Comparison of Radar Rainfall Retrieval Algorithms in Convective Rain During TOGA COARE

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Abstract

We compare two deterministic and two stochastic rain retrieval algorithms by applying them to 14 GHz reflectivity profiles acquired during TOGA COARE. The first deterministic algorithm corrects the $k - R$ relation, while the second corrects the $Z - R$ relation and is equivalent to correcting the calibration constant. The stochastic algorithms are based on applying an Extended Kalman Filter to the reflectivity. *One* algorithm employs only radar data, while the other employs both radar and path attenuation. We find that the deterministic algorithm which corrects the $Z - R$ relation and the two stochastic algorithms indicate a smaller mean diameter than would be expected for widespread, light or moderate rainfall. This finding seems in agreement with independent observations of the DSD in tropical convective rain. Only the algorithm which corrects the $k - R$ relation suggests larger drops. This, combined with observation that the $Z - R$ relation is much more variable than the $k - R$ relation indicates that the $Z - R$ relation should be corrected when using a deterministic algorithm.

1 Introduction

One of the challenges in estimating rainfall from spaceborne radars is the presence of attenuation at the higher frequencies planned for these systems. The Precipitation Radar (PR), for example, on the Tropical Rainfall Measuring Mission (TRMM) will operate at 14 GHz (Simpson et al. 1988). Several algorithms for rainfall retrieval for attenuating radars have been discussed in the literature. These include algorithms *in* which the rain rate profile is viewed as a deterministic quantity (Meneghini et al. 1983, Iguchi and Meneghini 1994, Marzoug and Amayenc 1991) and in which the rainfal profile is viewed as a random process (Haddad et al. 1996a, Haddad et al. 1996 b). Previous algorithm comparisons have focused on the different deterministic algorithms (Iguchi and Meneghini 1994, Amayenc et al. 1996, Testud et al. 1996). It is the pupose of this work to compare deterministic and stochastic algorithms using data acquired by the NASA/JPL Airborne Rain MApping radar (ARMAR). This system operates with the same downward-looking geometry and 14 GHz frequency as the TRMM PR and is described in detail by Durden et al. (1994). The ARMAR data used here were acquired in convective rain during Tropical Oceans Global Atmosphere Coupled Ocean Response

Experiment (Lukas and Webster 1992). In the next section we review the rainfall profiling algorithms. Following this, we compare the retrieved profiles using the different algorithms.

2 Rainfall Retrieval Algorithms

One approach to rain retrieval views both the measured reflectivity Z_m and and the desired rain rate R *as* deterministic functions of range. For radars operating at attenuating wavelengths, Z_m is related to the true reflectivity Z by

$$Z_m(r) = Z(r)10^{-0.2\int_0^r k(s)ds} \tag{1}$$

where r is range and k is the specific attenuation. Both Z and k can be calculated directly from the drop-size distribution (DSD) parameters or from power laws relating them to rainrate, i.e. $k = \alpha R^\beta$ and $Z = aR^b$. Hitschfeld and Bordan (1954) showed that by using the $Z - R$ and $k - R$ relations, (1) could be re-written as a first-order ordinary differential equation which has an exact analytical solution. Kozu et al. (1991) also used this technique to derive a similar analytical solution using DSD parameters rather than k - R and $Z - R$ relations. Unfortunately, as shown by Hitschfeld and Bordan and other subsequent authors, errors in the radar calibration or in the assumed rainfall parameters can cause the error in the retrieved rain to grow rapidly as a function of range. A solution to this problem is to use the path attenuation as a constraint. This can be derived either from radiometer or from the surface reference technique (SRT), in which a radar measurement of the ocean surface in a clear area is compared with the measurement in the raining area (Meneghini et al. 1983).

