# AN OVERVIEW OF GPS OCCULTATIONS FOCUSING ON RETRIEVING WATER VAPOR

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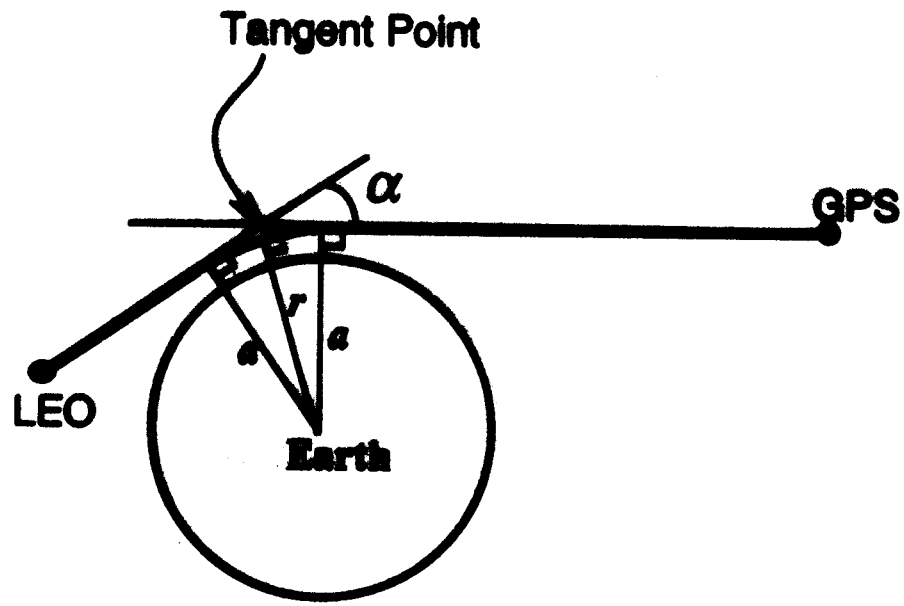
# OCCULTATION GEOMETRY AND THE ABEL TRANSFORM
## LEO OBSERVING A GPS SATELLITE




- Assuming spherical symmetry:

Forward propagation

Abel inversion

$$\alpha = -2a\int_{r_0}^{\infty} \frac{d\ln(n)/dr}{\sqrt{r^2n^2-a^2}}dr \quad \Rightarrow \quad \ln(n(r)) = \frac{1}{\pi}\int_{r}^{\infty} \frac{\alpha}{\sqrt{a^2-r^2n^2}}da$$

# OBTAINING GEOPHYSICAL VARIABLES FROM REFRACTIVITY

$$N = (n-1)\times 10^6 = \underbrace{77.6\frac{P}{T}}_{\text{Dry}} + \underbrace{3.73\times 10^5\frac{P_W}{T^2}}_{\text{Moist}} - \underbrace{40.3\times 10^6\frac{n_e}{f^2}}_{\text{Ionosphere}}$$

+ higher order ionospheric terms

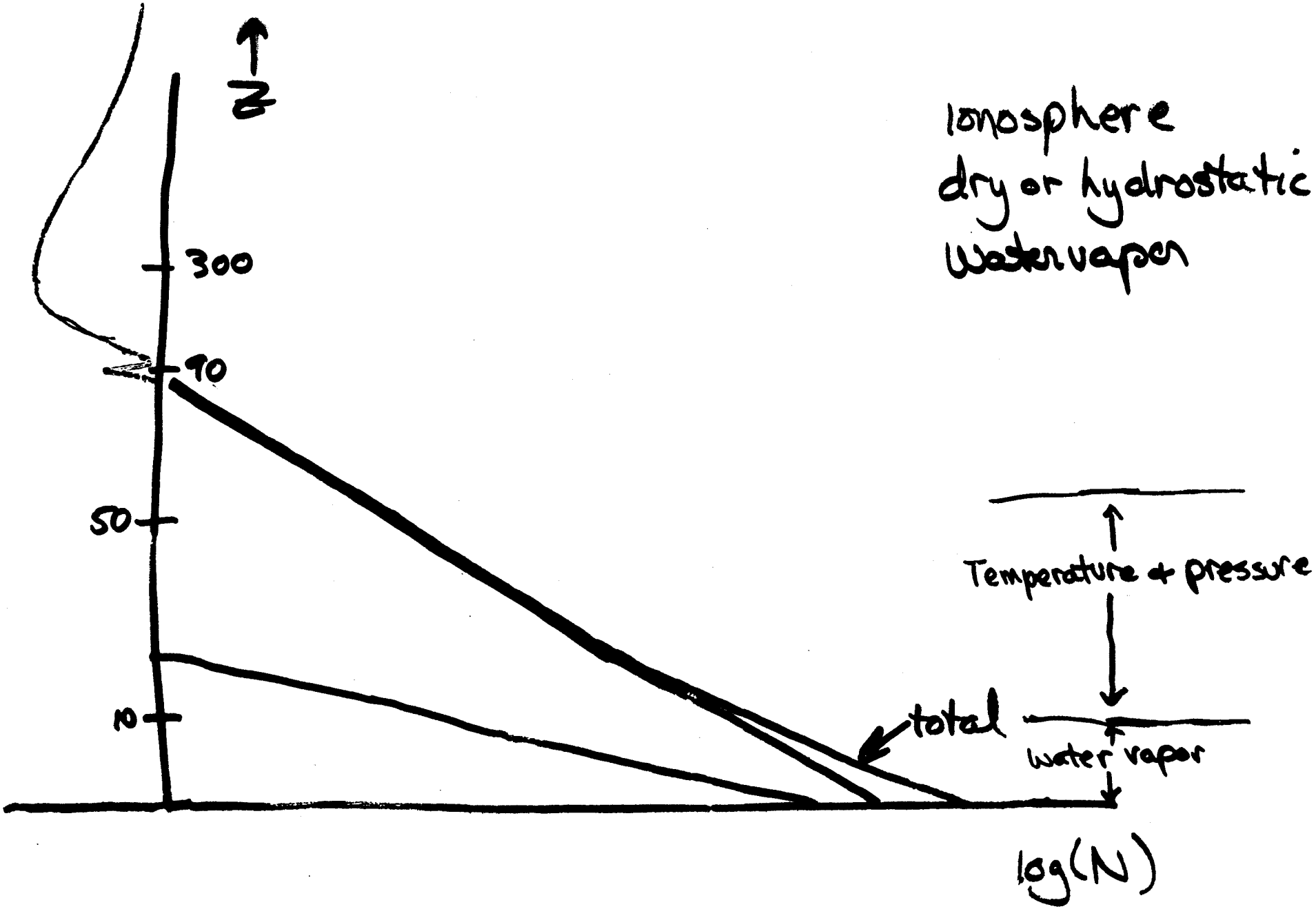
- Equation of state

$$\rho = 0.3484\frac{P}{T}$$

- Hydrostatic equilibrium equation

$$\frac{\partial P}{\partial h} = -g\rho$$

$n$ = index of refraction
$N$ = refractivity
$P$ = pressure
$T$ = temperature
$P_W$ = water vapor pressure
$n_e$ = electron density
$f$ = operating frequency
$\rho$ = density
$h$ = height
$g$ = gravitational acceleration


z
300
90
50
10
ionosphere
dry or hydrostatic
water vapor
Temperature & pressure
total
water vapor
log(N)

# Key GPS Features

GPS constellation provides 24 occultation signal source at 20,000 km altitude and ~55° inclination

24 GPS sources plus a single GPS receiver in low Earth orbit yield 400 to 700 globally distributed, daily occultations

Small, simple, inexpensive and low power receiver instrumentation

Density of occultations increased by placing additional receivers (and transmitters) in orbit

# Remote Sensing Characteristics

Simple technique

Very accurate measurement of atmospheric delay (1 part in $10^6$ near surface)

Low optical depth and insensitivity to condensed water at ~20 cm wavelengths

High vertical resolution of Fresnel diffraction (~1 km)

=> **Accurate, routine limb sounding of troposphere**

Observations provide very accurate temperature and geopotential estimates in stratosphere and cold troposphere (T<250 K)

Observations should yield accurate estimates of water vapor in the warmer regions of the mid- to lower troposphere

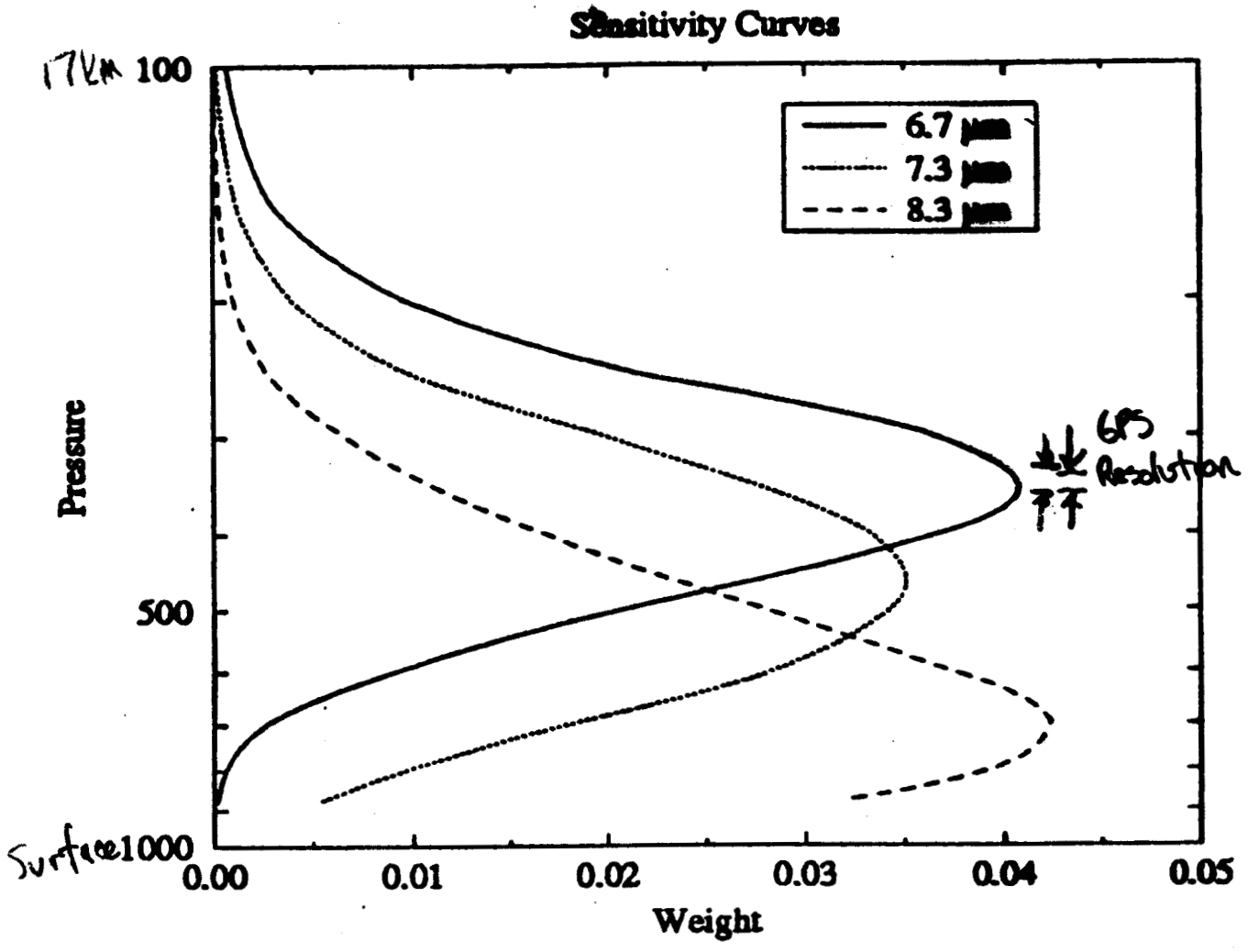



**Figure 1.** Sensitivity of $T_{6.7}$ (solid), $T_{7.3}$ (dotted), and $T_{8.3}$ (dashed) to local perturbations in relative humidity in thin layers equally spaced in the logarithm of pressure ($\Delta \ln p = dp/p \cong 0.04$). The curves are normalized such that the sum of weights over pressure is equal to unity.

corresponding clear-sky radiosonde climatology containing the identical sampling restrictions present in the satellite data. This study suggested that the restriction to clear and partially cloudy conditions introduces a slight dry bias into the TOVS climatology, typically about 4% and usually less than 10% when expressed in terms of the relative humidity. We note, however, that these results were heavily weighted toward the

humid
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sensiti
temper

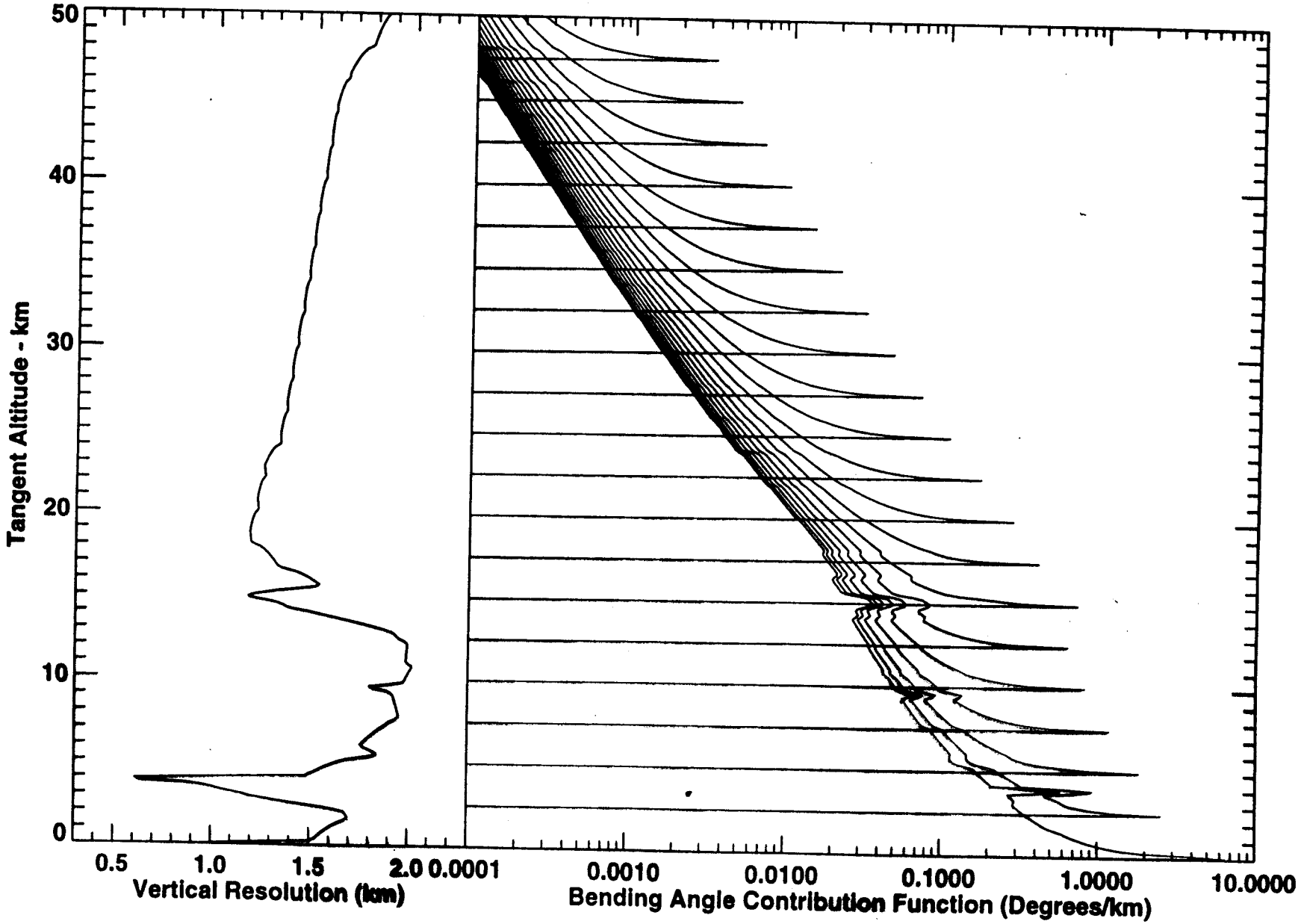
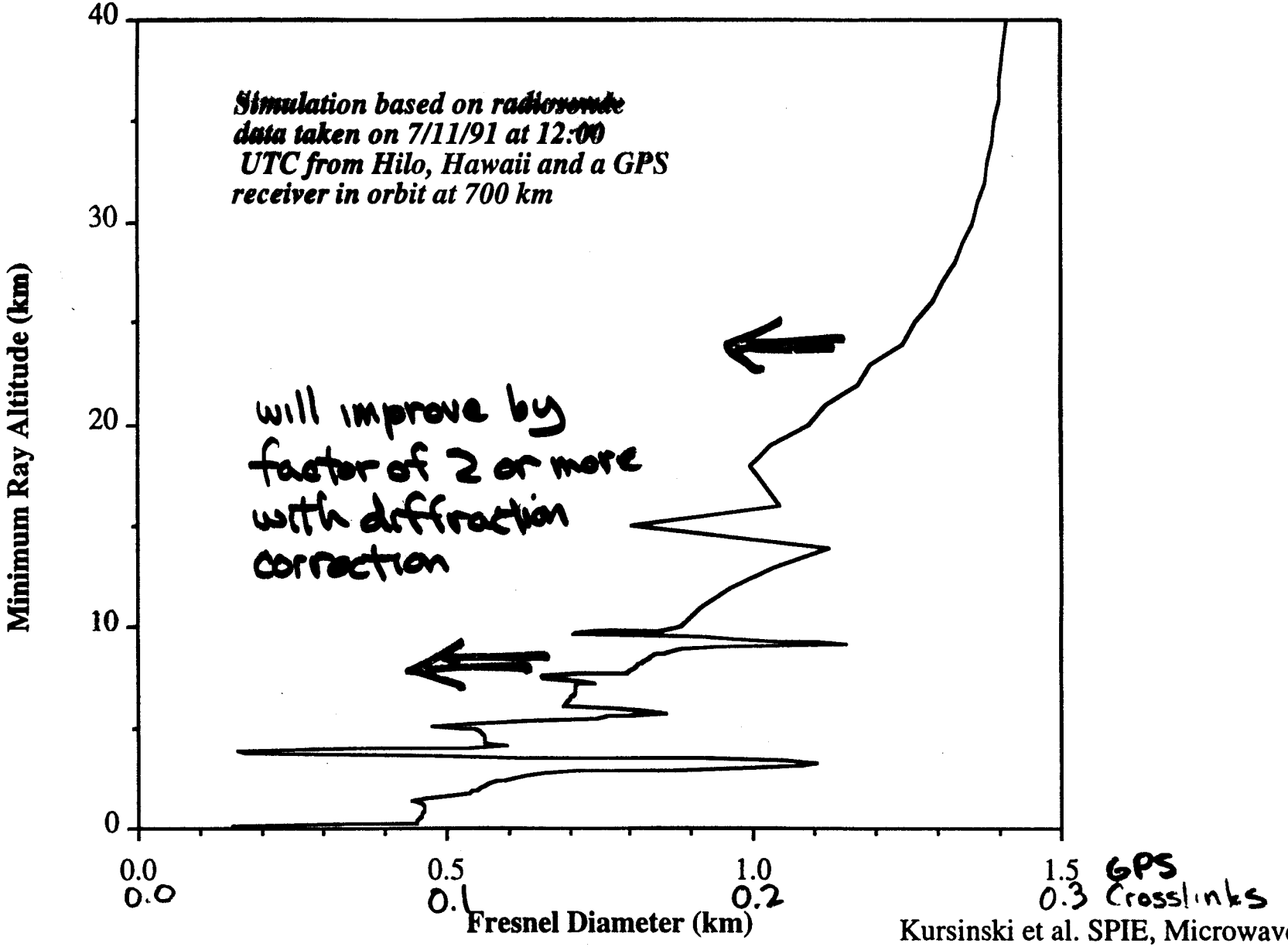

Tangent Altitude - km
50
40
30
20
10
0
0.5
1.0
1.5
2.0
Vertical Resolution (km)
0.0001
0.0010
0.0100
0.1000
1.0000
10.0000
Bending Angle Contribution Function (Degrees/km)

