmholtz solver to allow accurate simulations of diffractive optical elements
- Grating - reasonable agreement with RCWA diffraction efficiencies
- Computer generated holograms - quantitative calculation showing breakdown of Fourier optics model for wavelength-size features
- Cylindrical lens - calculation of fields and focal plane intensity