The measured path attenuation can be used in a variety of ways. When used only as a boundary condition, one gets the *kZS* algorithm, which compensates for an unknown attenuation to the first range bin (Marzoug and Amayenc 1991). Here, we consider algorithms where the path attenuation is used to find an adjustment in either the radar-rain relationships or the radar calibration parameters. The reflectivity profile derived using the Hitschfeld-Bordan approach is

$$Z(r) = \frac{Z_m(r)}{\left(1 - \frac{0.2\ln(10)\alpha\beta}{a^{\beta/b}b}\int_0^r Z_m(t)^{\beta/b}dt\right)^{b/\beta}} \tag{2}$$

The denominator is the correction of the measured profile for attenuation. At the surface (range r_s) this correction should be equal to the independently measured path attenuation A. This requires

$$A^{\beta/b} = 1 - \frac{0.2\ln(10)\alpha\beta}{a^{\beta/b}b} \int_0^{r_s} Z_m(t)^{\beta/b} dt \tag{3}$$

where ϵ is a parameter introduced to allow the equality. Solving for ϵ gives

$$\epsilon = \frac{1 - A^{\beta/b}}{\frac{0.2\ln(10)\alpha\beta}{a^{\beta/b}b} \int_0^{r_s} Z_m(t)^{\beta/b} dt} \tag{4}$$

Refering to (3), it can be seen that ϵ multiplies a term containing both the k - R and Z – R parameters and also the measured reflectivity Z_m. Consequently, ϵ can be considered aa a correction for any one parameter or distributed over all parameters. If it is believed that only the $k - R$ relation coefficient is in error, α is replaced by $\epsilon\alpha$. This yields the rain profile

$$\epsilon(r) = \frac{(Z_m(r)/a)^{1/b}}{\left(1 - \frac{0.2\epsilon\ln(10)\alpha\beta}{a^{\beta/b}b} \int_0^{r} Z_m(t)^{\beta/b} dt\right)^{1/\beta}} \tag{5}$$

and is known as the a-adjustment method (Meneghini et al. 1983). However, if the error is, instead, believed to be in the Z – R relation coefficient, a is replaced by $a/\epsilon^{b/beta}$. In this case the Z profile is the same as in the a-adjustment case, but the rain rate is different, since a appears in the numerator of (5) and must be replaced by $a/\epsilon^{b/beta}$. The resulting rain rate profile using a-adjustment is thus a factor $\epsilon^{1/\beta}$ times the rain rate derived from (5). It is also possible to adjust the measured reflectivity Z_m to compensate for calibration errors. In this C-adjustment procedure (Meneghini et al. 1983) the Z profile is a factor of $\epsilon^{b/\beta}$ times that found using the α- or a-adjustment. This is because we must replace Z_m in (2) with $Z_m\epsilon^{b/\beta}$ in both the denominator and the numerator. This was shown by Iguchi and Meneghini (1994); note that in their notation β is equivalent to our β/b. However, when the C-adjustment Z profile is converted to rain rate using the Z – R relation, the rain rate is $(Z_m/a)^{1/b}\epsilon^{1/\beta}$. This is identical to the rain rate produced by the a-adjustment method. Thus. the rain profiles retrieved from the a- and C-adjustment algorithms are identical and differ from the a-adjustment by a constant factor. We refer to the α-adjustment as algorithm D1. Rain retrieved from adjusting a or the calibration constant is greater by a factor $\epsilon^{1/\beta}$ and is refered to as D2.

An alternative approach is to view the measurements and the desired rain profile as stochastic processes (Haddad et al. 1996a). Specifically, one can model the rain profile as a Markov process, while observation of this process is described by the radar equation (1), with an additional term for observation noise. The minimum variance estimate of the rain rate profile is then given by the mean of the rate rate profile conditioned on the observed radar data. Were the radar equation linear in R, a standard Kalman filter could be applied. Its nonlinearity, however, requires use of the Zakai equation for the full probability density function (PDF) (Haddad et al. 1996a) or use of an Extended Kalman Filter (Haddad et al. 1996b) for the conditional mean and covariance. The latter approach is significantly faster, while still allowing additional information to be used in the framework of Bayesian estimation. Specifically, as shown in Haddad et al. (1996 b), one can incorporate ***a priori*** statistics of the DSD parameters, as well as the SRT-observed path attenuation. Here, we test two stochastic algorithms, one which uses only radar data and a ***priori*** DSD statistics (algorithm S1), the other which additionally uses the path attenuation (algorithm S2). The details of these algorithms are described in Haddad et al. (1996 b). The only modification has been to use the new DSD parametrization discussed in Haddad et al. (1997) Traditionally, the gamma DSD has been described by the parameters N_0, μ and Λ:

$$N(D)dD = N_0 D^{\mu} e^{-\Lambda D} dD \text{ drops of diameter } D \text{ mm, per m}^3 \tag{6}$$