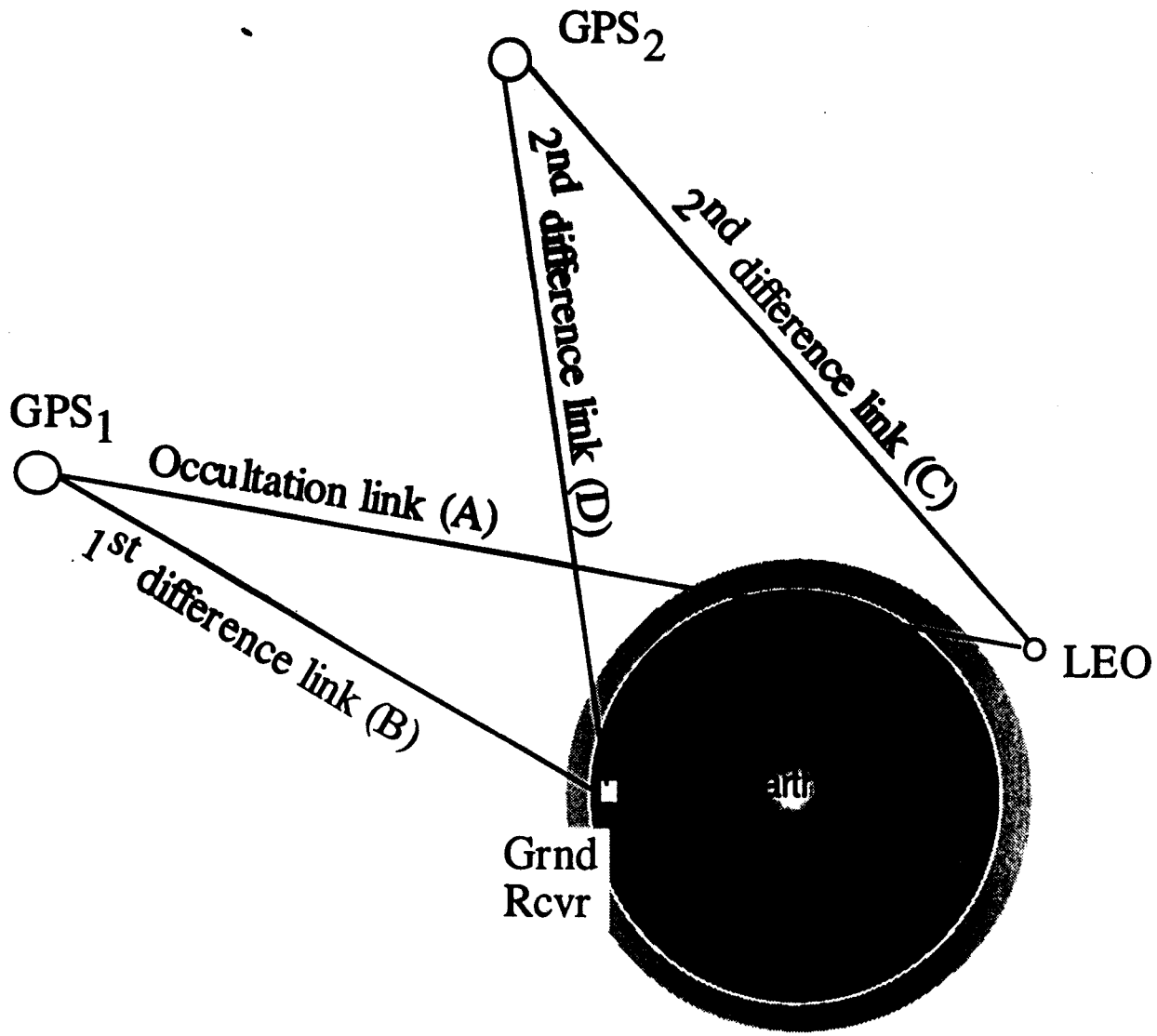
Figure 8

## Vertical Resolution
## (Fresnel Zone Diameter)




Kursinski et al. SPIE, Microwave Instrumentation for Remote Sensing of the Earth, April 1993

# Ground Network: Geometry for calibrating clock errors



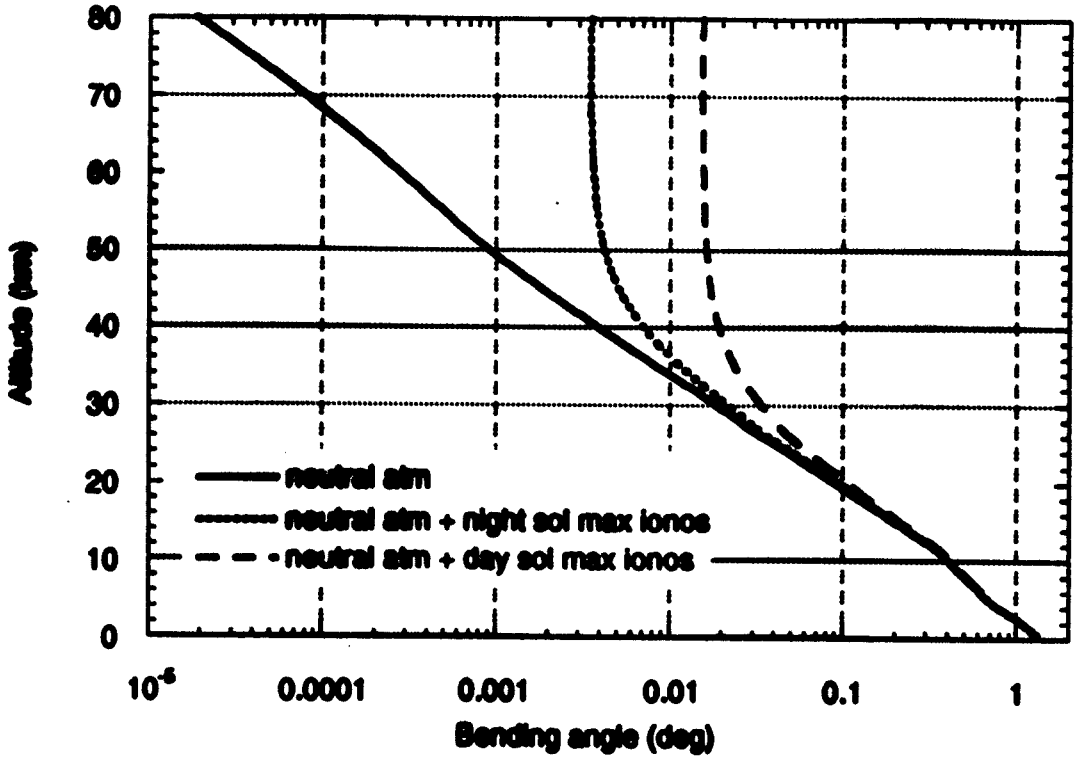



**Figure 2.6.** Atmospheric and ionospheric bending for limb ray paths plotted as a function of ray path tangent height. Solid line: bending calculated for U.S. Standard Atmosphere (Champion et al., 1985). Long dashed line: bending calculated for U.S. standard atmosphere plus typical mid-latitude ionosphere during daytime, solar maximum conditions. Short dashed line: bending calculated for U.S. standard atmosphere plus typical mid-latitude ionosphere during nighttime, solar maximum conditions.

# Ionosphere Calibration

GPS has two signal frequencies to calibrate the dispersive ionospheric refractivity, $\sim 1/f^2$, where f is the radio signal frequency.

## Upward Looking Receiver:

- The ionosphere free path length is $L = c_i L_1 - (c_i - 1) L_2$

  where L is the optical path length along a signal path defined as $L_{opt} = \int_{path} n\, dl = \int_{path} (1 + N)\, dl$
  $L_1$ and $L_2$ are the optical path lengths at the two GPS signal frequencies, 1.57542 and 1.2276 GHz respectively
  the coefficient, $c_i = \left(1 - f_1^2/f_2^2\right)^{-1}$ equals 2.5457...

## Occultation Receiver:

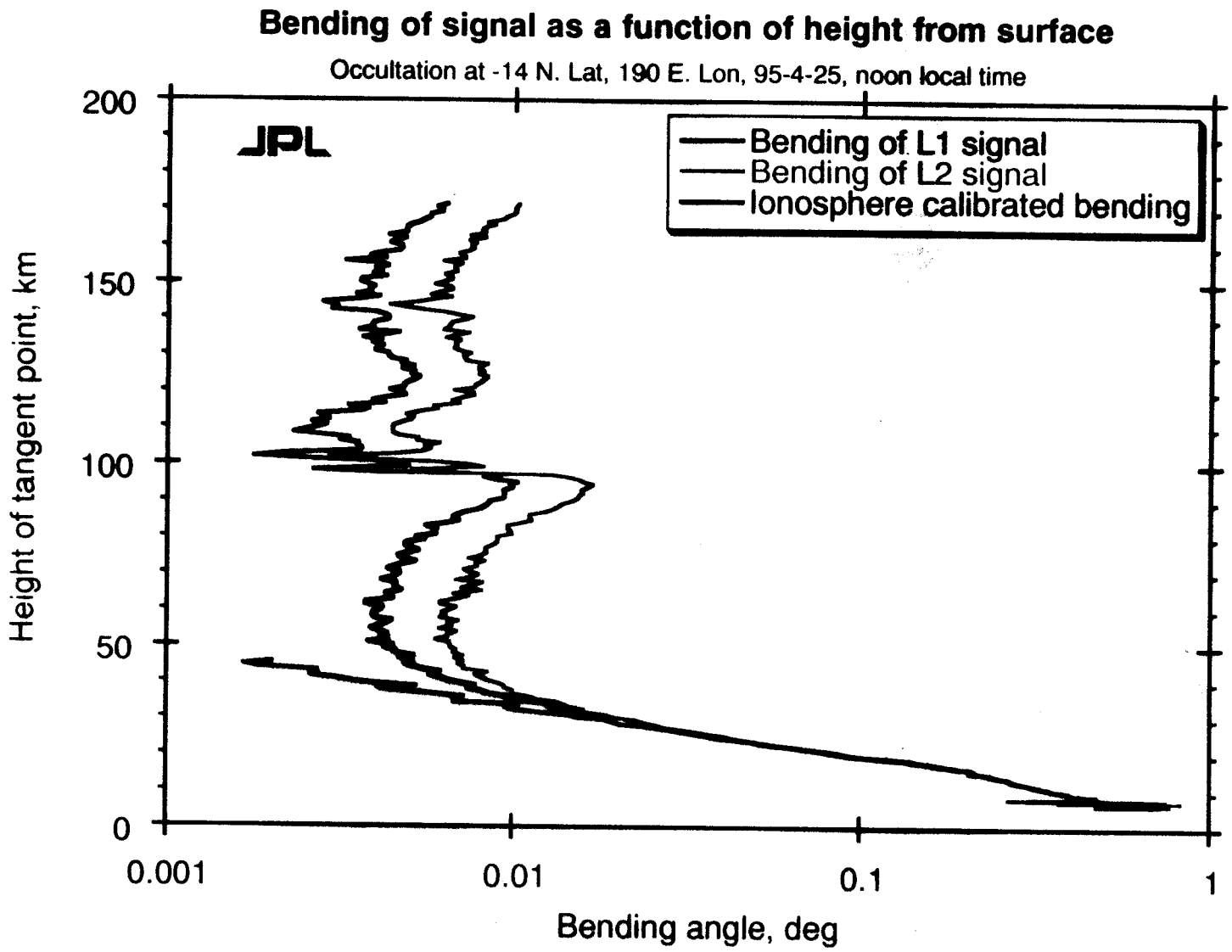
- The ionosphere-free bending is $\alpha_{neutral}(a) = c_i\, \alpha_1(a) - \left(c_i - 1\right) \alpha_2(a)$ (1)

  where $\alpha_1$ and $\alpha_2$ are the total bending angles for the two GPS signal frequencies.

- In the occultation geometry, the dispersive refractivity causes the two GPS signal paths to separate.
- An solution developed by Vorob'ev and Krasil'nikova [1993] based on separating the charge-neutral atmosphere and ionospheric bending angle contributions (assuming a spherically symmetric index of refraction) :

$$\alpha(a) = 2\,a \int_{r_0}^{\infty} \frac{\frac{dn_{neut}}{dr}\, dr}{n_{neut}\sqrt{\left(n_{neut} r\right)^2 - a^2}} + 2\,a\,\frac{40.3}{f^2} \int_{r_0}^{\infty} \frac{\frac{dn_e}{dr}\, dr}{n_{neut}\sqrt{\left(n_{neut} r\right)^2 - a^2}}$$

- Application of (1) requires interpolation from the discrete set of $\alpha_2(a_2)$ to the set of $a_1$ values.


Bending of signal as a function of height from surface
Occultation at -14 N. Lat, 190 E. Lon, 95-4-25, noon local time
JPL
Bending of L1 signal
Bending of L2 signal
Ionosphere calibrated bending
Height of tangent point, km
200
150
100
50
0
0.001
0.01
0.1
1
Bending angle, deg

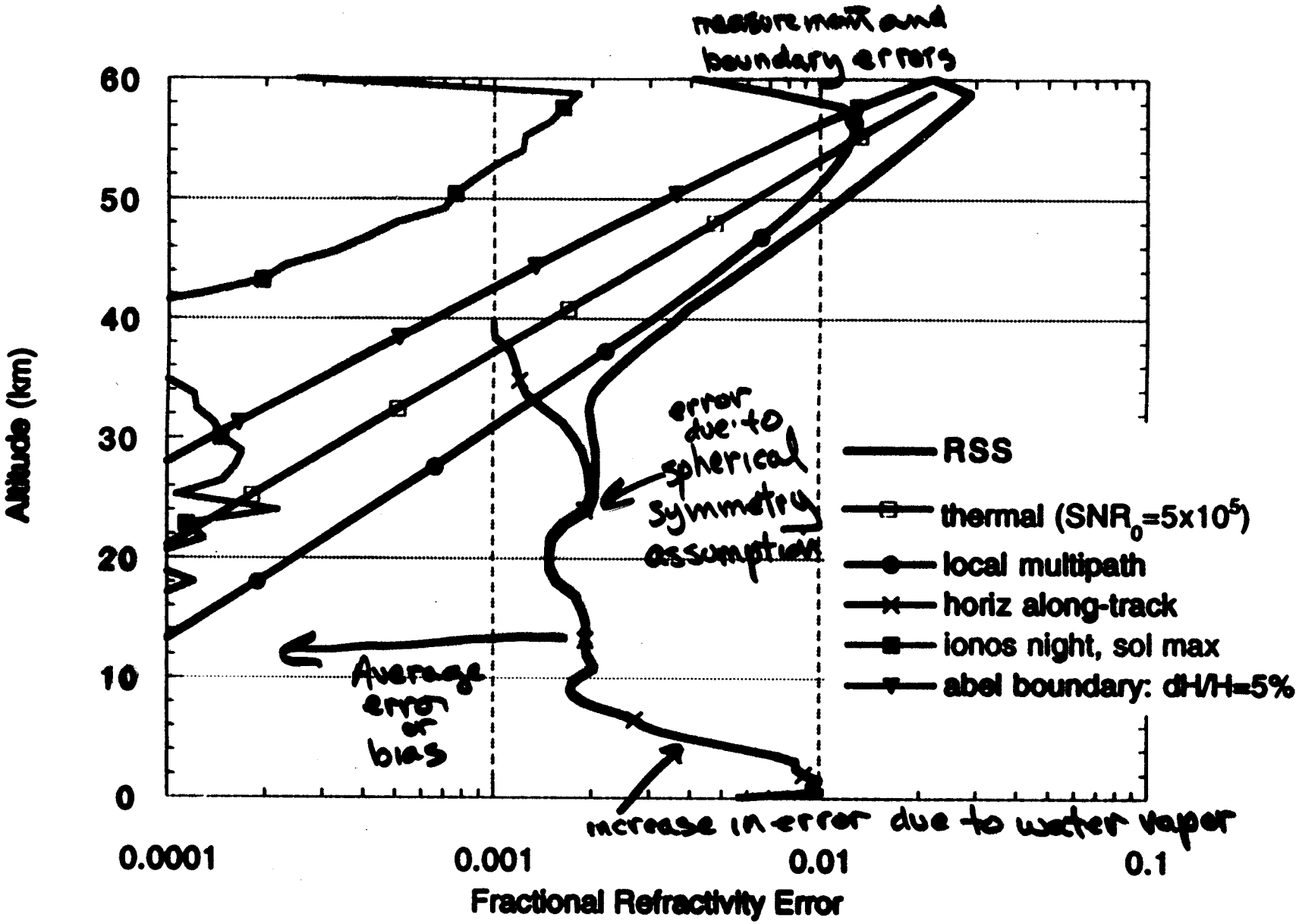



Figure 18

Kursinski et al., 1996-7
Observing Earth's atmosphere

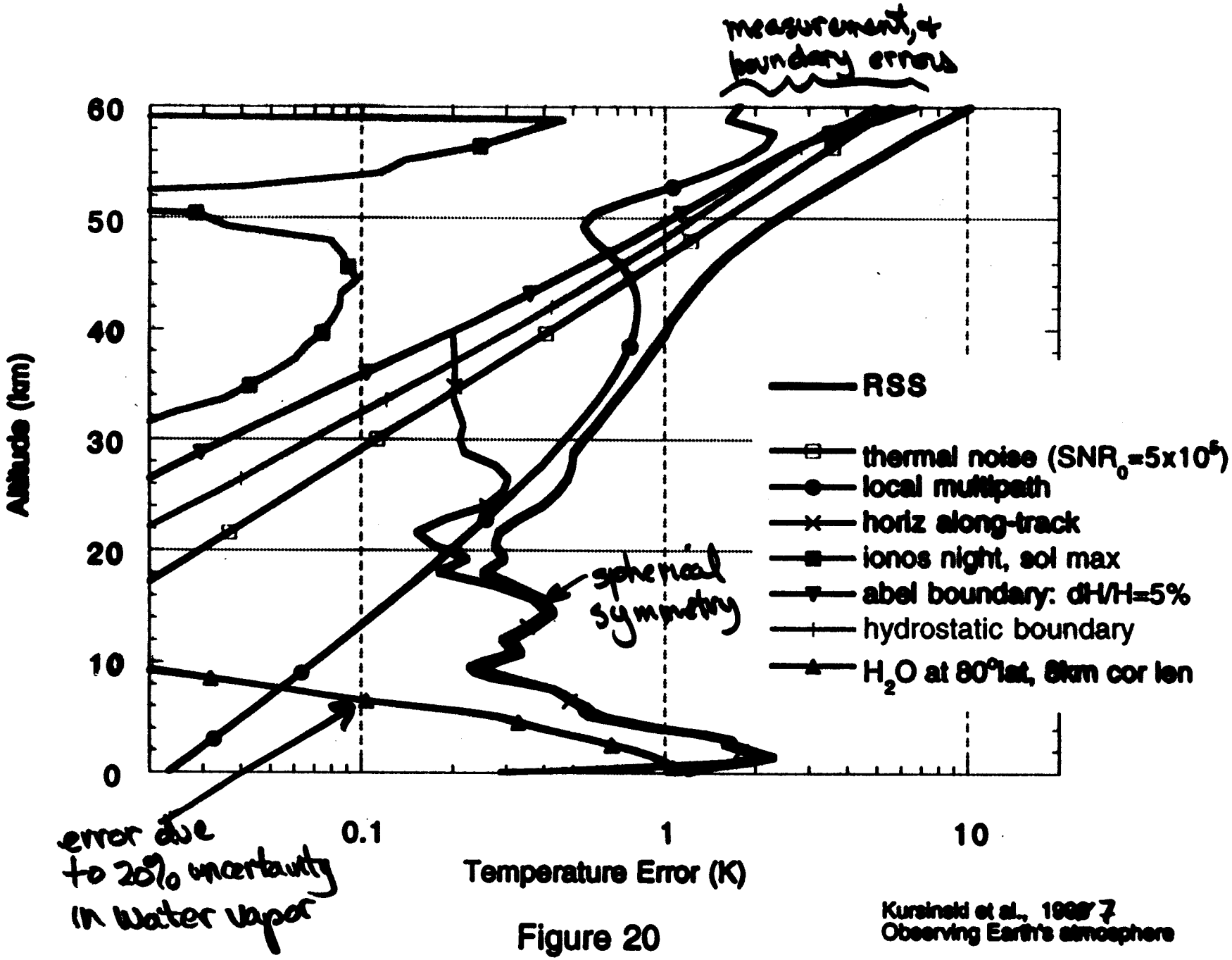



Figure 20

Kursinski et al., 1997
Observing Earth's atmosphere

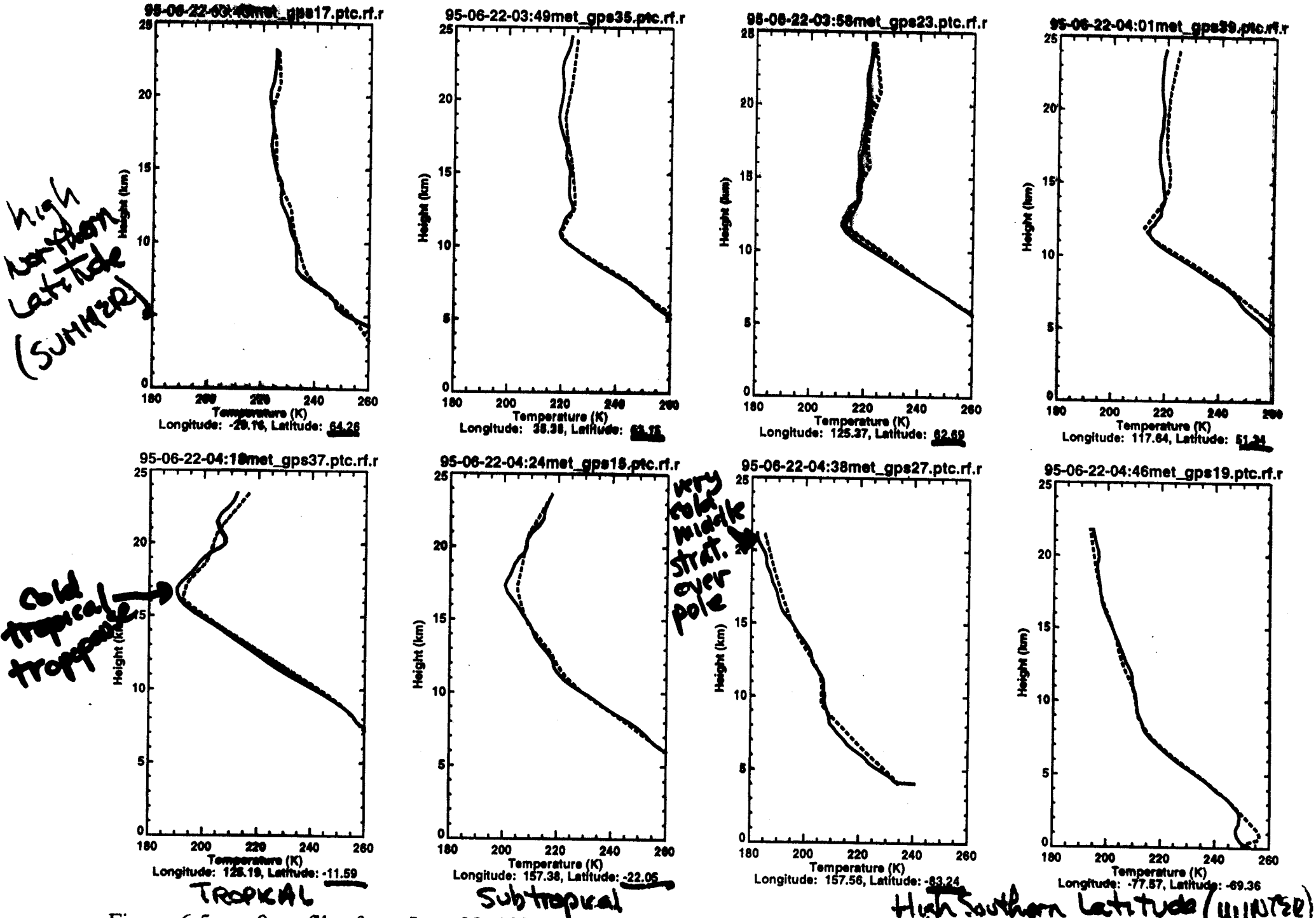



Figure 6.5 8 profiles from June 22, 1995 demonstrating the range of temperature structure observed by GPS-MET Solid lines are occultation profiles. Dashed lines are interpolated from ECMWF analyses

