

# Self-Complementary Structures and Their Application in Grid Amplifiers

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**Abstract** An extension to Deschamps's theorem for a class of 3-terminal bounded structures with one axis of symmetry is presented. For these structures, a simple relationship between the impedance matrix of the odd mode excitation of the original structure and the admittance matrix of the even mode excitation of its complement exists. Using this, a self-complementary grid amplifier is designed and the measured results are presented.

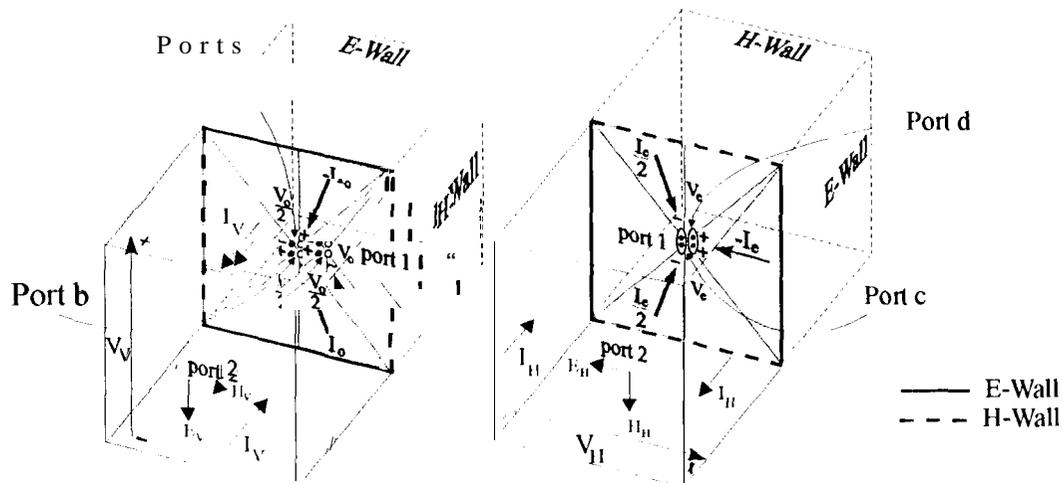
## MOTIVATION

Previous grid-array amplifiers have used crossed-dipole antennas for input and output [1, 2]. The current distributions on these dipole antennas do not permit a simple solution for the unit cell. Consequently, previous grid amplifiers have been modelled using an approximate transmission line equivalent circuit model [2]. The crossed-dipole construction has the additional drawback that it requires the use of differential pairs of transistors that must be specially fabricated for the application. Periodic arrays of self-complementary symmetrical structures offer a unit cell that can be more readily modelled. This configuration allows the use of a single transistor per unit cell while maintaining cross-polarized input and output.

## PROPERTIES OF SELF-COMPLEMENTARY STRUCTURES

In a 1959 paper Deschamps showed that the field solutions for the complement of a structure are equivalent to the field solutions for the dual of that structure. The complement of a structure is obtained by replacing the metal parts of it with free space and the apertures with metal. Using this property he presented equations relating the voltages and currents of a complementary structures to the voltages and currents of the original structure [3].

Here we present an extension to the Deschamps's theorem for a symmetrical, 3-terminal rectangular structure, bounded with electric and magnetic walls such as shown in Fig. 1. Adjacent walls are of opposite type. Fig. 1 is the waveguide representation of this bounded planar structure. The symmetry we refer to in this discussion is with respect to an axis parallel to either boundary. In the following discussion, an odd-mode excitation is defined as the mode where equal voltages with opposite polarity appear at ports a and b of the structure as shown in Fig. 1 (a). An even-mode excitation is defined as the mode where equal voltages with the same polarity appear at ports c and d of the structure as shown in Fig. 1 (b). Consider a source configuration at the terminals of the structure in Fig. 1 (a) that results in an odd-mode excitation. At low frequencies where the structure only supports a single TEM mode, this odd-mode excitation generates a vertically polarized electric field ( $E_V$ ) at the far-field. The complement of this structure is shown in Fig. 1 (b). According to Deschamps's theorem the far-field electric field is horizontally polarized ( $E_H$ ) and the