These parameters, however, are mutually correlated, so that it is not possible to vary one independently of the others. A parameter set which is uncorrelated consists of the rain rate R, the parameter s“, related to the DSD width, the parameter D'', related to the mean diameter. The old parameters are found from the new using

$$\begin{aligned}
\mu &= \frac{1}{s''^2 D''^{0.33} R^{0.074}} - 4 \\
\Lambda &= \frac{1}{s''^2 D''^{1.33} R^{0.23}} \\
N_0 &= 55 \frac{\Lambda^{\mu+4}}{\Gamma(\mu+4)\,(1-(1+0.53/\Lambda)^{-\mu-4})} R
\end{aligned} \tag{7}$$

It waa found that s“ is essentially constant, 0.39. The Kalman filter is thus run for a range of ***D“*** values. The resulting rain rate for each run is then averaged over the ***a priori*** Gaussian density function for ***D“.***

3 Results

The data used here were acquired in moderate to intense convective rain events during TOGA COARE. We chose profiles based on local maxima of the SRT-measured path attenuation; i.e. we attempted to choose profiles from near the center of each convective cell. Figure 1 shows the reflectivity profile for an event which had a path attenuation (l-way) of 18 dB. Figure 2 shows the retrieved rain rate for the deterministic algorithms D1 and D2. Figure 3 shows the retrieved rain profiles and the uncertainties (corresponding to one standard deviation) for the stochastic algorithms. We find that all four algorithms produce high rain rates at altitudes up to 2 km and much lower rain rates above 2 km. When the path attenuation information is not used (S1), the profile shows a large maximum at 2 km altitude. However, as shown in Figure 3, the standard deviation estimate for S1, is large. When the path attenuation is used (Dl, D2 and S2), the peak at 2 km is less pronounced. The main differences between D1, D2 and S2 for this case is the magnitude of the rain rate (S2 is approximately 4 times that of D1 and twice that of D2). As shown in Figure 3, the use of the path attenuation substantially reduces the uncertainty in the stochastic estimate. This implies that there are many rain profiles which could fit the radar-only data but much fewer profiles which can fit both the radar and path attenuation measurements.

To better understand the source of the differences between algorithms, we examine the resulting k - R and $Z - R$ relations, For the D1 and D2 algorithms the initial relations are from Nakamura et al. (1990).

$$k = 0.032R^{1.124} \qquad (1o)$$

$$Z = 372.4R^{1.54} \qquad (11)$$

and are expected to be most appropriate for widespread, light-to-moderate rainfall. The correction factor ϵ found when using the above relations in D1 and D2 is 2.0, meaning that either the initial α is too small or that the initial a is too large. Table 1 shows the resulting $k - R$ and $Z - R$ coefficients for algorithms D1 and D2 when applying ϵ = 2.0. Also shown in Table 2 are the $k - R$ and $Z - R$ relations found by S1 and S2. The relations for S2 correspond to a mean drop size smaller than found by S1, when the path attenuation is not used.

The above analysis was extended to 20 profiles from intense convective cells, acquired on seven separate TOGA COARE flights. The mean one-way path attenuation for these profiles is 12.7 dB, with a maximum of 21.5 dB. Those cells with the largest path attenuations tended to have significant reflectivities (>30 dBZ) at altitudes well above 4.8 km, which is the typical altitude of the zero degree isotherm, based on dropsonde measurements. We computed the average percentage difference between rain rates for the 20 reflectivity profiles. This difference is defined as the rain rate difference at each range bin, normalized by the greater of the two rain rates. The rain rates produced by D1 are 43.5% less than the D2 rain rate; i.e., D2-D1 normalized by D2 is 43.5$%0, independent of range. Figure 4 shows the differences versus altitude for D2 and S1, D2 and S2, and S1 and S2.