EQUATORIAL WAVES

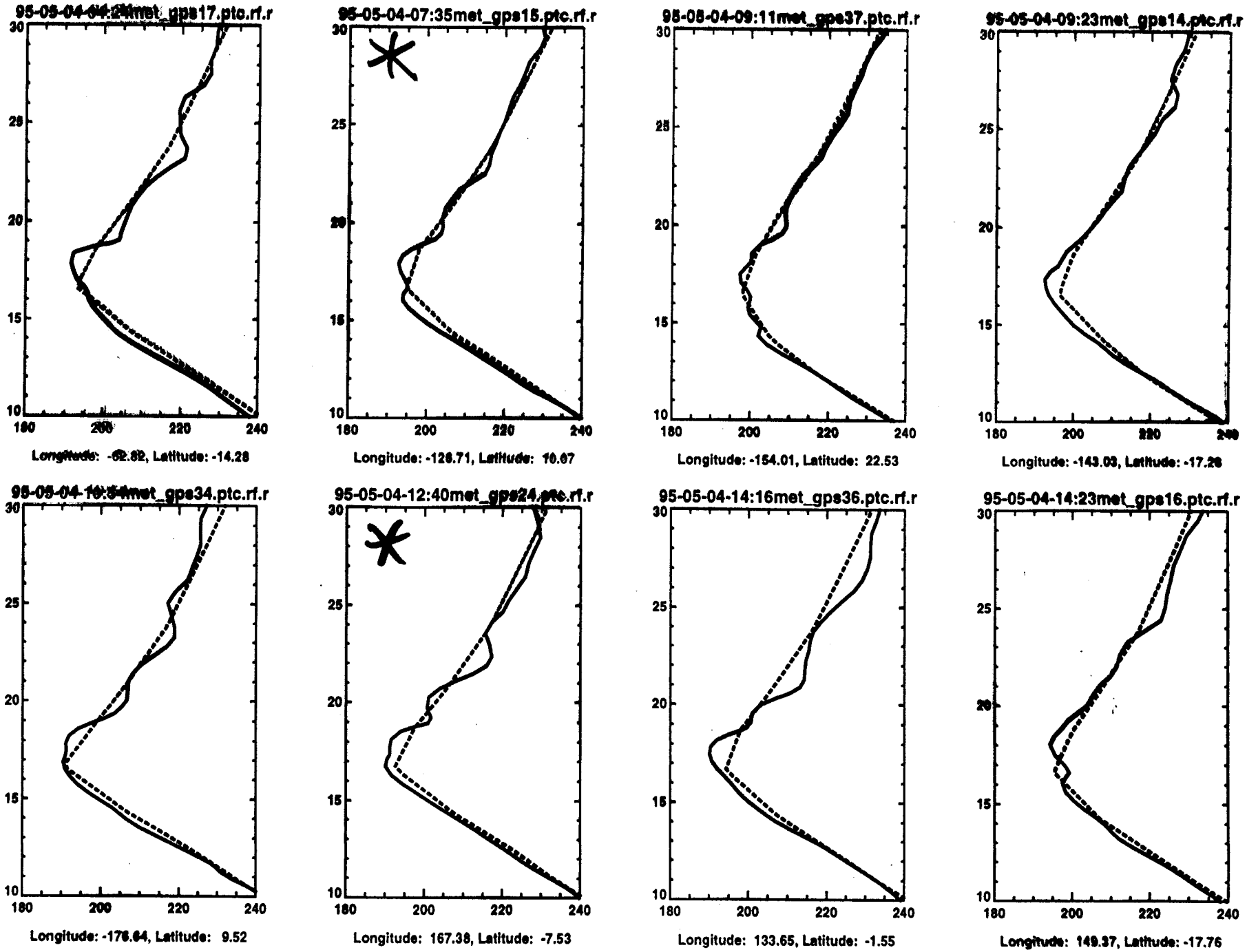
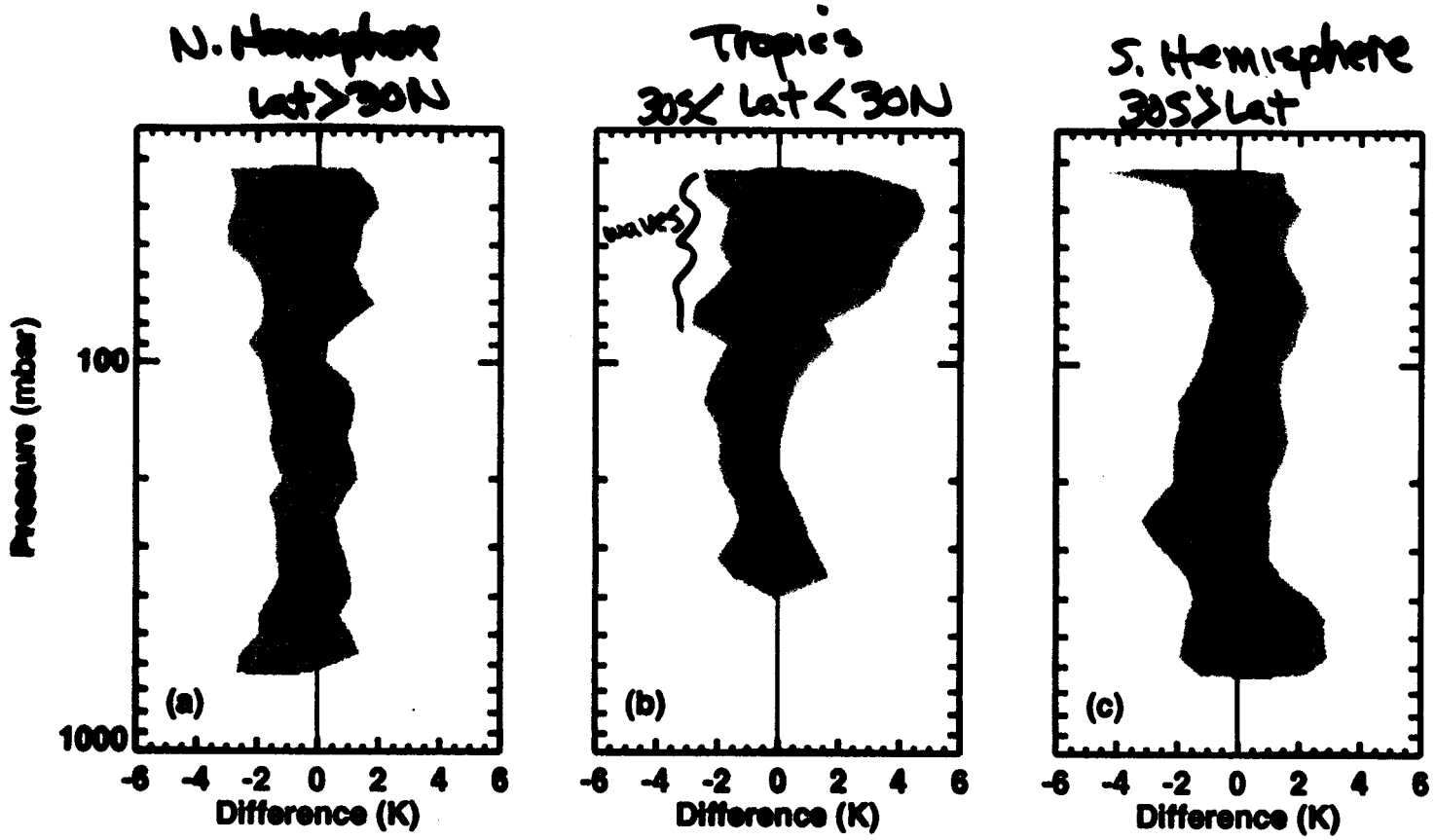



Figure 6.6 Examples of low latitude temperature profiles. Solid lines of occultation profiles. Dashed lines are interpolated from daily NMC stratosphere analyses.

TEMPERATURE DIFFERENCES
APR.-MAY 1995 OCC - ECMWF GLOBAL ANALYSIS




KURSINSKI et al.
SCIENCE, 271, 1107 (1996)

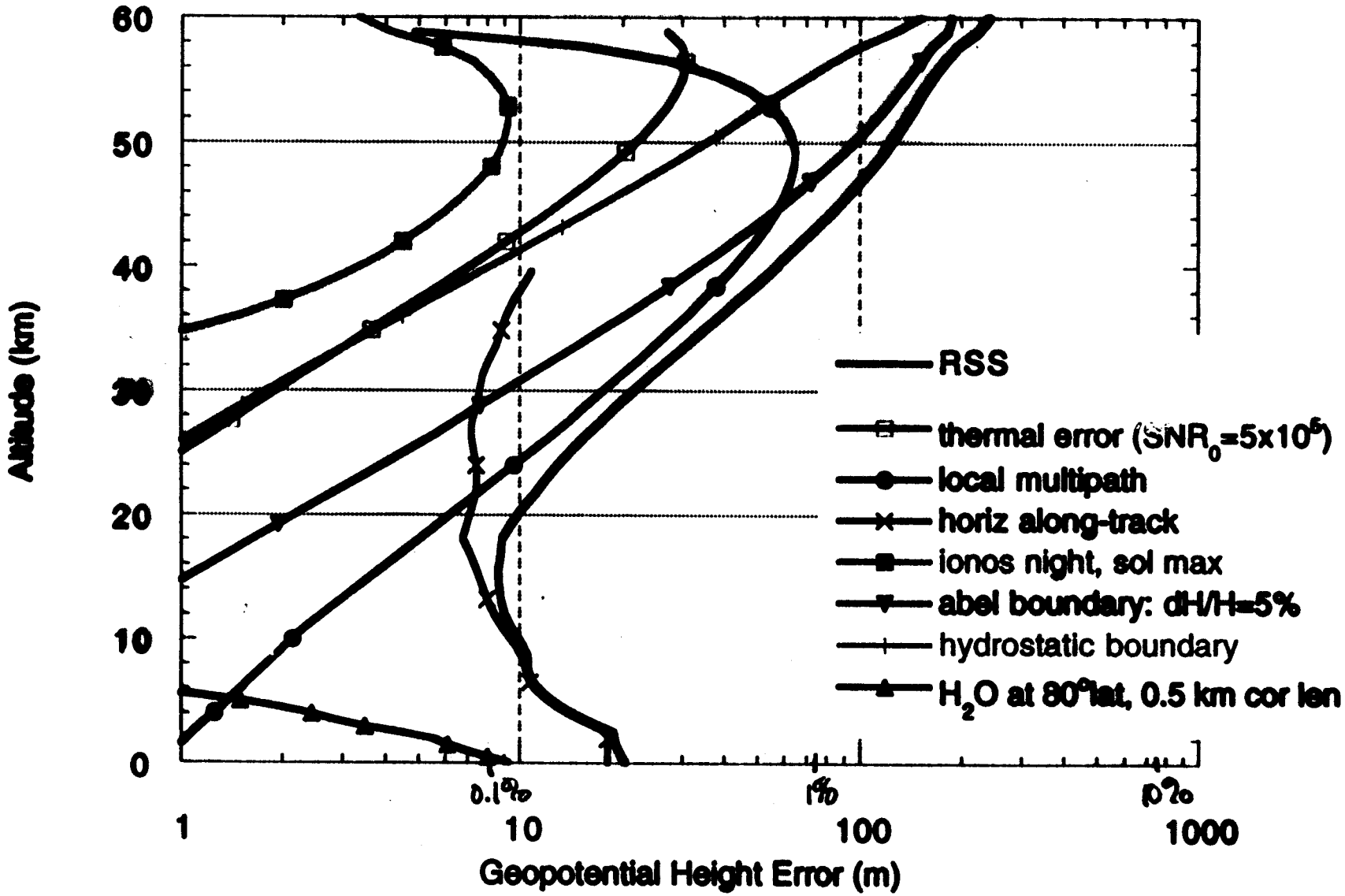
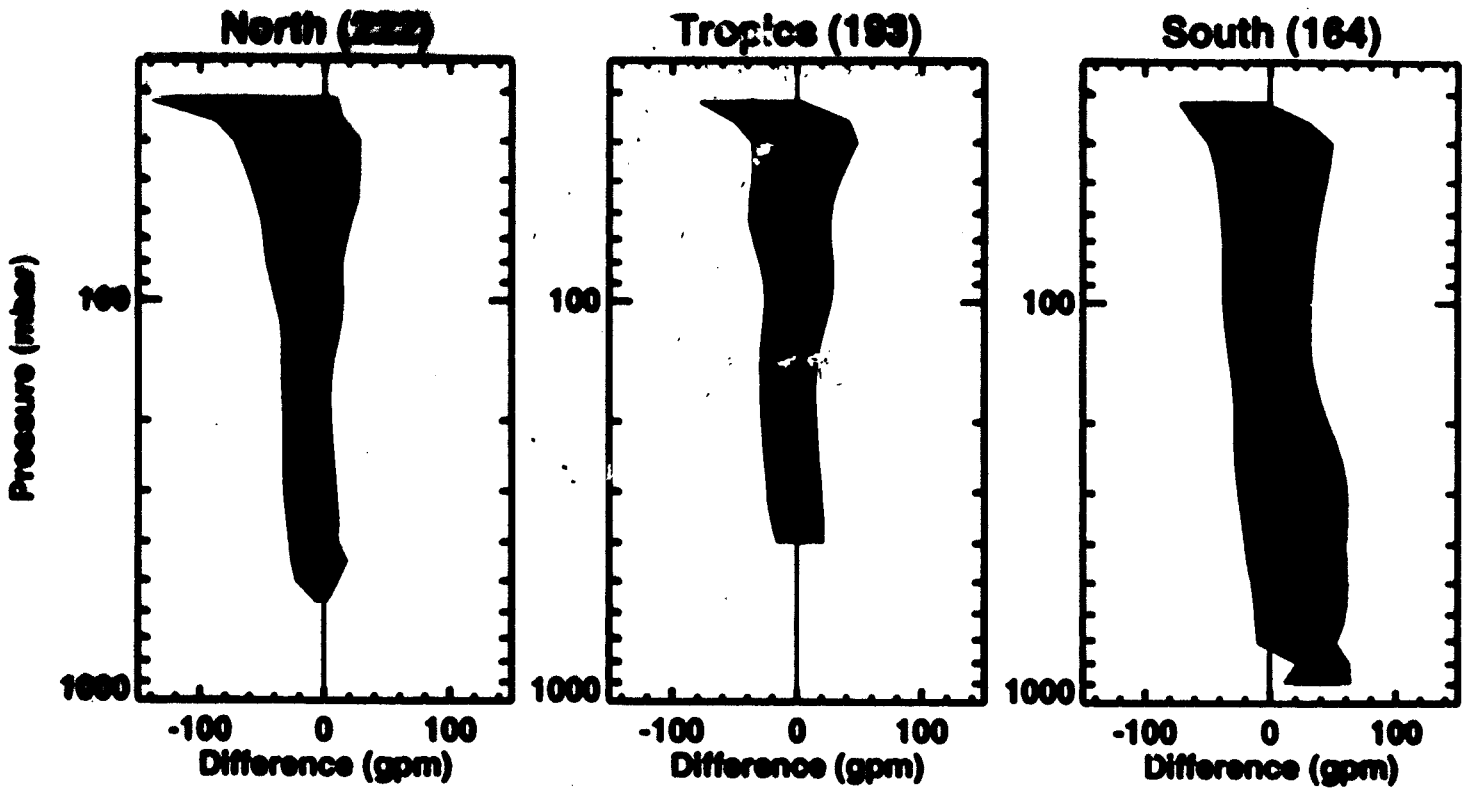



Figure 19

Kursinski et al., 1996
Observing Earth's atmosphere

# Comparison of GPS/MET Retrievals to ECMWF

## Summer AS-off Period




JPL: Kursinski, Hajj, Bertiger, Leroy, Romans, Schofield, et al. (12 March 1996)

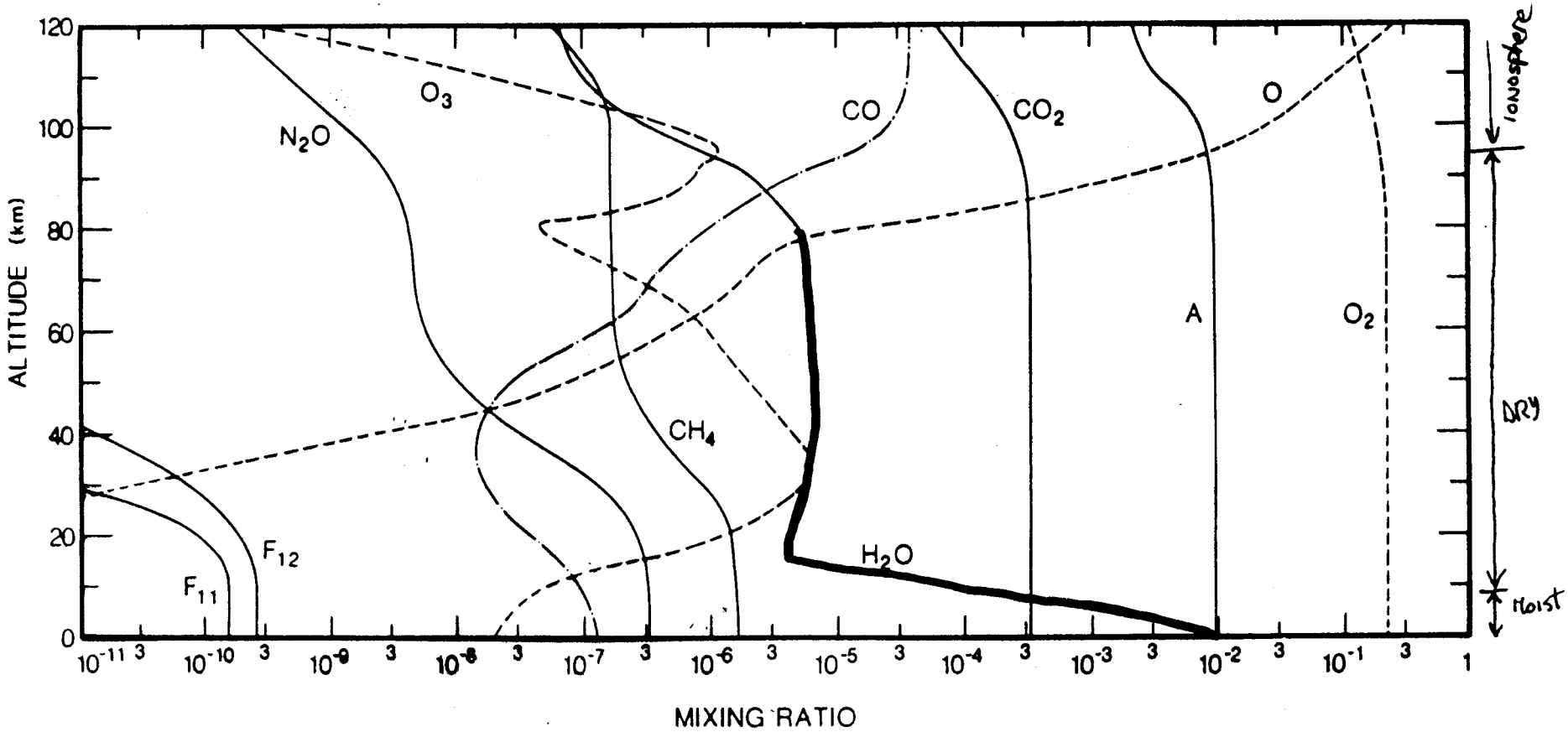



**FIG. 1.5.** Vertical profiles of mixing ratio of selected species at equinox. After Allen et al. (1981, 1984).

# Estimated Accuracy of Specific Humidity

NOT RELATIVE HUMIDITY

- Specific humidity (water vapor mass mixing ratio): $$q = \left[\frac{m_d}{m_w}\left(\frac{P}{P_w} - 1\right) + 1\right]^{-1}$$