**Fig. 1.** Waveguide representation of a bounded symmetrical structure. (a) odd mode. Port 1 is the series combination of ports a and b, shown by two hollow dots. (b) Even mode. Port 1 is the parallel combination of ports c and d, shown by two ovals. Port 2 in (a) and (b) is the parallel combination of the two ports of the waveguide.

excitation generating it is an even-mode polarization shown in Fig. 1 (b). We define port 1 of Fig. 1(a) as the series combination of ports a and b, and port 1 in Fig. 1(b) as the parallel combination of ports c and d. Two hollow dots in Fig. 1 (a) and two ovals in Fig. 1 (b) show port 1. We will refer to port 1 of Fig. 1 (a) and 1(b) as the odd and even-mode device ports. The waveguides in Figs. 1(a) and 1(b) extend to both sides of the structures. However, due to reciprocity and to simplify the analysis of this 3-port structure these parallel ports are combined into one port. The new port (port 2) has half the characteristic impedance of free space. We will refer to the parallel combination of these two ports as the waveguide port.

For the ports 1 and 2 shown in Fig. 1(a) and 1(b) a simple equation relates the Z-parameter matrix of tile structure in Fig. 1(a) and the Y-parameter matrix of Fig. 1(b). This relation is derived in [4] and is shown below:

$$\frac{Z_{oij}}{Y_{eij}} = \eta_o^2/4 \quad i = 1, 2 ; j = 1, 2 \quad (1)$$

where  $\eta_o$  is the characteristic impedance of free space.  $Z_{oij}$  ( $i = 1, 2; j = 1, 2$ ) are the elements of the Z-matrix relating the odd-mode excitation of the device port to the vertically polarized field at the waveguide port (Fig. 1(a)) and  $Y_{eij}$  ( $i = 1, 2; j = 1, 2$ ) are the elements of the Y-matrix relating the even-mode excitation of the device port to the horizontally polarized field at the waveguide port (Fig. 1(b)).

The results stated above apply to any symmetrical structure with previously described boundaries and its complement. In the case of a self-complementary structure, there are additional simplification. A self-complementary structure is one which looks the same when the metal part is exchanged with the non-metal part. For a self-complementary structures this means that the odd-mode Z-parameters are related to the even-mode Y-parameters of the same structure by (1).

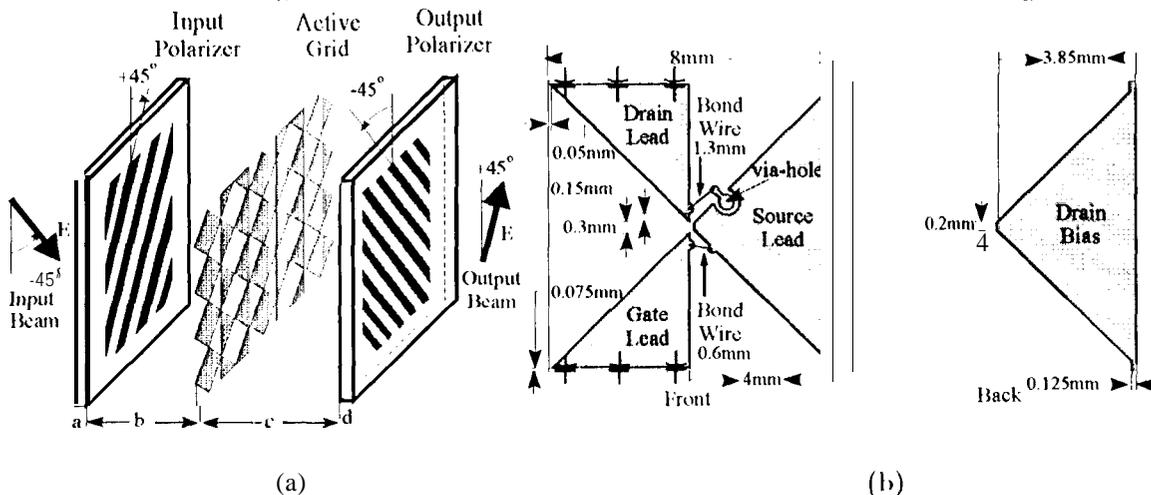
## SELF-COMPLEMENTARY GRID AMPLIFIERS

Consider the self-complementary grid amplifier configuration shown in Fig. 2. To make the active grid a self-complementary structure we assume that the grid is suspended in free space.