We also looked at the average $k - R$ and $Z - R$ relations found by the different algorithms. These values are shown in Table 2, and are, coincidentally, the same as the $k - R$ and $Z - R$ relations in Table 1. The coefficients for the various profiles did, however, differ from those in Table 2, and we have also shown the standard deviations. The $k - R$ and $Z - R$ relations for D1 and D2 are based on the the average ϵ of 2.0. It requires either that the initial α be doubled, as shown in Table 2 for Dl, or that the initial a be reduced by a factor of 0.4 as shown for D2. A similar correction factor was noted by Amayenc and Tani in their analysis of ARMAR data from 6 February during TOGA COARE. The relations for S2, shown in Table 2, correspond to a mean drop size smaller than found by S1, when the path attenuation is not used.

4 Discussion

Previous measurements of raindrops in tropical convective rain suggest that the drops are smaller than drops observed in widespread, moderate rain, such as described by the Marshall-Palmer distribution (Tokay and Short 1996). The correction found by the deterministic algorithms implies either a larger α or smaller a. A larger α would imply a distribution with larger drops than expected in widespread rain. A smaller a would imply the converse and would appear to be in agreement with independent DSD measurements in intense convective rain. The presence of relatively small drops is also indicated by the stochastic algorithms. The $Z - R$ relation found by S2 has a coefficient roughly a factor of four less than in (11), while the exponent is similar. This indicates that the reflectivity for a given rain rate is generally smaller in the convective rain, implying smaller drop sizes.

Most previous authors have considered adjusting either the coefficient of the $k - Z$ relation or the calibration constant, i.e., the α- or C-adjustment methods. However, as noted in Section 2, when we consider the retrieved rain rate rather than the profile of Z or k, adjusting the calibration turns out to be identical to adjusting the coefficient in the $Z - R$ relation, i.e. and u-adjustment. Consequently, proof that the radar is well calibrated is not grounds for applying the correction to the $k - R$ relation. In fact, a survey of the literature suggests that the $Z - R$ relation is more variable and therefore more likely to be in error than the $k - R$ relation. This is due to the dependence of Z on the sixth moment of the DSD in the Rayleigh scattering regime, while k depends only on the third moment. This inherent variability of the $Z - R$ relation, combined with the fact that correction of the $Z - R$ relation in D2 provides estimates closer to S1 and S2, suggests using the path attenuation to correct the $Z - R$ relation rather than the $k - R$ relation when employing a deterministic algorithm.

5 Conclusions,

We have compared two deterministic and two stochastic rain retrieval algorithms by applying them to 14 GHz reflectivity profiles acquired during TOGA COARE. The first deterministic algorithm corrects the k-R relation, while the second corrects the $Z - R$ relation. The stochastic algorithms are based on applying an Extended Kalman Filter to the reflectivity. One algorithm employs only radar data, while the other employs both radar and path attenuation. We find that the deterministic algorithm which corrects the $Z - R$ relation and the two stochastic algorithms indicate a smaller mean diameter than would be expected for widespread, light or moderate rainfall. This finding seems in agreement with independent observations of the DSD in tropical convective rain. Only the algorithm which corrects the $k - R$ relation suggests larger drops. This, combined with observation that the $Z - R$ relation is much more variable than the $k - R$ relation indicates that the $Z - R$ relation should be corrected when using a deterministic algorithm.

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Table 1: Z - R and k - R Relations for 22 Feb.