- Changes in $q$, pressure $P$, temperature $T$ and retrieved refractivity $N$ are related:

$$dq = (C+q)\frac{dN}{N} + (C+2q)\frac{dT}{T} - (C+q)\frac{dP}{P}$$

where C is 77.6 T $m_w$ / 3.73e5 $m_d$ [C = 32 g/kg at 230 K, = 39 g/kg at 300 K]

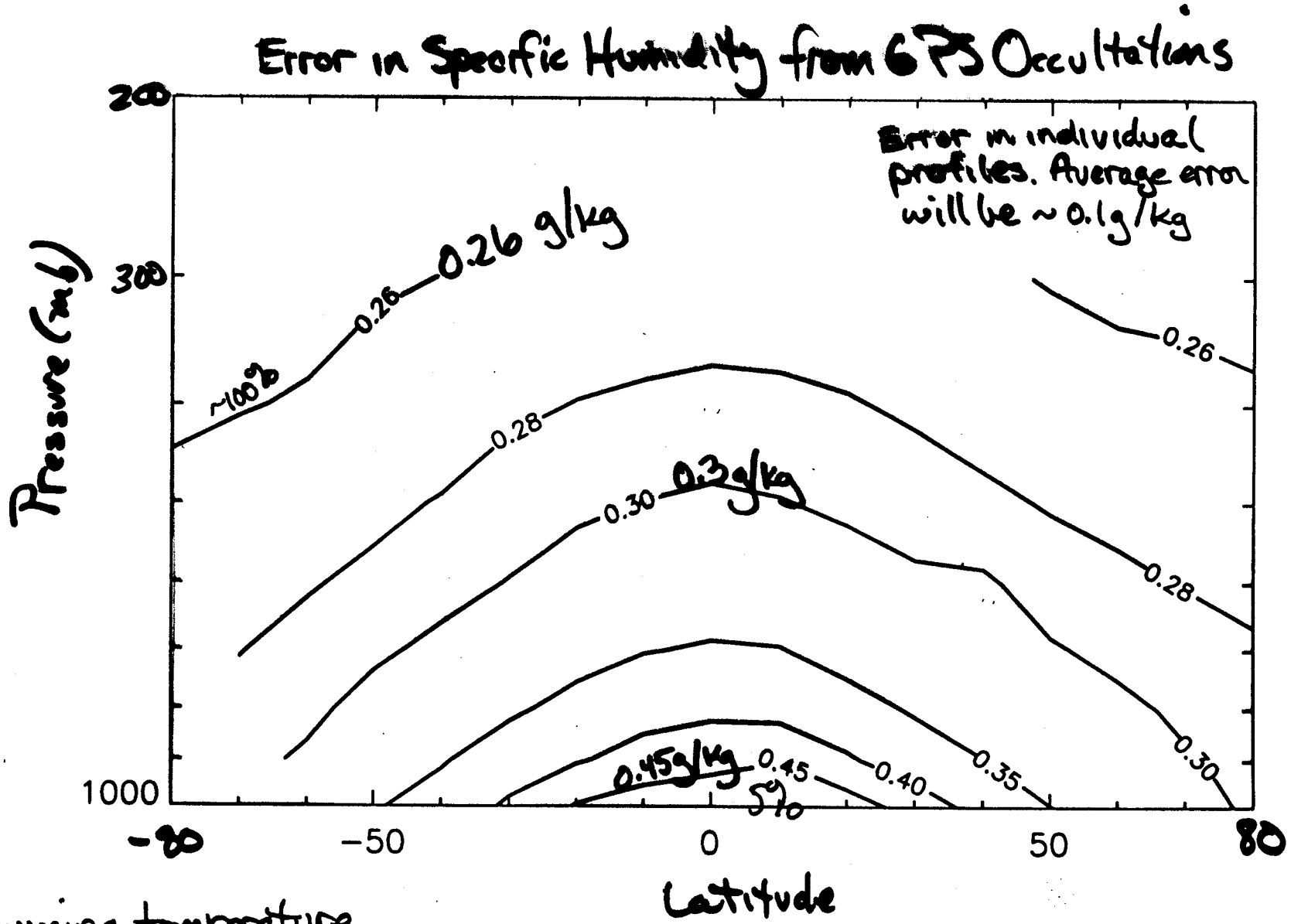
- **Assuming independent** errors in $N$, $T$ and the surface pressure, $P_S$, yields

$$\sigma_q = \left[(C+q)^2\left(\frac{\sigma_N}{N}\right)^2 + (C+2q)^2\left(\frac{\sigma_T}{T}\right)^2 + (C+q)^2\left(\frac{\sigma_{P_s}}{P_s}\right)^2\right]^{1/2}$$

- RMS error in $q$ varies little ranging from 0.2 to 0.5 g/kg
  - Higher altitude $q$ error dominated by the $T$ error (~1.5K)
  - Near surface $q$ error has largest contribution by $N$ error (~1%)

- **Mean** specific humidity, $\bar{q}$, accuracy is limited by **mean** $T$ error

($\varepsilon_T$ ~ 0.5 K => $\varepsilon_q$ ~ 0.1 g/kg )



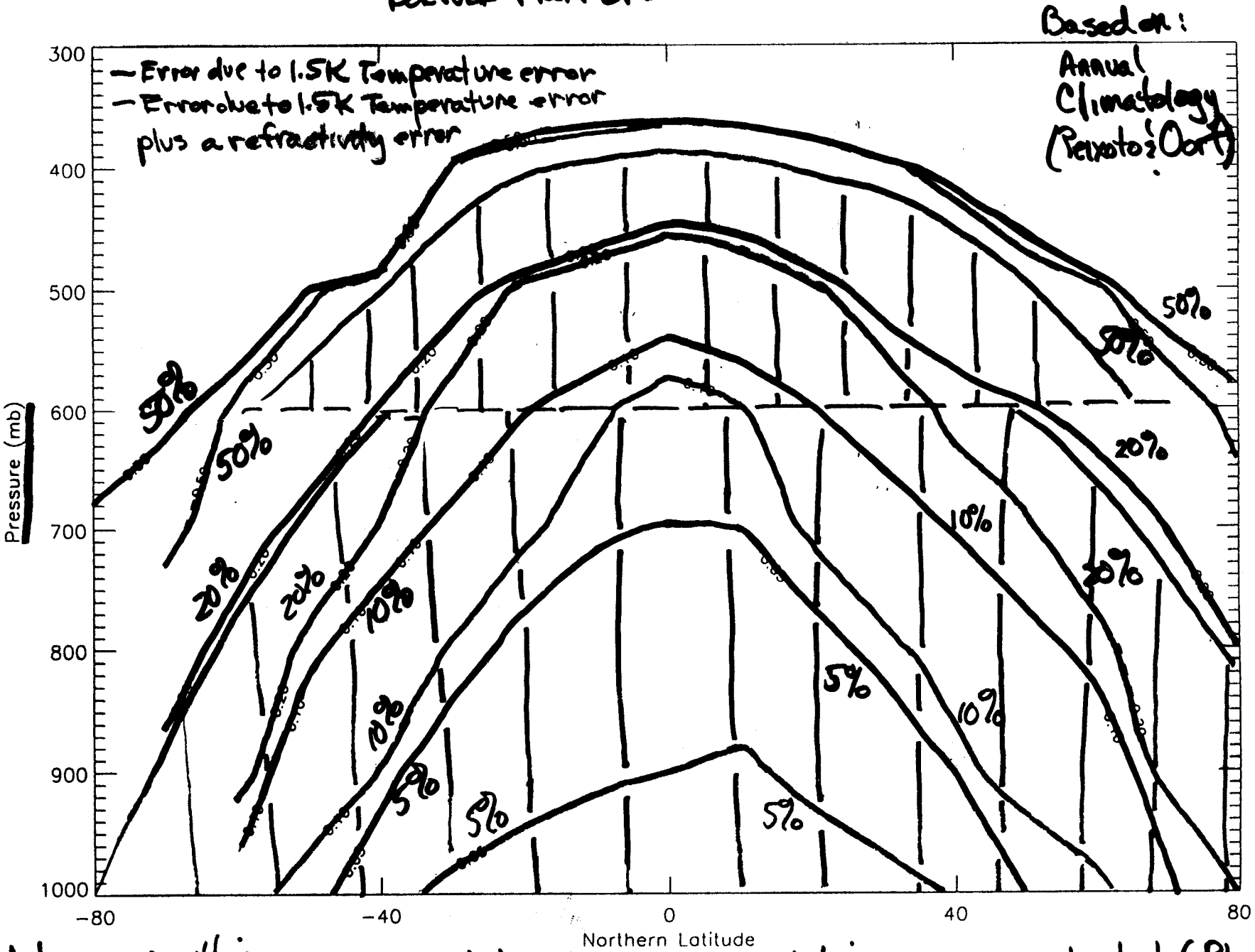

Assuming temperature is known to ~1.5K

Refractivity error of 0.2% at 300mb & above
1% near surface

Figure 0

Kursinski & Hajj
An Examination of Water Vapor...
JGR 5/99

# EXPECTED ACCURACY OF WATER VAPOR DERIVED FROM GPS OCCULTATIONS

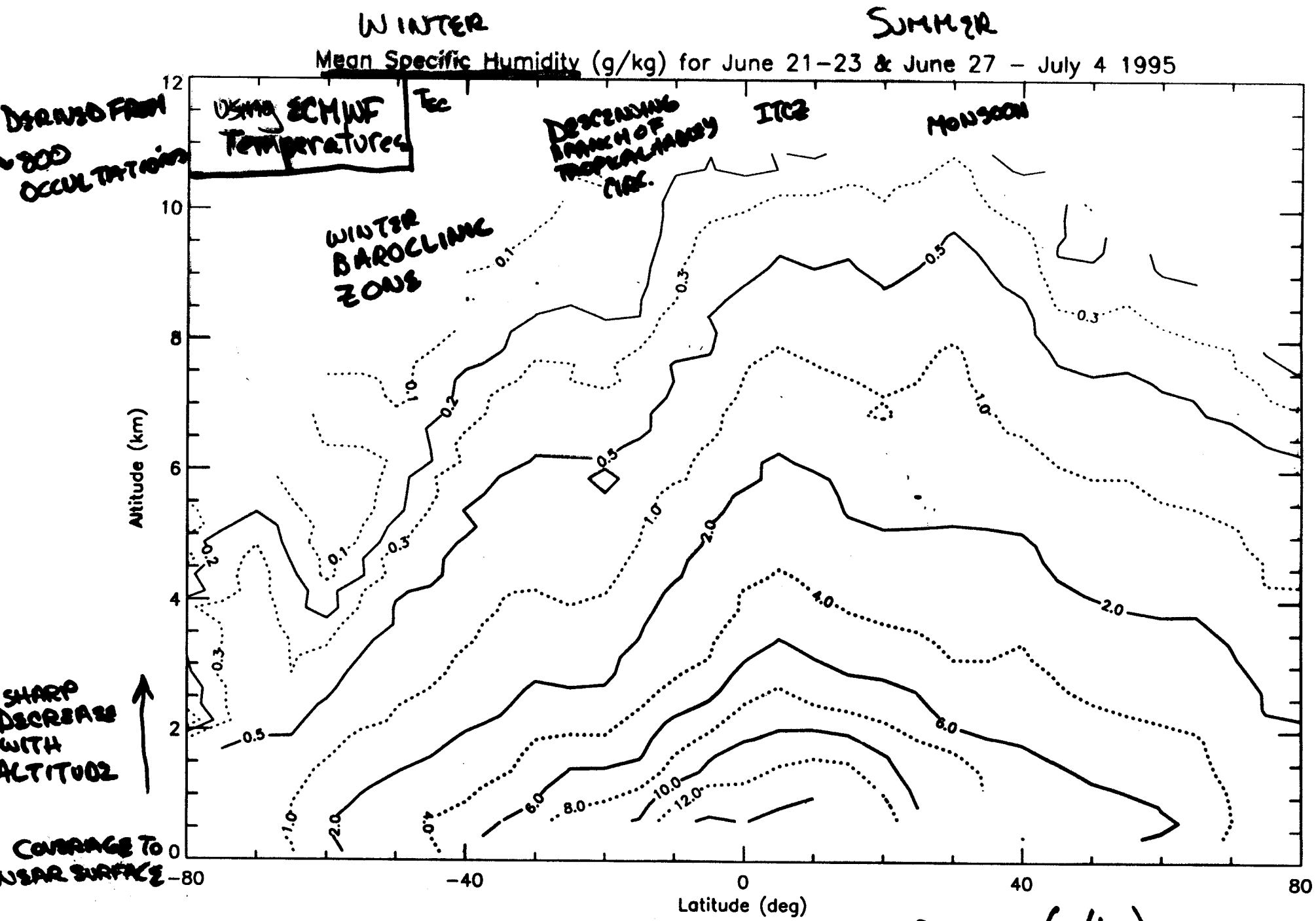



red shaded region is within NPOESS specs.

— divide by ~3 to get representative estimate of climatological

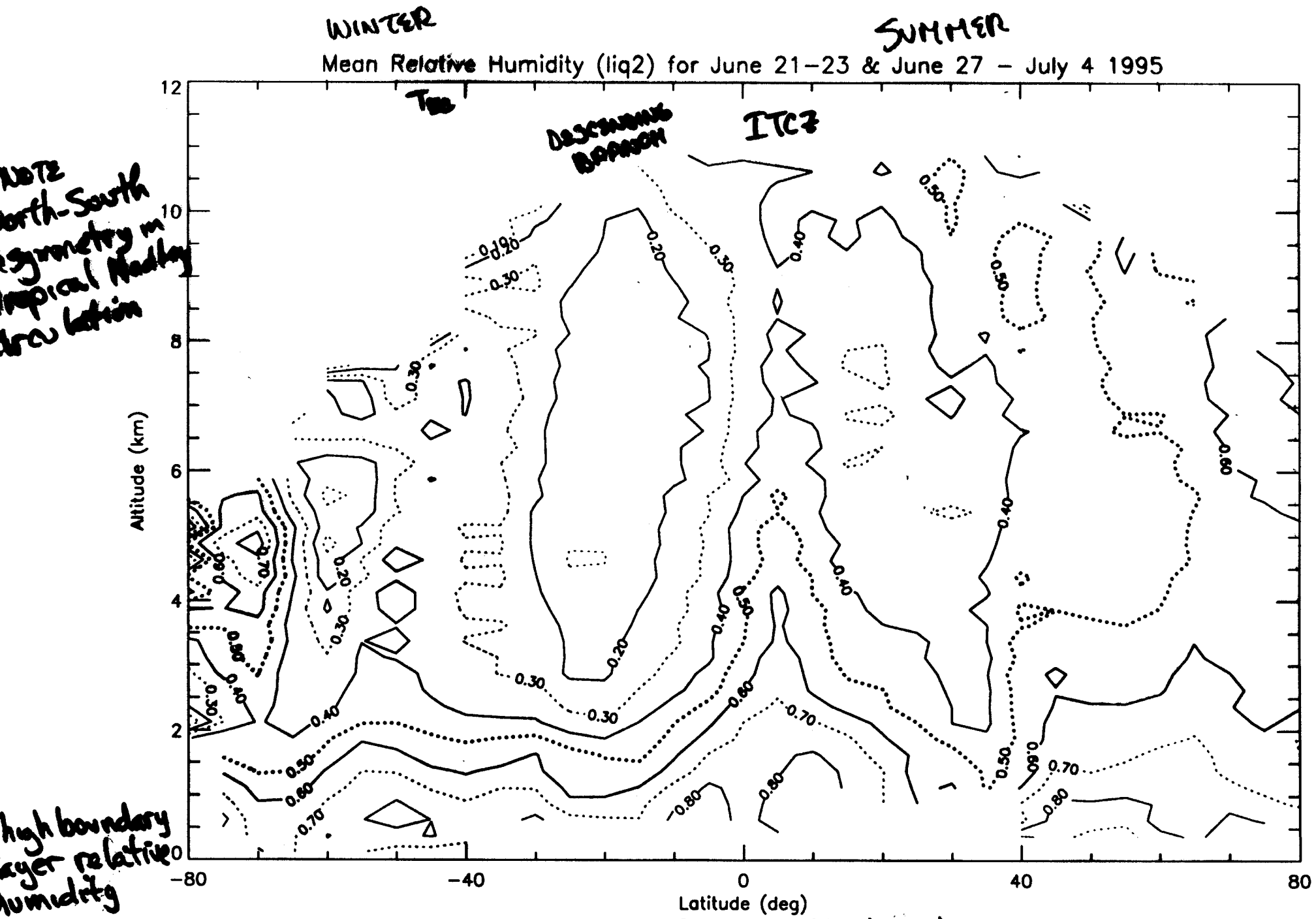
Kursinski et al. GRL 1995

ERK 12/96




MEAN WATER VAPOR MIXING RATIO (g/kg)
DERIVED FROM GPS REFRACTIVITY & ECMWF TEMPERATURES

KUROINSKI & HAJJ 1999

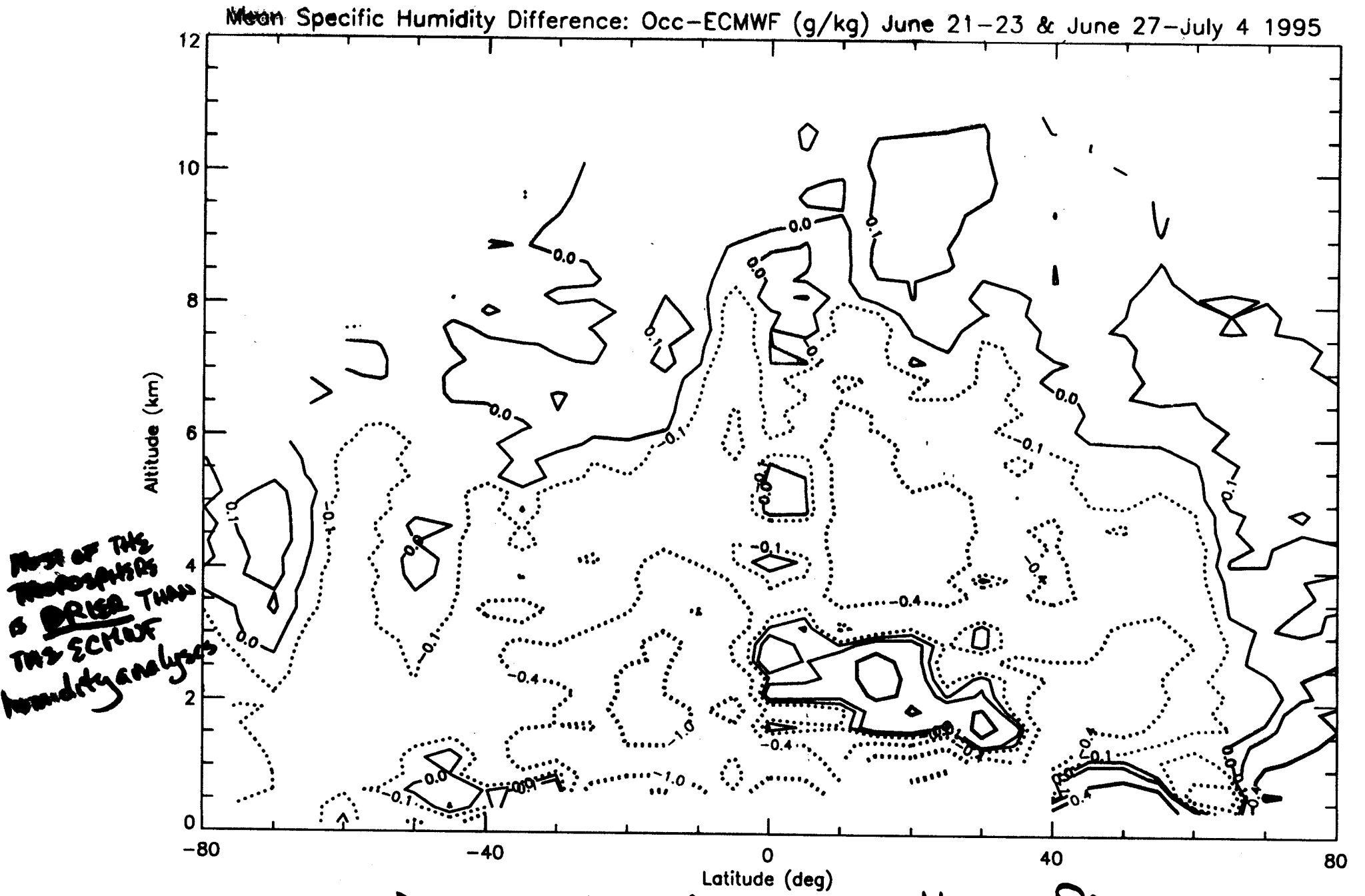



NOTE
North-South
asymmetry in
tropical Hadley
circulation

high boundary
layer relative
humidity

ZONAL MEAN RELATIVE HUMIDITY
(over liquid)