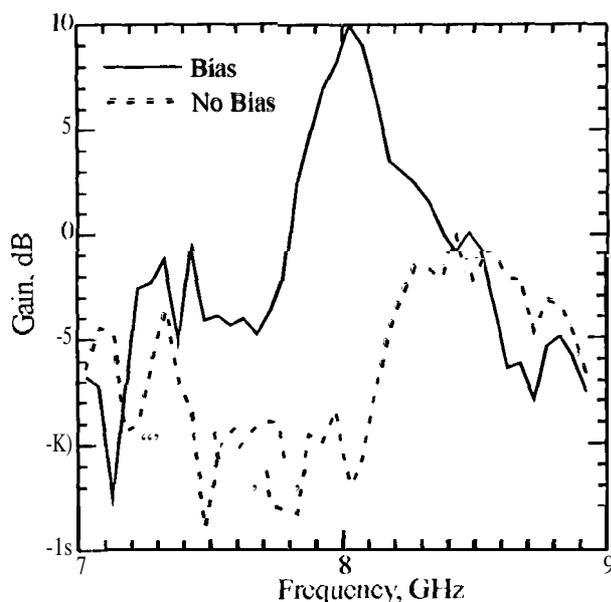
In practice this is achieved by constructing the grid on a very thin (10-mil-thick) substrate with a low dielectric constant (relative dielectric constant of 2.2). To keep the input and output signals orthogonal the polarizer metal strips are  $\pm 45^\circ$  from the vertical. The input signal that is incident from the left enters the input polarizer normal to its strips (Fig. 2(a)) and generates RF currents on the input gate leads of the grid (Fig. 2(b)). Fig. 2(b) shows the unit cell of the grid amplifier. Currents on the output drain leads generate an output signal that passes through the output polarizer. Similar to the conventional grid amplifiers, the polarizers provide tuning of the amplifier's input and output circuits. To provide bias to the transistors a double-sided design is used. The source leads in each column are connected to each other and biased from the front of the grid. A wire bond connects each drain lead to a via-hole that is connected to the back side of the grid. The drain bias leads in one column are connected to each other and biased at the back side of the grid. Each gate is shorted to the source through a bond wire. To DC isolate the gate of one cell from the drain of the cell below, a 0.15-mm-slot is used to separate the two. Three 20-pF chip capacitors are used to make an RF connection between the gate and drain of neighboring cells. The transistors are Fujitsu FLK012XP MESFET's. Bond wires at the gate and the drain of the FET are used to resonate with the input and output capacitance of the FET and present the appropriate load for the unit cell.

The solution for the self-complementary grid amplifier is divided into solving for the odd and even-mode excitations. CAD analysis of the unit cell is simpler for the odd-mode excitation [4]. Hewlett-Packard's High Frequency Structure Simulator (HFSS) [5] is used to derive the Z-parameters of the odd-mode excitation of the self-complementary unit cell [4]. The Y-parameters of the even-mode excitation are derived using (1) and the Z-parameters of the odd-mode. The equivalent circuit model of the grid is derived by combining these odd and even-mode Z and Y-parameters and a transmission-line equivalent circuit model [4].

The gain of the grid is shown in Fig. 3. This gain is measured for a drain voltage of 1.85 V and a total drain current of 850 mA. The measured peak gain is 10 dB at 8 GHz. At peak gain the difference between the biased and unbiased gain is over 20 dB. For these bias conditions a linear dependence between the input and the output power indicates that the grid is stable. The 3-dB bandwidth of this amplifier is 210 MHz. This narrow bandwidth is due to the inductors used at the gate and the drain of the transistor in each unit cell. The grid suffered



**Fig. 2.** (a) Perspective view of a self-complementary grid amplifier. (b) Front and back side of the grid amplifier unit cell. The drain diamonds are behind the source diamonds.



**Fig. 3.** Measured gain of the amplifier versus frequency.

from random oscillations at higher drain voltages. The oscillation frequency and power were a function of bias voltage and time and covered a range of frequencies around 8.5 GHz.

### CONCLUSIONS

The properties of periodic arrays of self-complementary structures are used to design a self-complementary grid amplifier. The amplifier has a gain of 10 dB at 8 GHz. The stability of the amplifier is a problem and more work will be needed to fix it.

### ACKNOWLEDGEMENTS

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