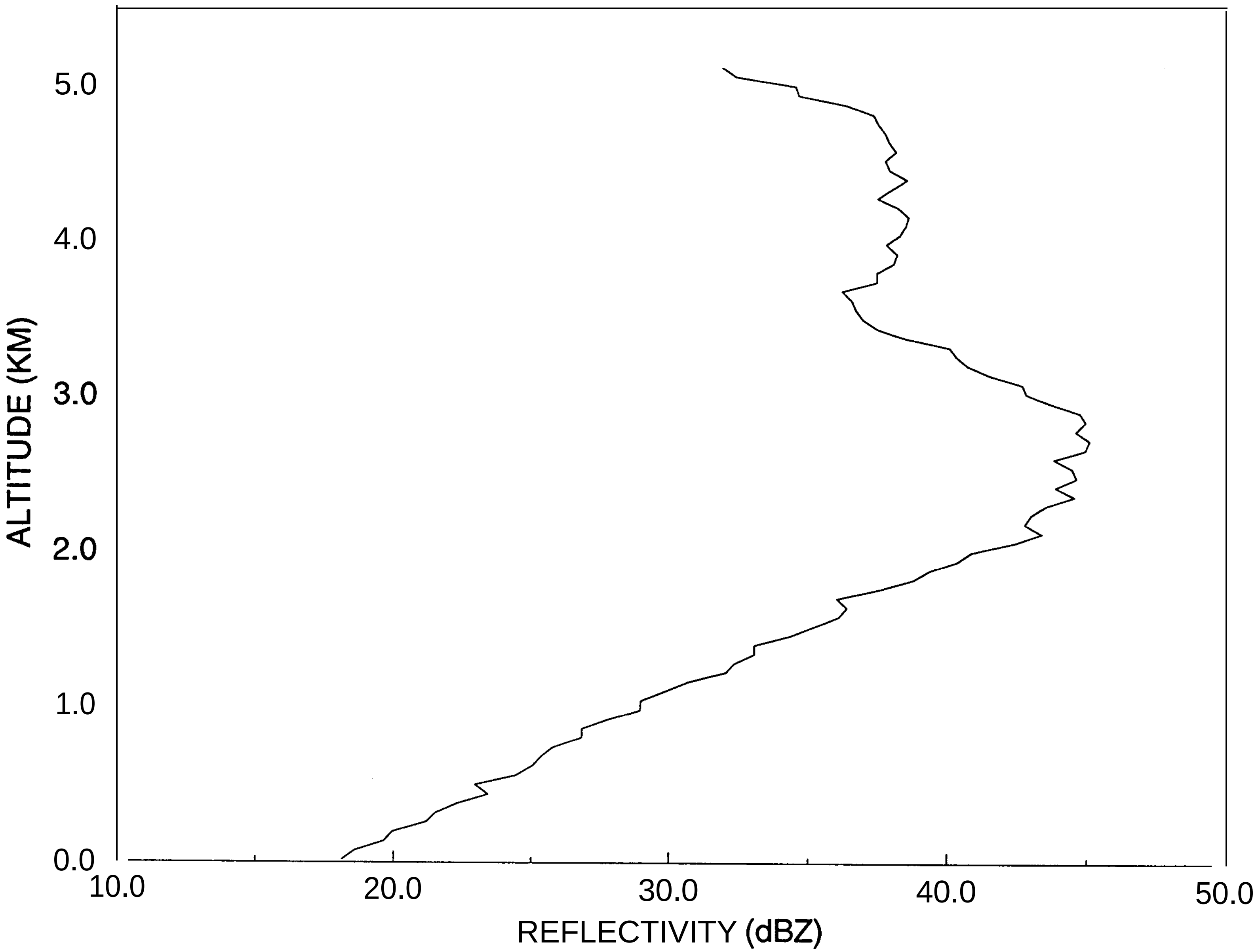
Parameter	D1	D2	S1	S2
a	372.4	144.1	192.7	94.4
b	1.54	1.54	1.50	1.48
α	.064	.032	.023	.018
β	1.124	1.124	1.154	1.152