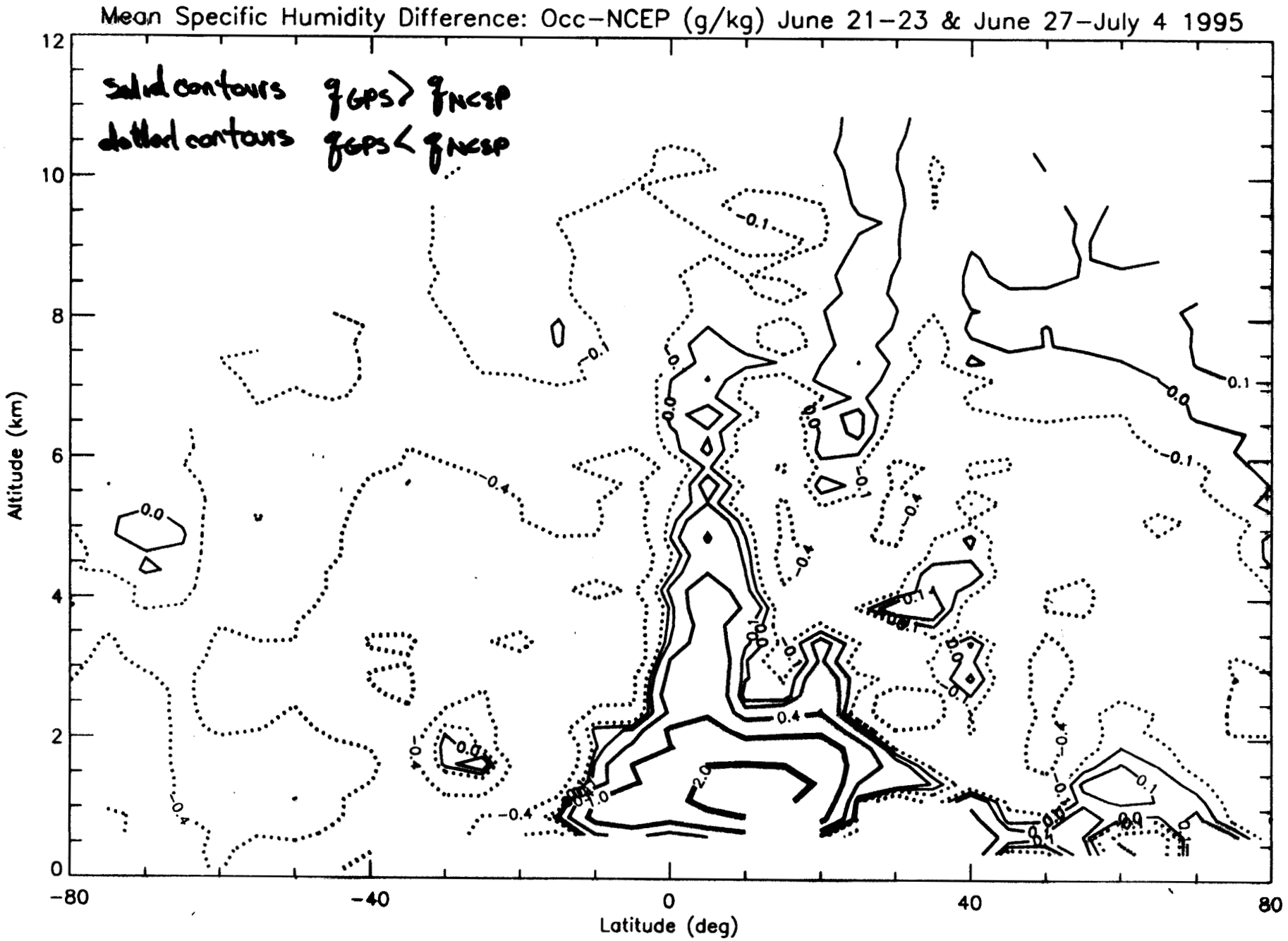
Kursinski & Hajj
1999




MUCH OF THE TROPOSPHERE IS DRIER THAN THE ECMWF humidity analysis

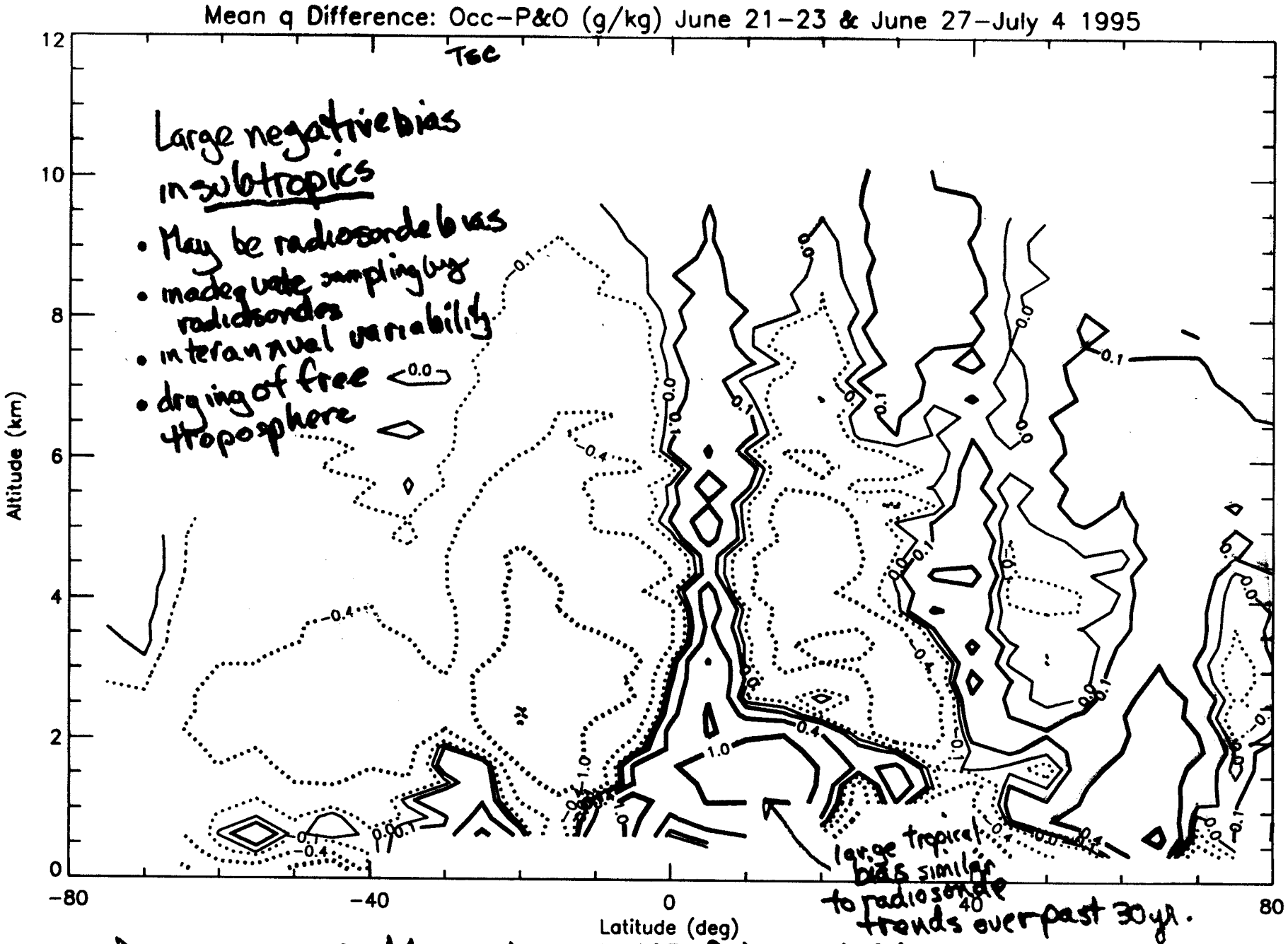
DIFFERENCE OF WATER VAPOR MIXING RATIO
[GPS − ECMWF]
g/kg

KUOSINSKI & HAJJ
5/99




DIFFERENCE OF WATER VAPOR MIXING RATIO (g/kg)
[occ − NCEP REANALYSES]

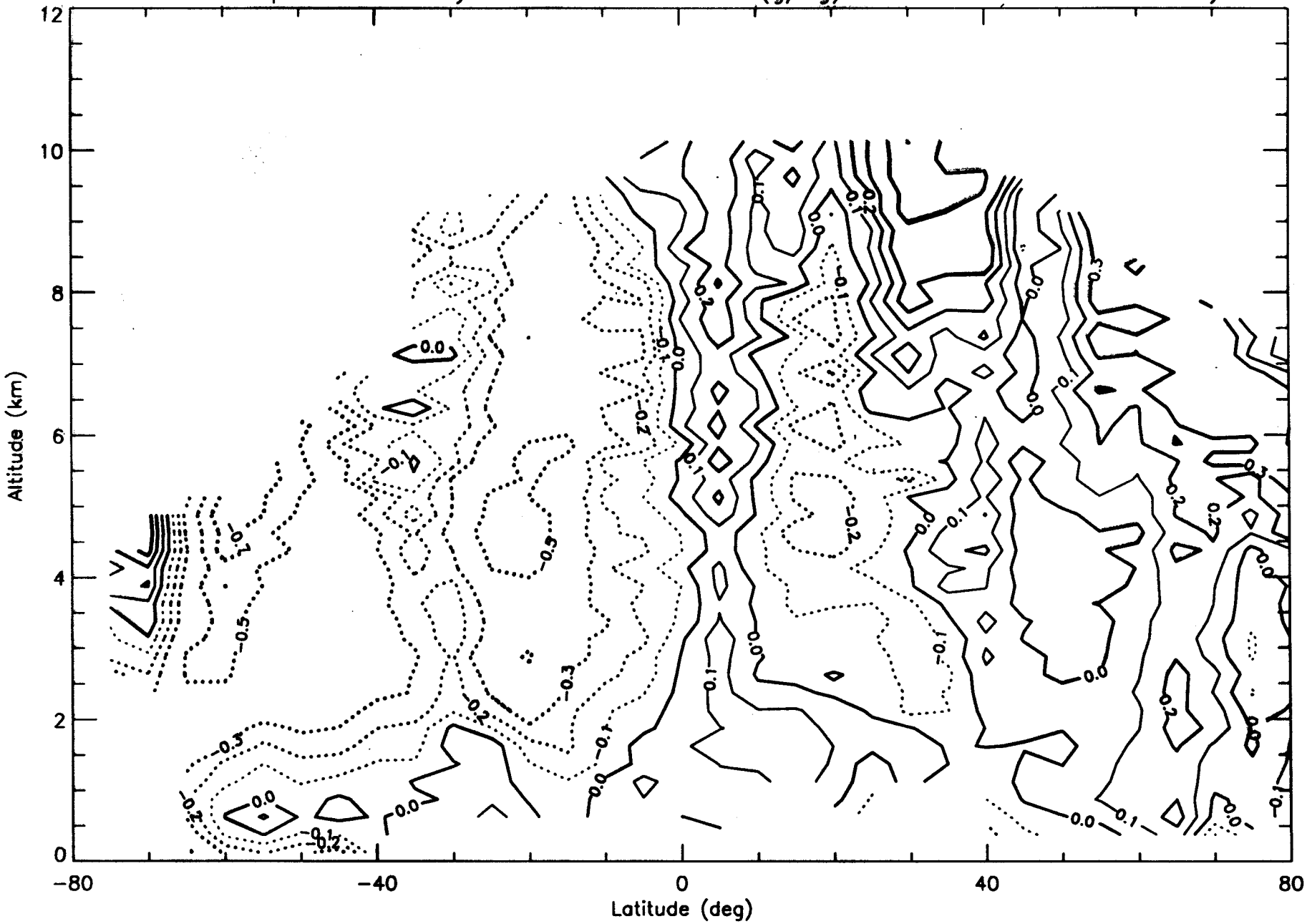
KURSINSKI & HA




DIFFERENCE OF MEAN WATER VAPOR MIXING RATIO
GPS (6/21/95 – 7/4/95) — PEIXOTO & OORT JUNE-JULY-AUGUST 1963-1973
(radiosonde)

KURSINSKI & HAJJ
5/99

Normalized Mean Specific Humidity Difference: Occ−P&O (g/kg) June 21−23 & June 27−July 4 199




Kursinski & Hajj 1999

# Separation of True Variability and Errors

Each observation can be decomposed into a true value plus an error such that the q from ECMWF ($\equiv q_E$) and that derived from GPS refractivity plus the ECMWF temperatures ($\equiv q_G$) can be written as

$$q_E = \bar{q} + q' + \overline{\varepsilon_E} + \varepsilon'_E$$

$$q_G = \bar{q} + q' + \overline{\varepsilon_G} + \varepsilon'_G$$

where $\varepsilon_E$ and $\varepsilon_G$ are the errors in the $q_E$ and $q_G$ estimates and overbars and primes refer to the zonal mean and variable parts of each term respectively.

The difference between the q estimates ($\equiv \Delta q$) is

$$\Delta q = q_G - q_E = \overline{\varepsilon_G} + \varepsilon'_G - \overline{\varepsilon_E} - \varepsilon'_E$$

The variance of the ECMWF and GEW q estimates and their differences can be written as

$$\sigma_{qE}^2 = \overline{q'^2} + 2\,\overline{q'\varepsilon'_E} + \overline{\varepsilon'^2_E} \tag{1a}$$

$$\sigma_{qG}^2 = \overline{q'^2} + 2\,\overline{q'\varepsilon'_G} + \overline{\varepsilon'^2_G} \tag{1b}$$

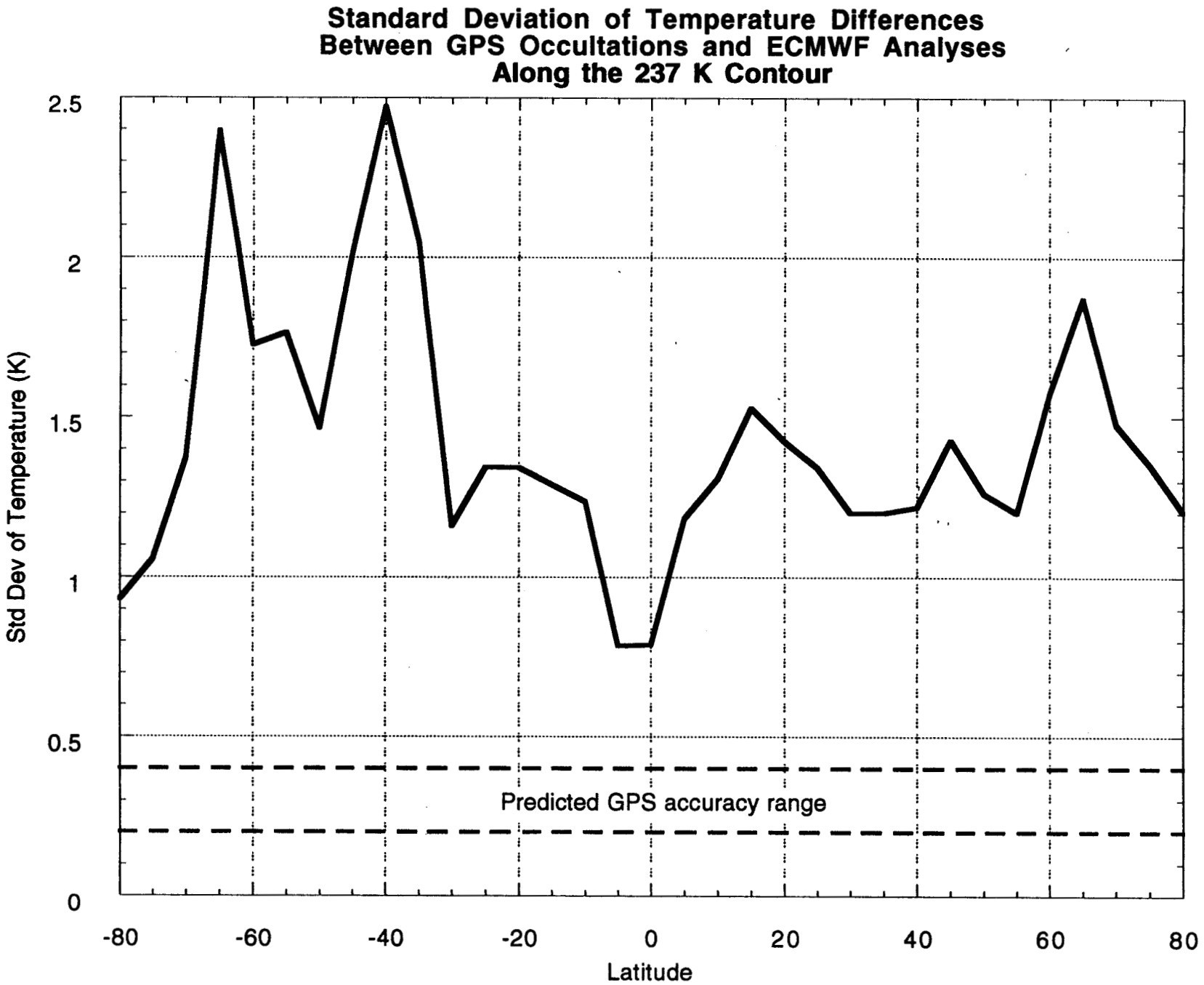
$$\sigma_{\Delta q}^2 = \overline{\varepsilon'^2_G} - 2\,\overline{\varepsilon'_G\varepsilon'_E} + \overline{\varepsilon'^2_E} \tag{1c}$$

(1a-c) provide three constraints sufficient to isolate the true variability, the error in the $q_G$ estimates and the error in the $q_E$ estimates **if the correlation terms are small**. The three combinations which would do so are

$$\frac{1}{2}\left(\sigma_{qE}^2 + \sigma_{qG}^2 - \sigma_{\Delta q}^2\right) = \overline{q'^2} + \overline{q'\varepsilon'_E} + \overline{q'\varepsilon'_G} + \overline{\varepsilon'_E\varepsilon'_G} \tag{2a}$$

$$\frac{1}{2}\left(\sigma_{qE}^2 - \sigma_{qG}^2 + \sigma_{\Delta q}^2\right) = \overline{\varepsilon'^2_E} + \overline{q'\varepsilon'_E} - \overline{q'\varepsilon'_G} - \overline{\varepsilon'_E\varepsilon'_G} \equiv L_E \tag{2b}$$

$$\frac{1}{2}\left(-\sigma_{qE}^2 + \sigma_{qG}^2 + \sigma_{\Delta q}^2\right) = \overline{\varepsilon'^2_G} - \overline{q'\varepsilon'_E} + \overline{q'\varepsilon'_G} - \overline{\varepsilon'_E\varepsilon'_G} \equiv L_G \tag{2c}$$