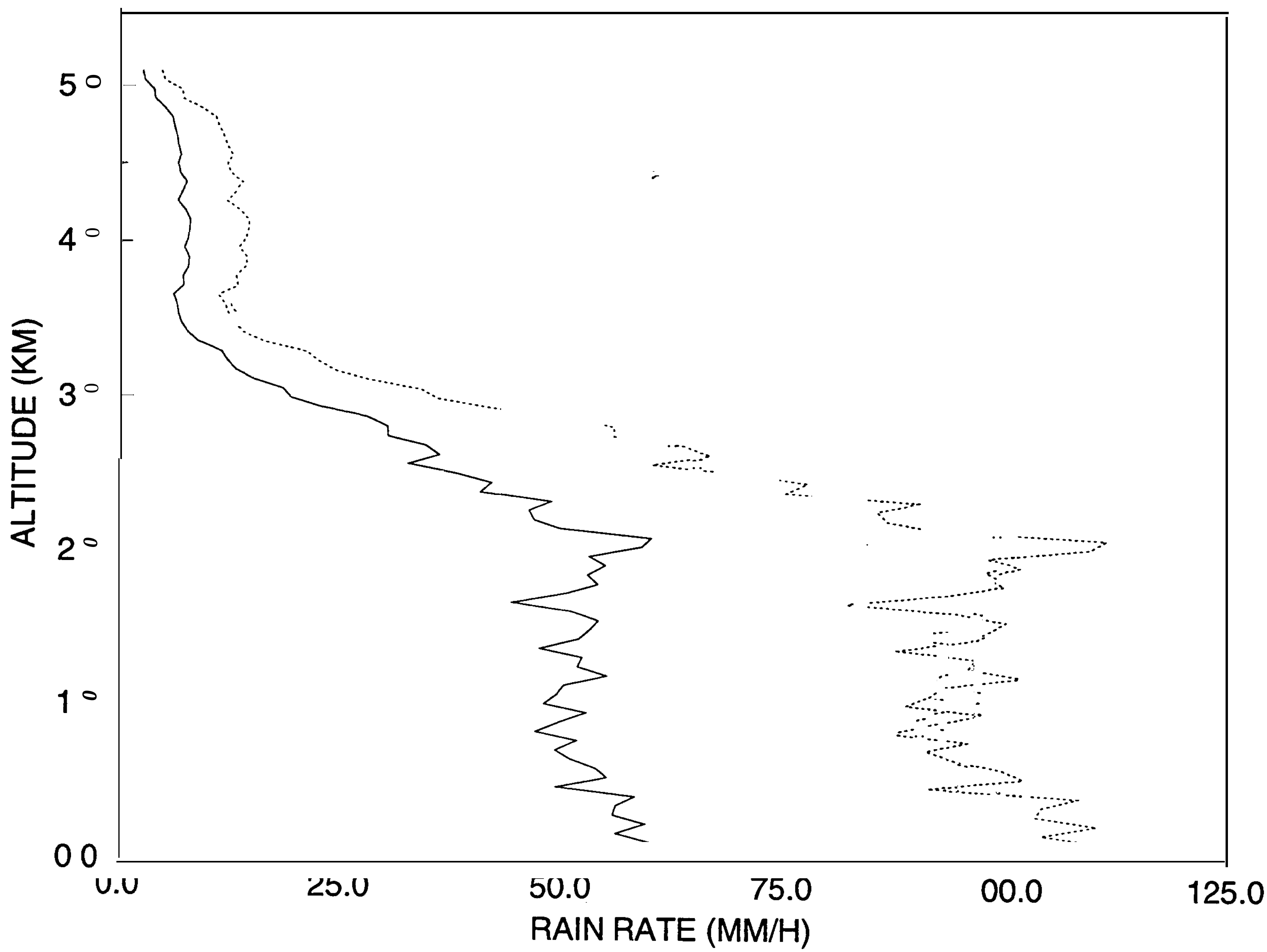
Table 2: Average $Z - R$ and $k - R$ Relations