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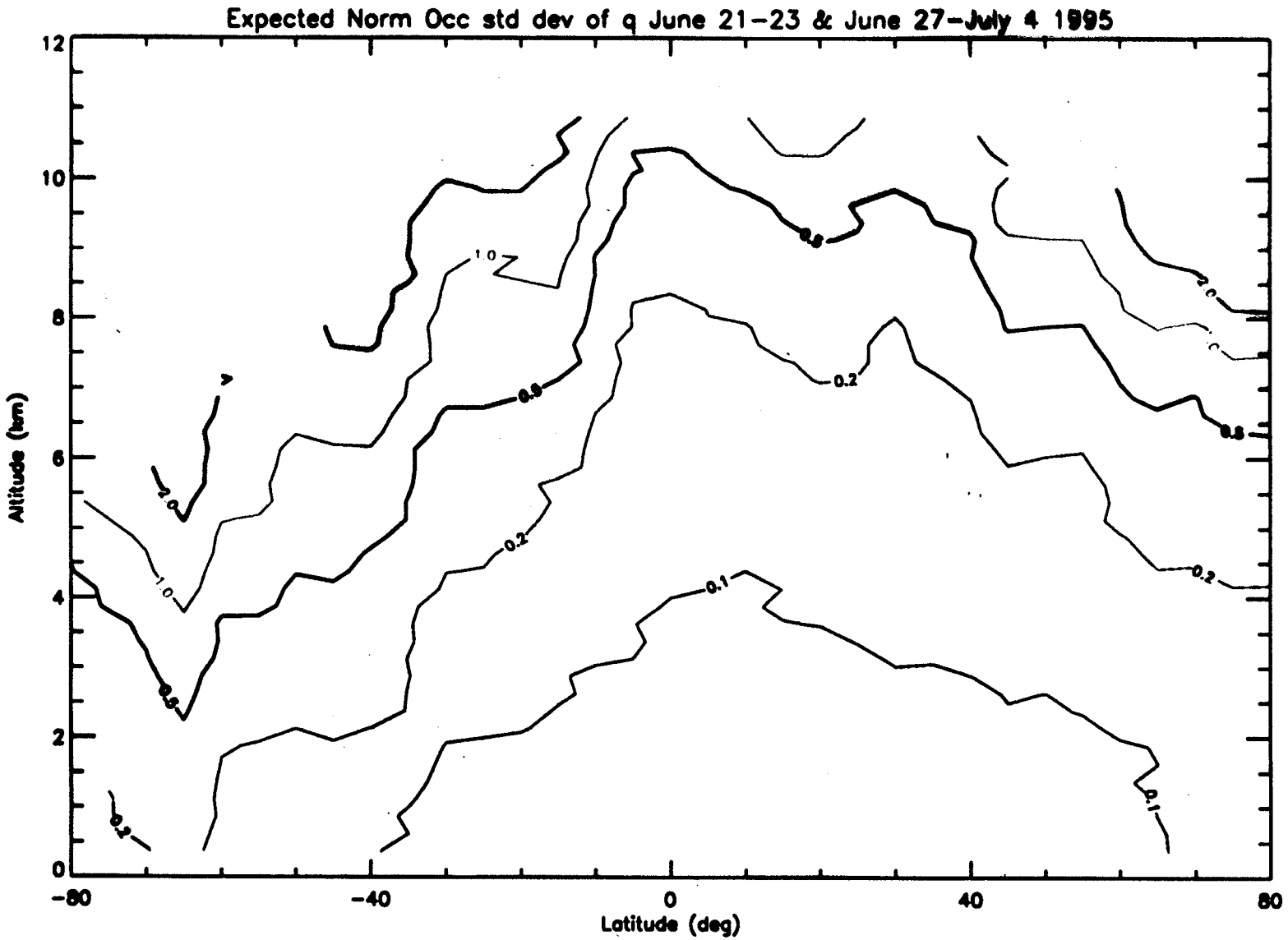
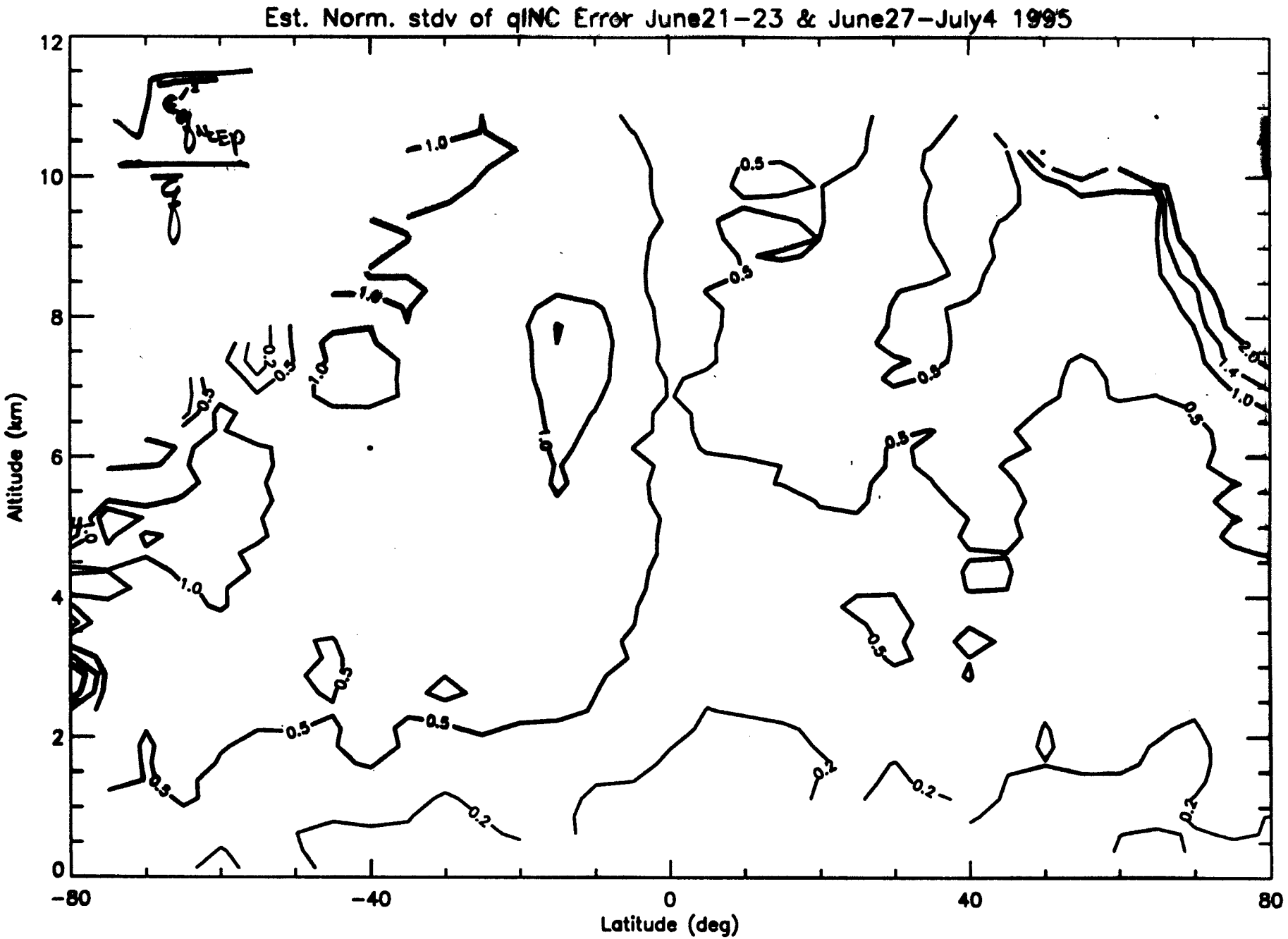
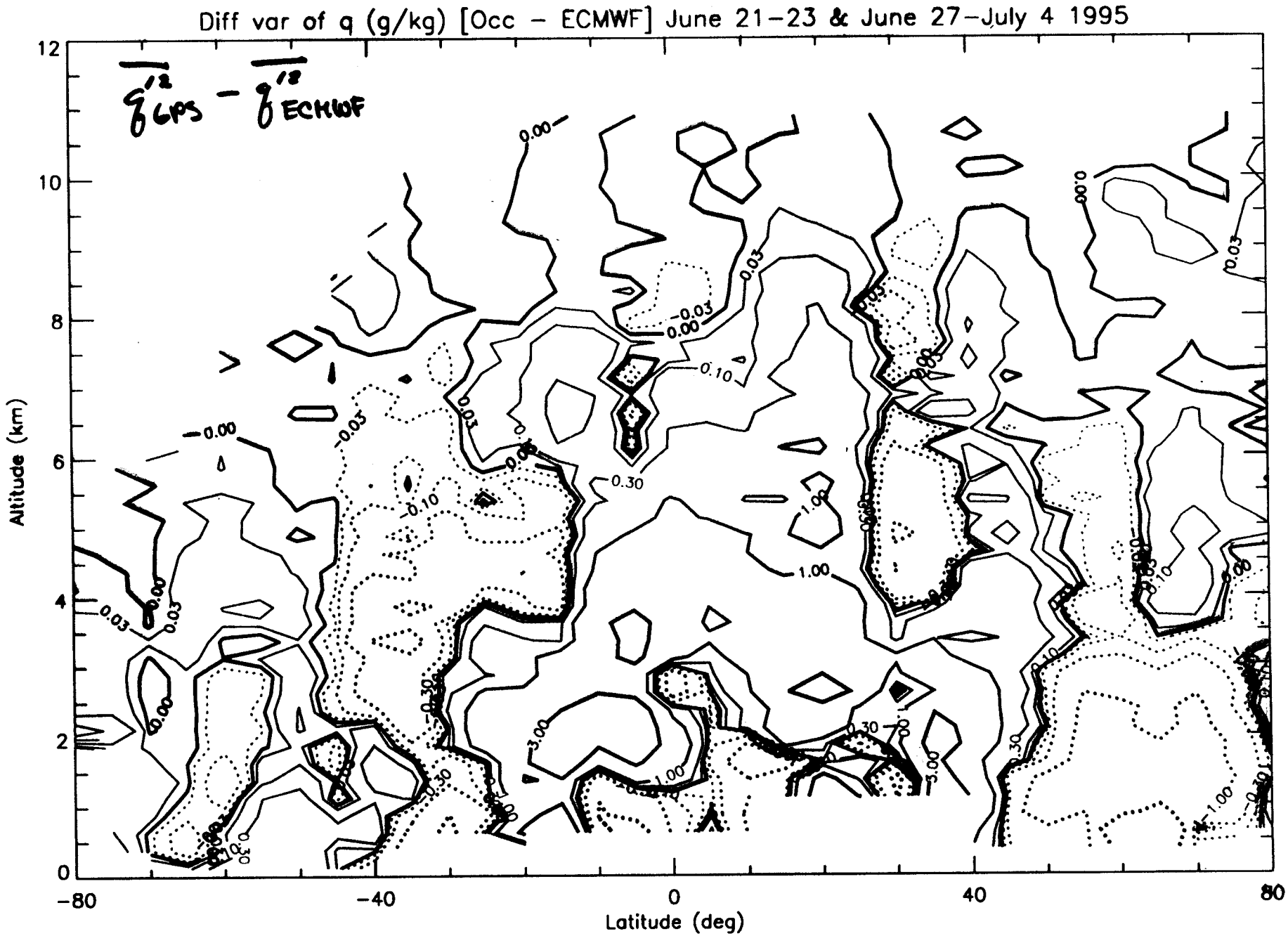
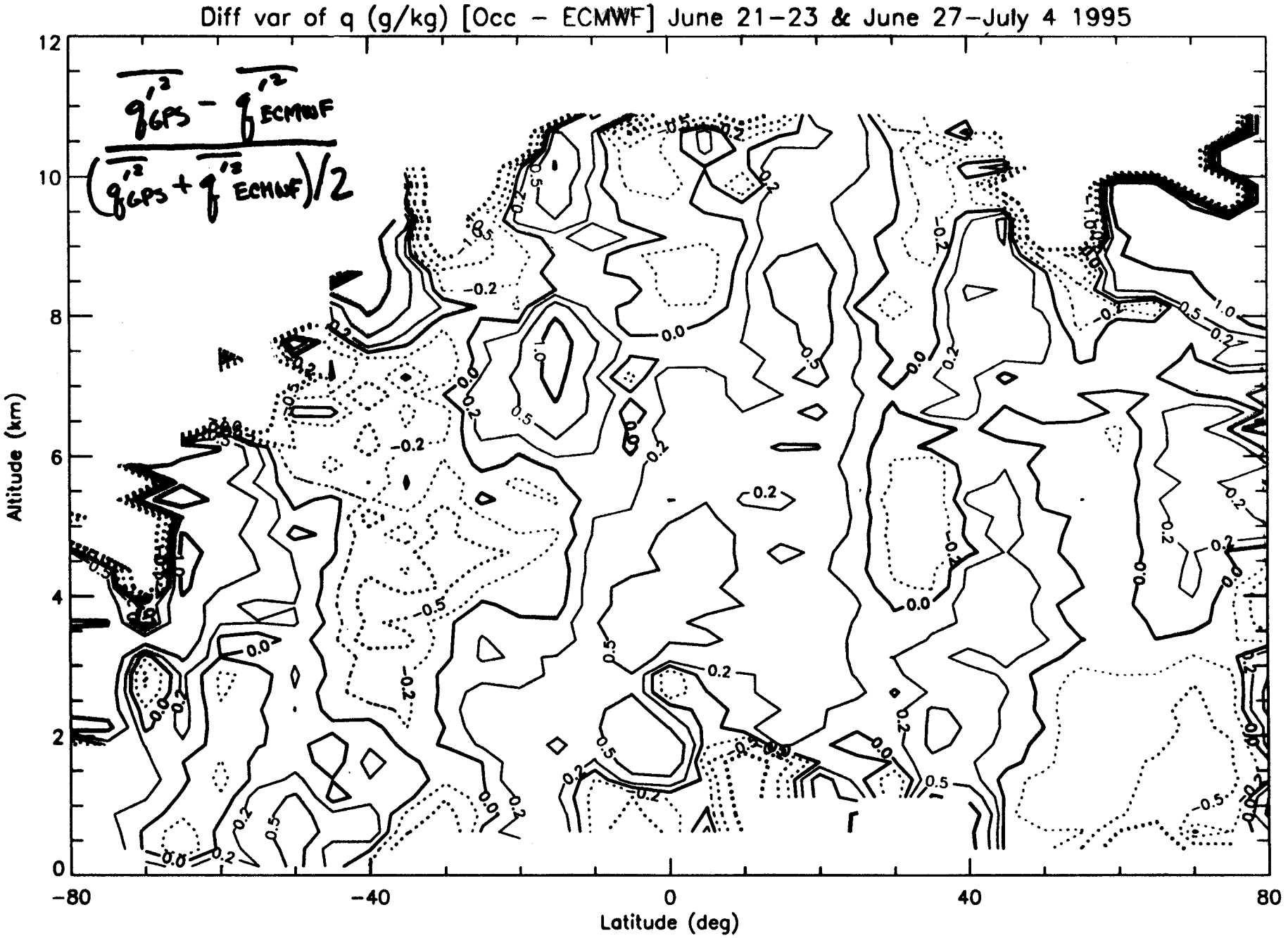
Kursinski 3/98

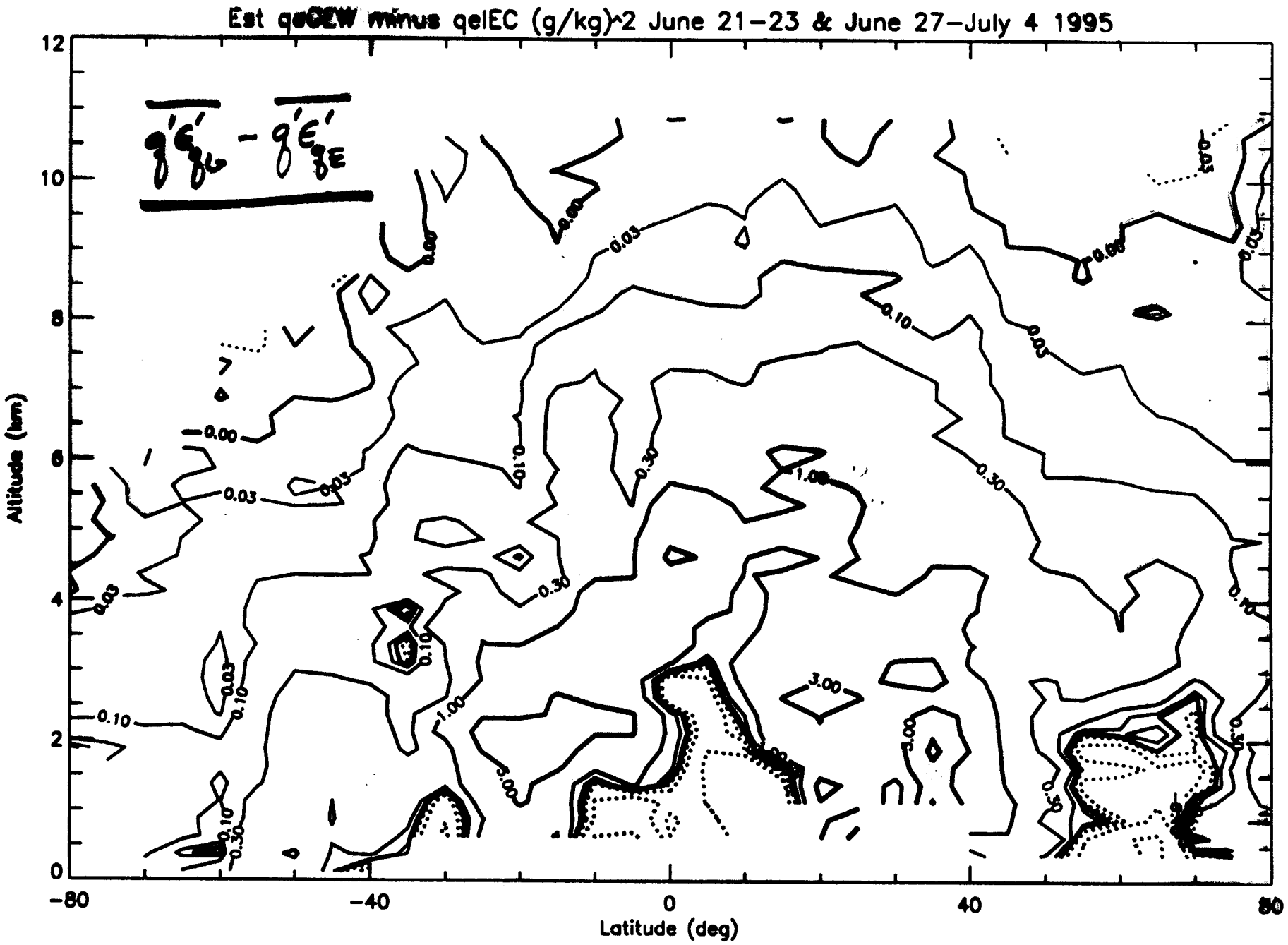

Expected Norm Occ std dev of q June 21-23 & June 27-July 4 1995
Altitude (km)
Latitude (deg)
12
10
8
6
4
2
0
-80
-40
0
40
80
1.0
0.5
0.2
0.1


Est. Norm. stdv of qINC Error June21–23 & June27–July4 1995
Altitude (km)
Latitude (deg)
12
10
8
6
4
2
0
-80
-40
0
40
80
-1.0
0.5
0.2
1.4
2.0


Diff var of q (g/kg) [Occ - ECMWF] June 21-23 & June 27-July 4 1995
$\overline{q'^2}_{GPS} - \overline{q'^2}_{ECMWF}$
Altitude (km)
Latitude (deg)
12
10
8
6
4
2
0
-80
-40
0
40
80
0.00
0.03
-0.03
0.10
-0.10
0.30
-0.30
1.00
-1.00
3.00
-3.00




Kursinski & Hajj
5/99

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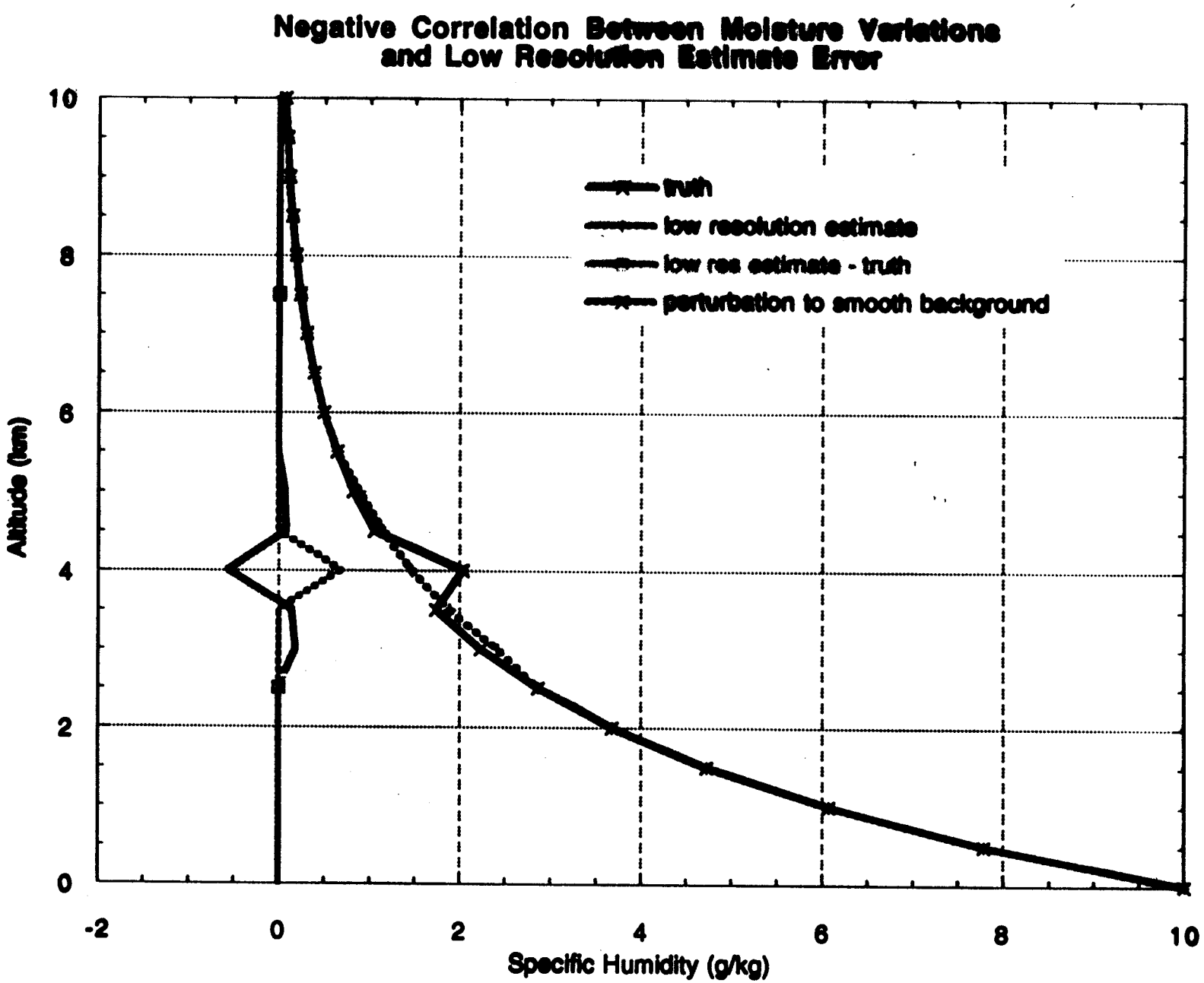
5/99




Kursinski? Hajj
5/99

# Assimilation of GPS data into weather predictions and analyses
## (from Palmer, Barnett and Eyre, AMS conf. 1998)

Problem: How to use a set of measurements to improve the estimate of the atmospheric state in a model

Approach: Define a penalty function and minimize it by adjusting the model variables.
Assuming errors are Gaussian, the maximum likelihood penalty function is then

$$\xi(x) = \underbrace{\left(y_{obs} - y(x)\right)^T S_y^{-1}\left(y_{obs} - y(x)\right)}_{\text{observation part}} + \underbrace{\left(x - x_a\right)^T S_a^{-1}\left(x - x_a\right)}_{\text{model part}}$$

where: $y_{obs}$ is the vector of measurements,
$y(x)$ is the simulated measurements based on the model state vector x,
$x_a$ is the apriori model state vector and $S_a$ is its expected error covariance.
$S_y$ is the measurement error covariance plus the covariance of the forward model.

Minimizing $\xi(x)$ means finding x such that

$$\nabla_x \xi(x) = S_a^{-1}\left(x - x_a\right) - K^T S_y^{-1}\left(y_{obs} - y(x)\right) = 0$$

where **K is $\nabla_x y$.**

y(x) is in general nonlinear such that we must linearize y(x) and iterate to find the solution which is

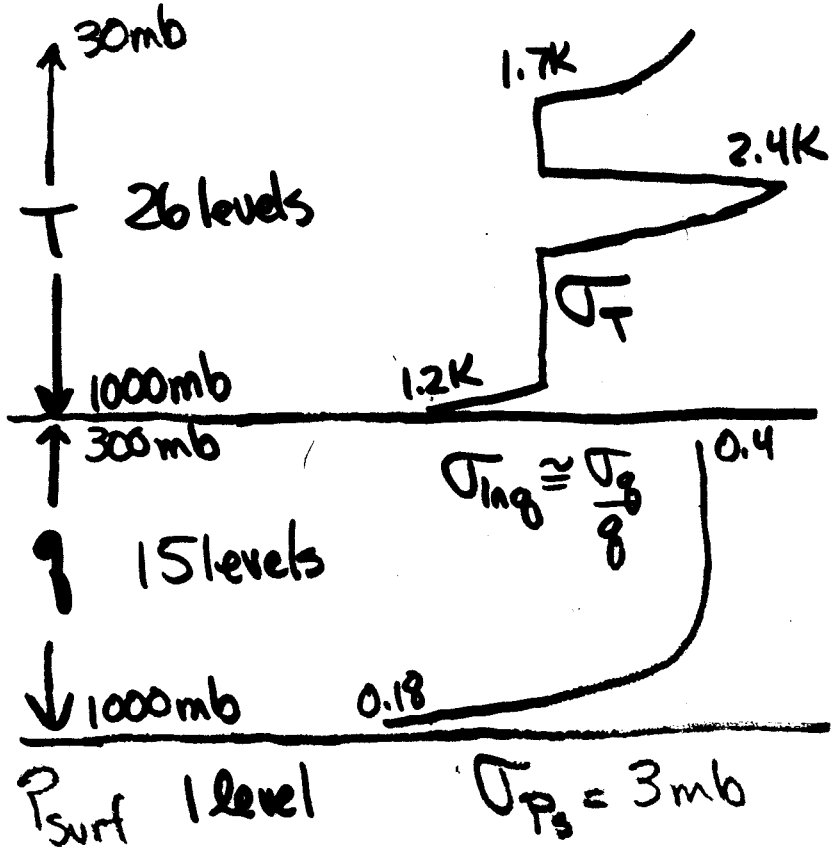
$$x_{n+1} = x_n - \left[H_x \xi(x)\right]^{-1} \nabla_x \xi(x_n)$$

where n is the iteration index and H is the Hessian operator which is obtained by differentiating $\nabla_x \xi(x_n)$ again:

$$H_x \xi(x) \cong S_a^{-1} + K^T S_y^{-1} K$$

The error covariance of x is

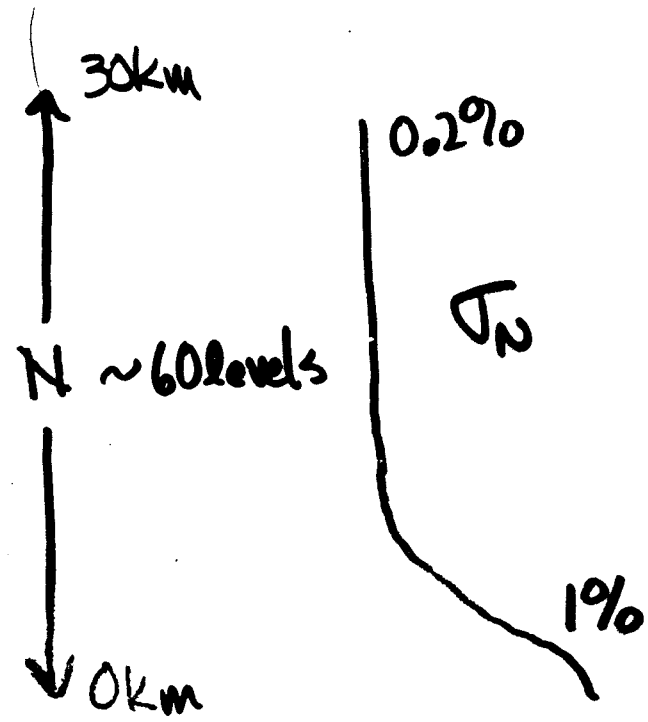
$$\hat{S} = \left[S_a^{-1} + K^T S_y^{-1} K\right]^{-1}$$