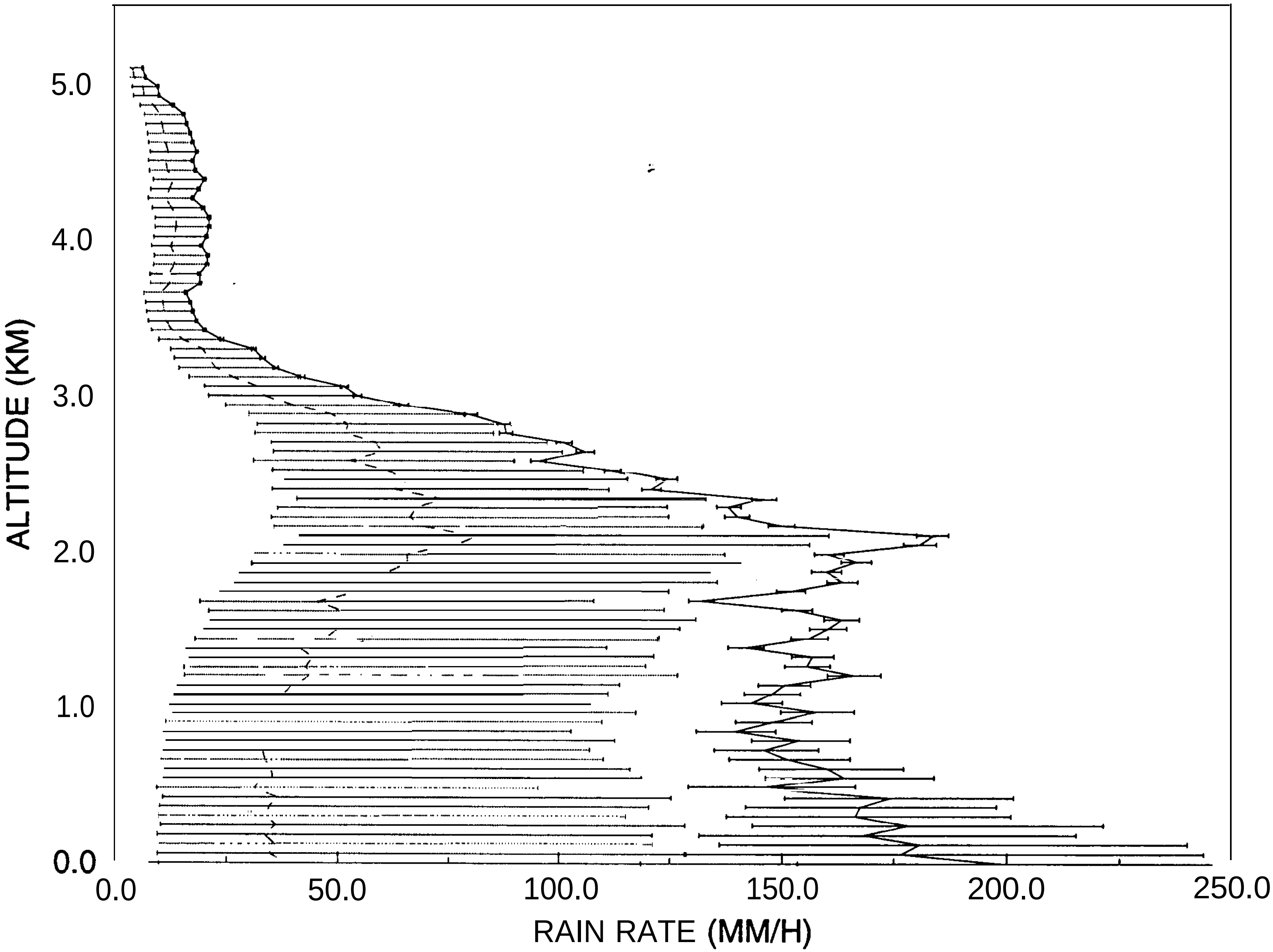
Parameter	D1	D2	S1	S2
a mean	372.4	144.1	192.7	94.4
b mean	1.54	1.54	1.50	1.48
α mean	0.064	0.032	0.023	0.018
β mean	1.124	1.124	1.154	1.152
a std. dev.	0.0	40.0	0.0	35.7
b std. dev.	0.0	0.0	0.0	0.03
α std. dev.	0.02	0.0	0.0	0.002
β std. dev.	0.0	0.0	0.0	0.007

Figure Captions

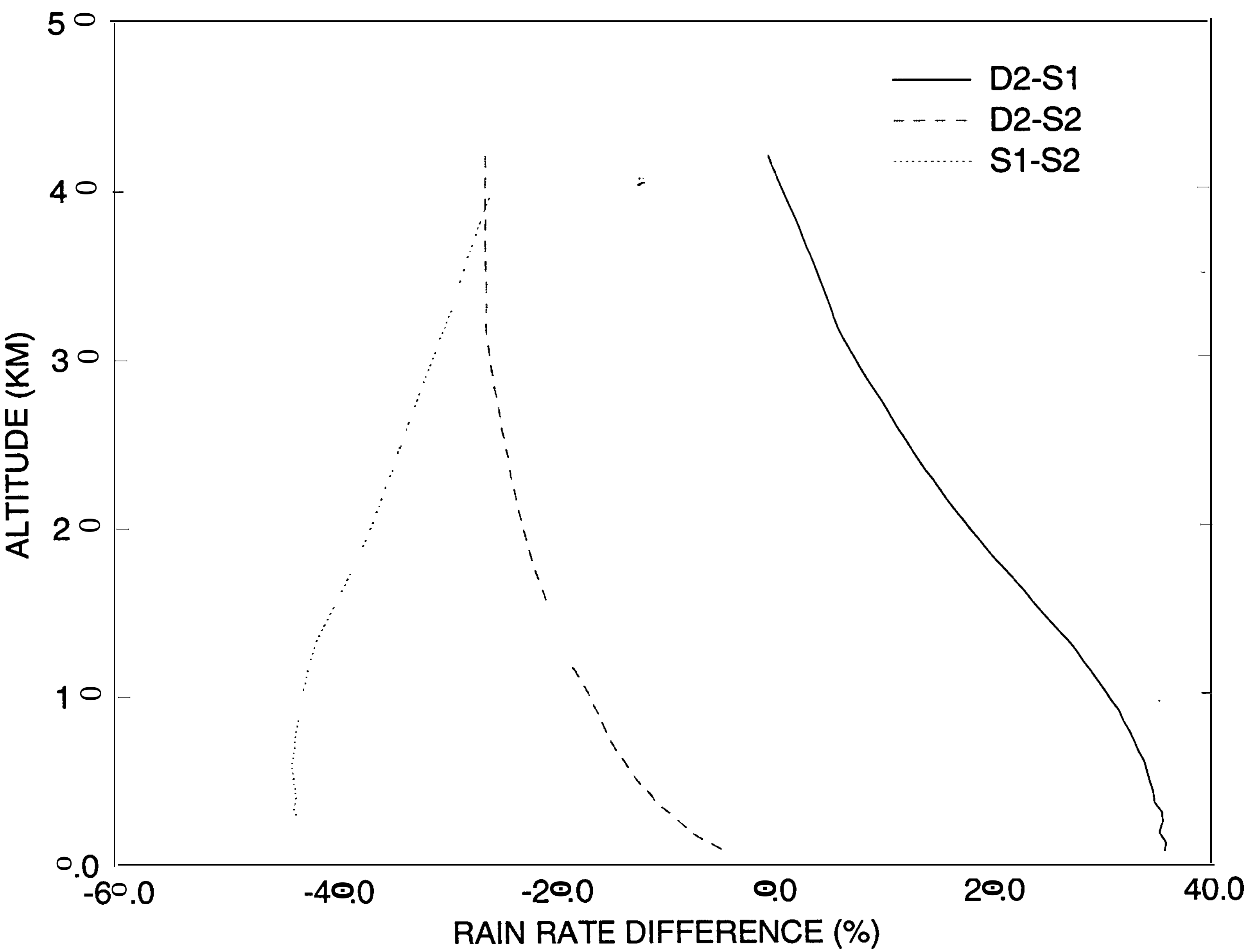
1. ARMARreflectivity profile for2220GMT on22 February 1993.
2. Rain rate profiles from D1 and D2(right).
3. Rain rate profiles and uncertainties from S1 and S2(right).
4. Average percentage difference between rain rates from various algorithms, versus altitude.


5.0
4.0
3.0
2.0
1.0
0.0
ALTITUDE (KM)
10.0
20.0
30.0
40.0
50.0
REFLECTIVITY (dBZ)


ALTITUDE (KM)
5.0
4.0
3.0
2.0
1.0
0.0
0.0
25.0
50.0
75.0
00.0
125.0
RAIN RATE (MM/H)



5.0
4.0
3.0
2.0
1.0
0.0
ALTITUDE (KM)
0.0
50.0
100.0
150.0
200.0
250.0
RAIN RATE (MM/H)


D2-S1
D2-S2
S1-S2
ALTITUDE (KM)
5.0
4.0
3.0
2.0
1.0
0.0
-60.0
-40.0
-20.0
0.0
20.0
40.0
RAIN RATE DIFFERENCE (%)