Diagonal Terms of $S_x$

Model State Vector
$\equiv \mathbf{x}$ solution
$\equiv \mathbf{x}_a$ apriori

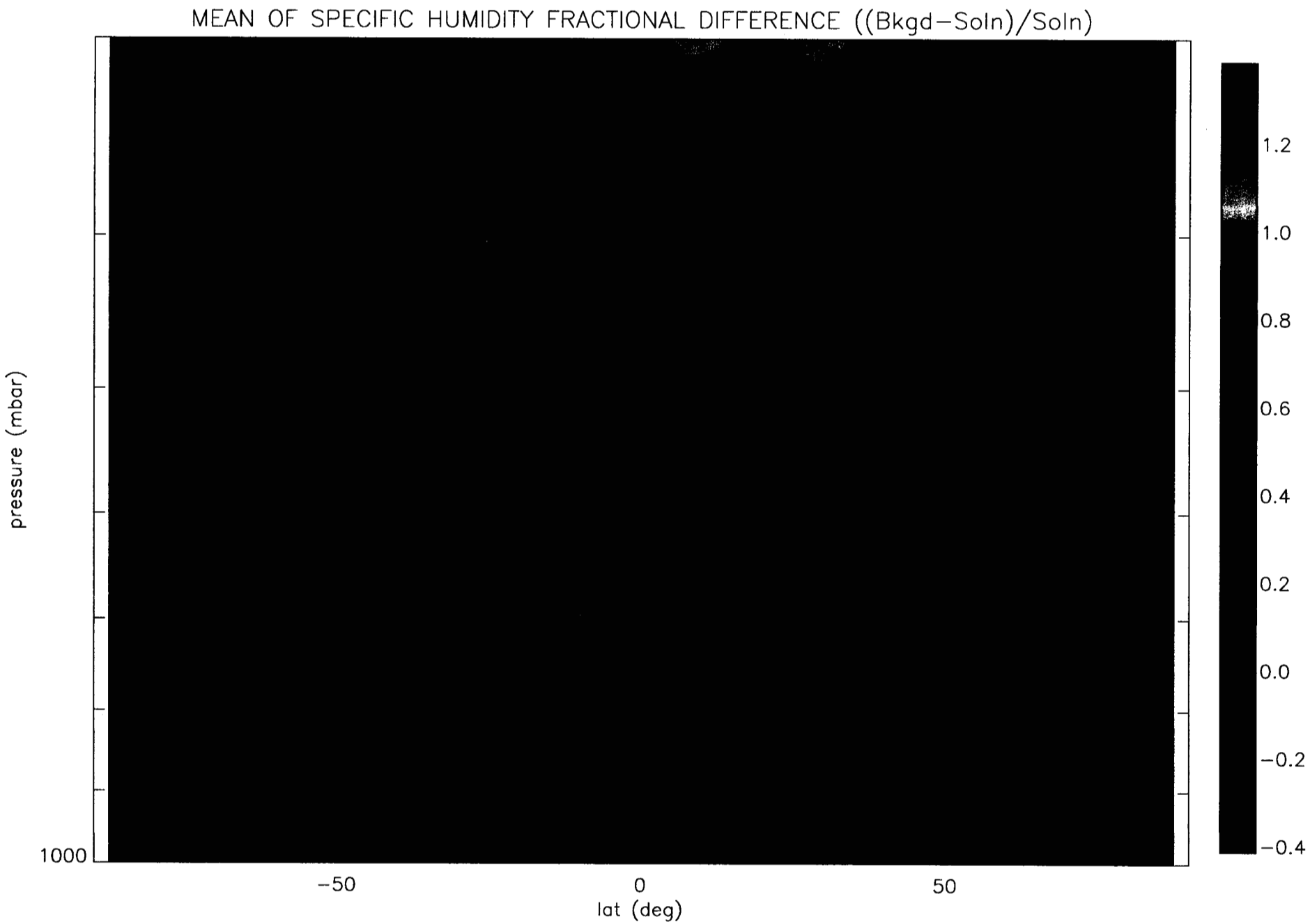
$X$

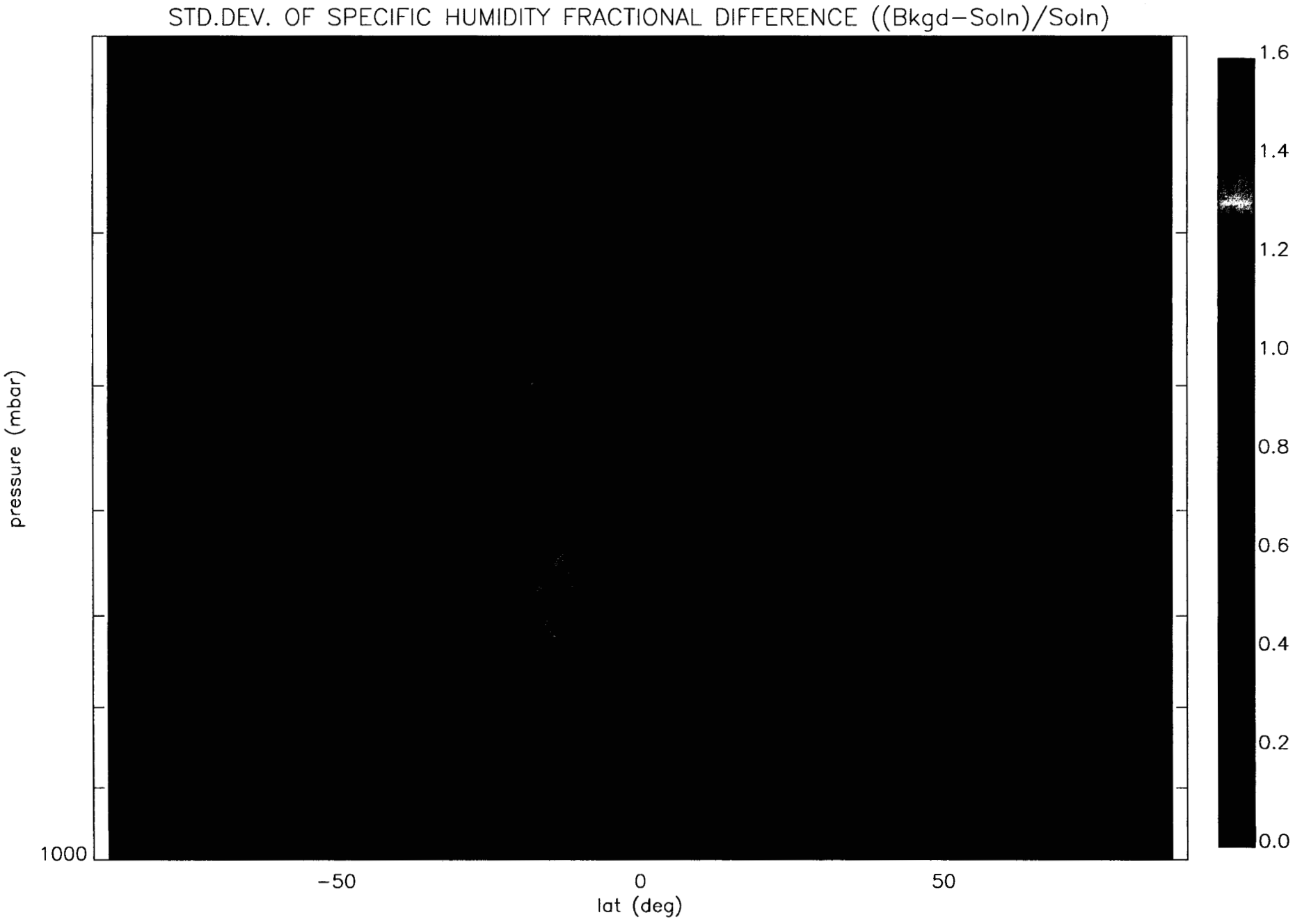


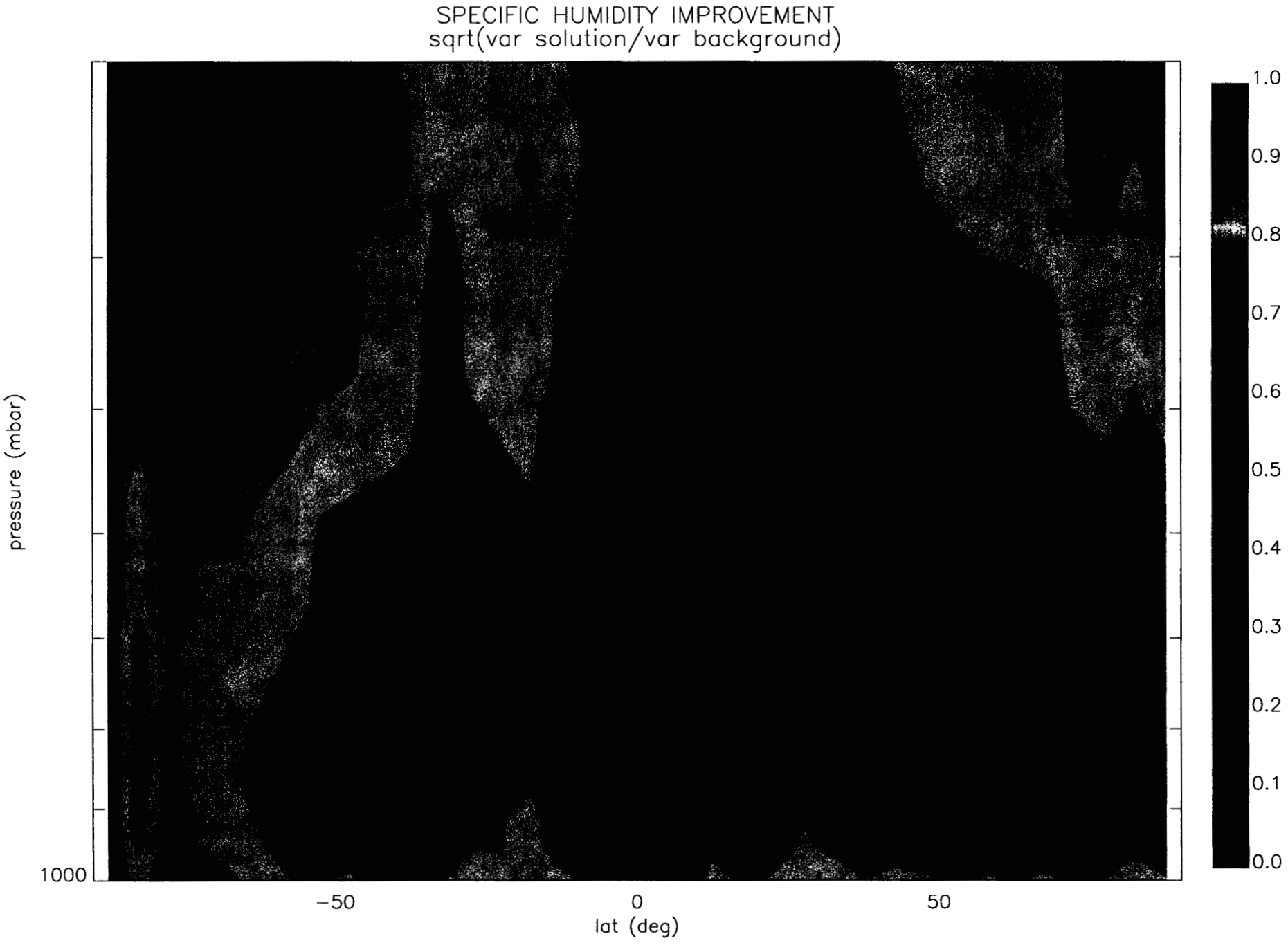

Diagonal Terms of $S_y$

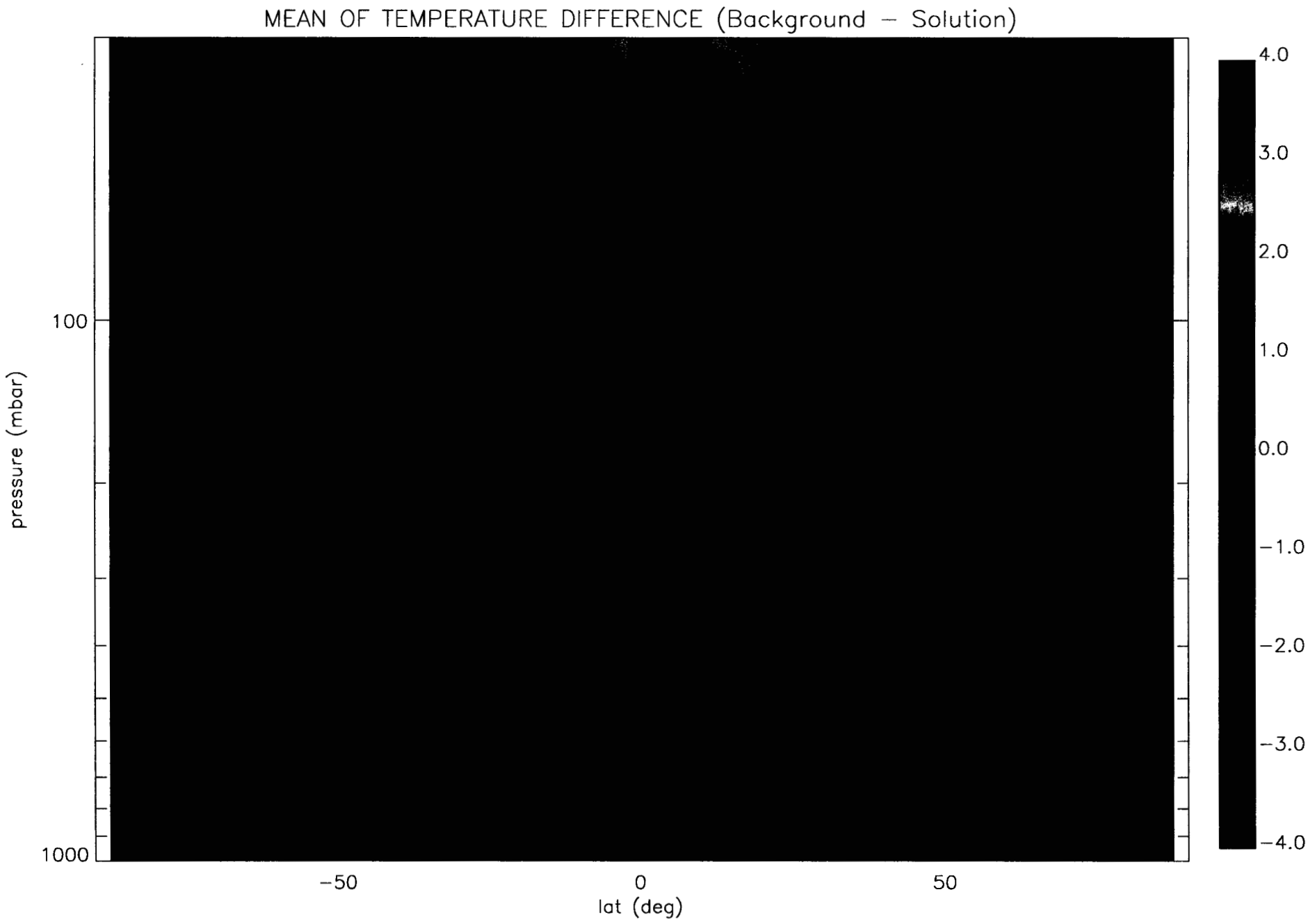
Observation Vector
$\equiv \mathbf{y}$

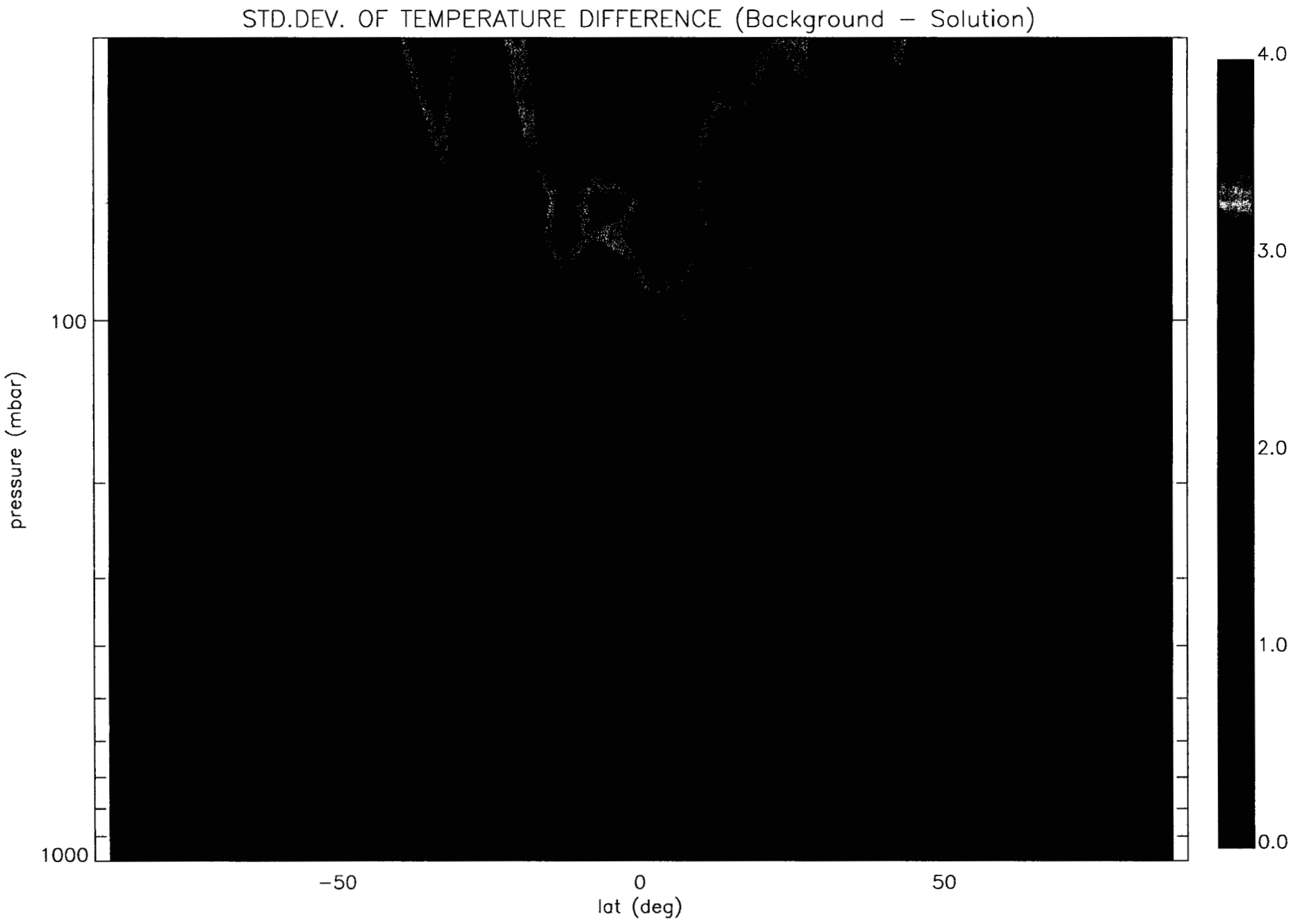
$Y$

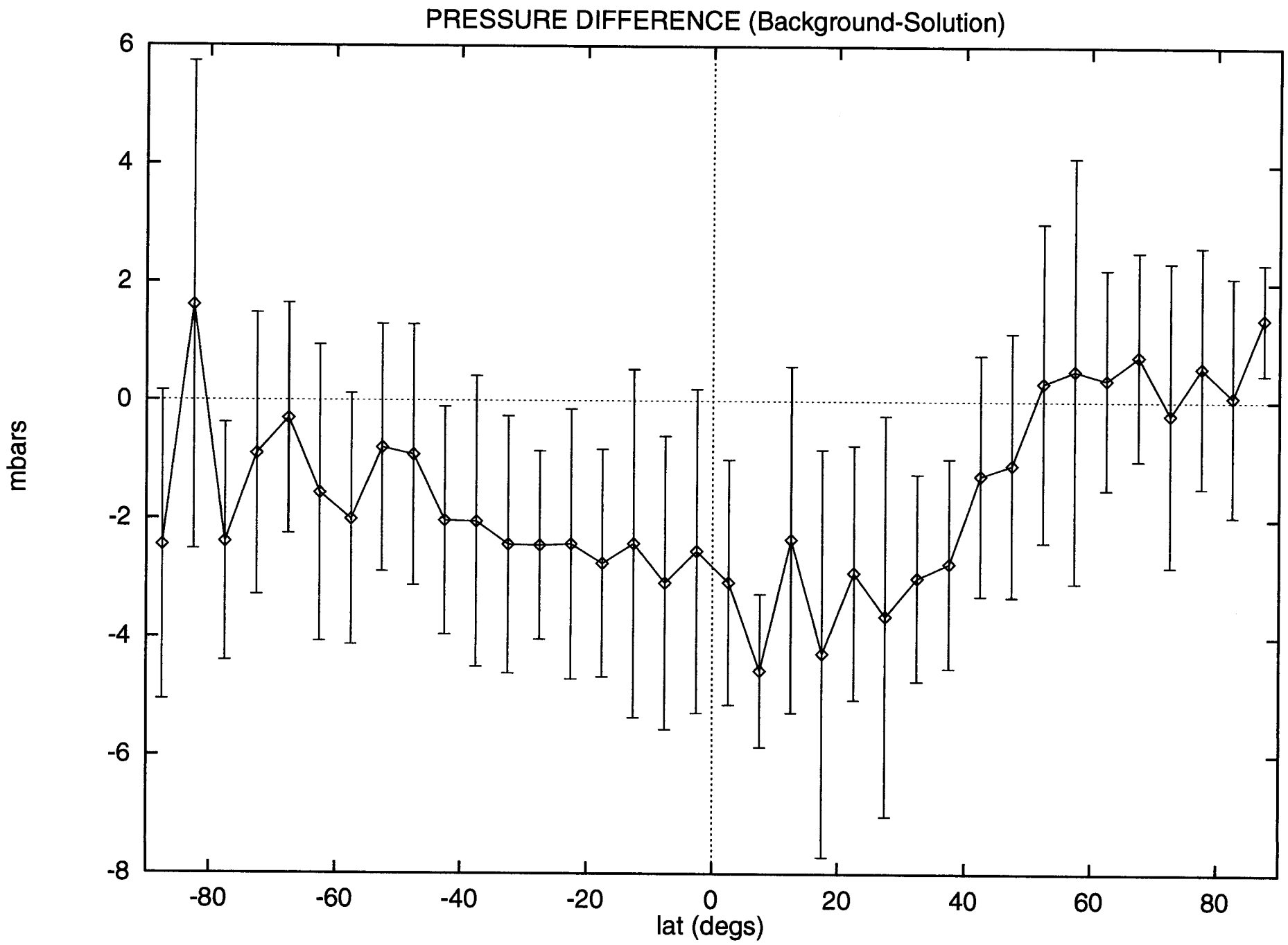

MEAN OF SPECIFIC HUMIDITY FRACTIONAL DIFFERENCE ((Bkgd−Soln)/Soln)
pressure (mbar)
1000
−50
0
50
lat (deg)
1.2
1.0
0.8
0.6
0.4
0.2
0.0
−0.2
−0.4

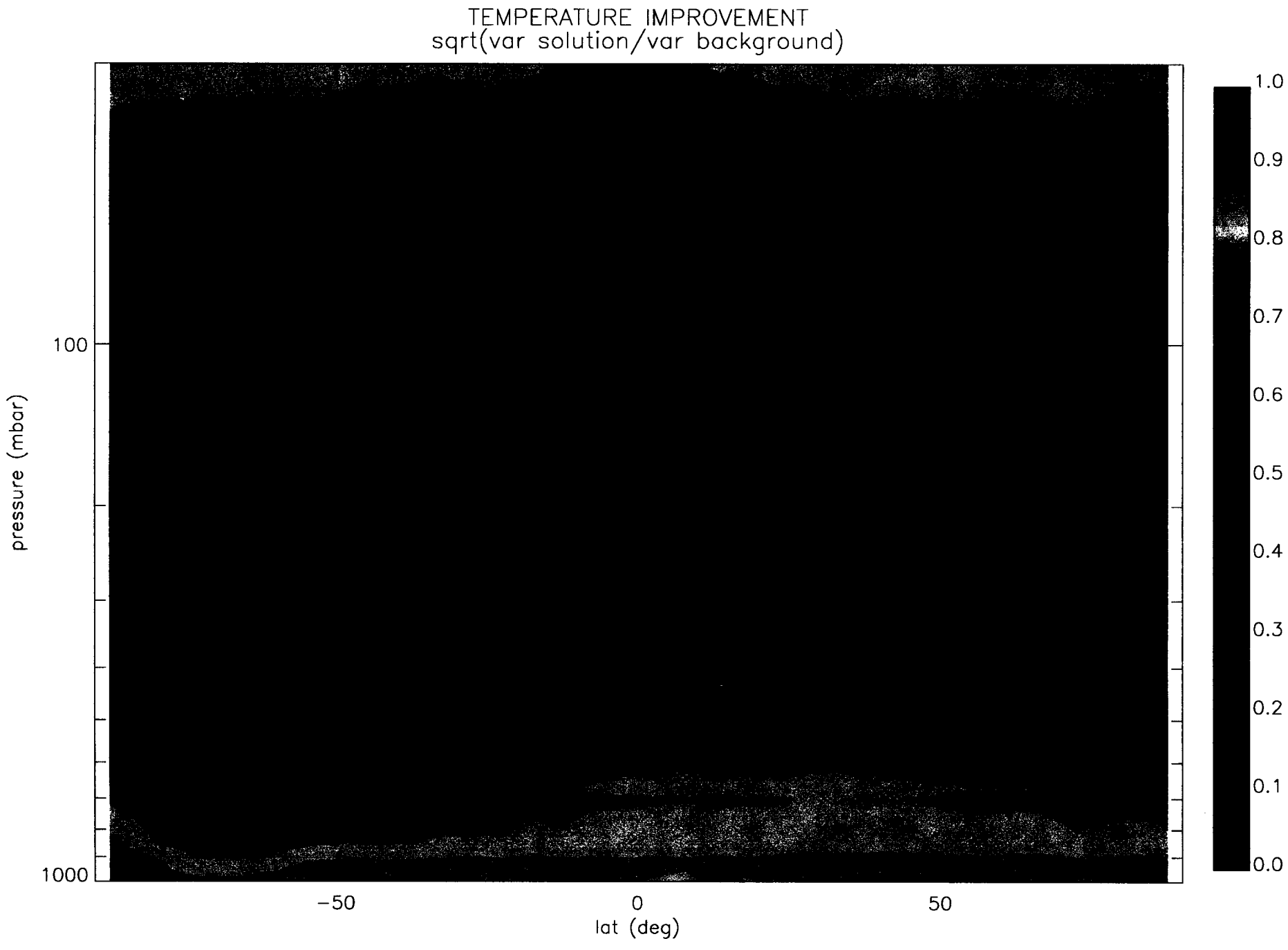

STD.DEV. OF SPECIFIC HUMIDITY FRACTIONAL DIFFERENCE ((Bkgd−Soln)/Soln)
pressure (mbar)
1000
−50
0
50
lat (deg)
1.6
1.4
1.2
1.0
0.8
0.6
0.4
0.2
0.0

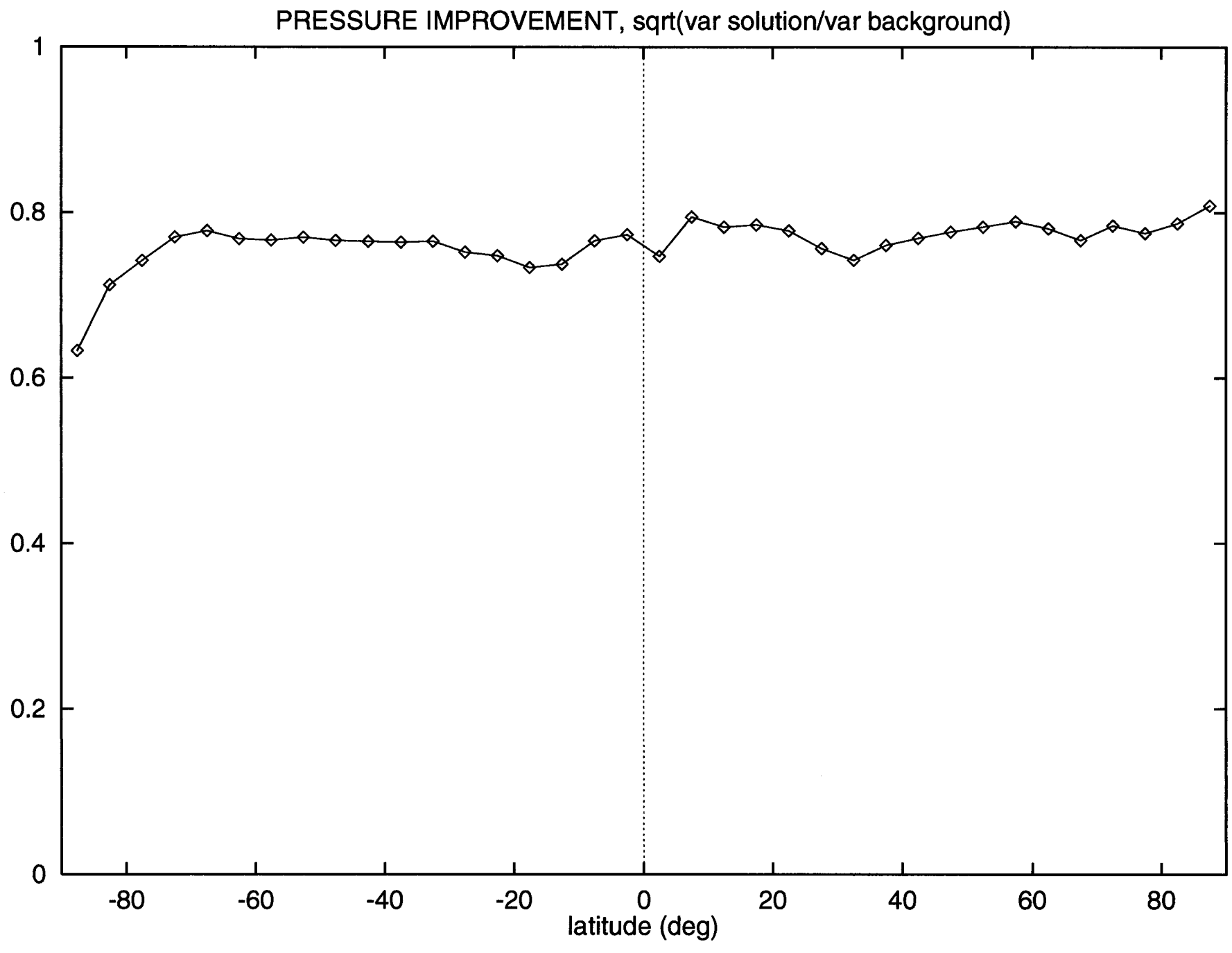

SPECIFIC HUMIDITY IMPROVEMENT
sqrt(var solution/var background)
pressure (mbar)
1000
−50
0
50
lat (deg)
1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0.0


MEAN OF TEMPERATURE DIFFERENCE (Background − Solution)
pressure (mbar)
100
1000
−50
0
50
lat (deg)
4.0
3.0
2.0
1.0
0.0
−1.0
−2.0
−3.0
−4.0


STD.DEV. OF TEMPERATURE DIFFERENCE (Background − Solution)
pressure (mbar)
100
1000
−50
0
50
lat (deg)
4.0
3.0
2.0
1.0
0.0


PRESSURE DIFFERENCE (Background-Solution)
mbars
6
4
2
0
-2
-4
-6
-8
-80
-60
-40
-20
0
20
40
60
80
lat (degs)


TEMPERATURE IMPROVEMENT
sqrt(var solution/var background)
pressure (mbar)
100
1000
−50
0
50
lat (deg)
1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0.0


PRESSURE IMPROVEMENT, sqrt(var solution/var background)
1
0.8
0.6
0.4
0.2
0
-80
-60
-40
-20
0
20
40
60
80
latitude (deg)

# Conclusions / Summary

Occultations yield specific (not relative) humidity

$\sigma_q \sim 0.2$ to $0.5$ g/kg , $\epsilon_q \sim 0.1$ g/kg

Zonal mean moisture examination June 21 - July 4, 1995

ECMWF analyses too moist below ~400 mb

NCEP reanalyses similar but also too dry at ITCZ

ECMWF drier and colder than NCEP above 400 mb

Peixoto & Oort is biased moist in subtropics, dry in ITCZ
(due to... ?)

# Conclusions / Summary

## 1D Var using occultation refractivity and ECMWF analyses

- Framework is simple
- Results are preliminary because covariances are not right
  initial $\sqrt{\chi^2}$ are within ~ factor of 2
  → covariances are not terrible
- Covariance improvements
  Single, global covariances are inadequate
  Tcovar increased in tropical, lower stratosphere
  Tcovar needs to reflect variation in tropopause with latitud
  qcovar needs to be increased in subtropics
  Ncovar needs to reflect dependence on horizontal structure
- Comments on Impact on model state vector
  Impact is underestimated due to covariance errors.
  Is surface pressure adjustment real? may be a gravity error?
  Tropospheric lapse rates are being adjusted
  presumably to get lower stratosphere densities to match N's