Geostrophic winds with a full representation of the Coriolis force: Application to IR observations of the Upper Jovian Troposphere.

M. de la Torre Juárez[1], *B.M. Fisher*[1], *G.S. Orton*
Jet Propulsion Laboratory/Caltech. Pasadena, CA 91109-8099
[1] N.R.C. 97-99

- Introduction.
- The full 3D and the primitive equations.
- Geostrophic balance in both approaches.
- Thermal winds.
- Comparing the approaches.
- Summary.

[DN.004] Geostrophic winds with a full representation of the Coriolis force. (DFD-53)

Jet Propulsion Laboratory
California Institute of Technology

Introduction

MOTIVATION:

- **Scientific:** Assessing the validity of the geostrophic origin of planetary winds.
- **Technical:** Remote sensing of velocity in planetary atmospheres.
  i) Tracking of visible features.
  ii) Geostrophic diagnostics using temperature maps.

OBJECTIVES:

1. **To formulate geostrophic balance with a full Coriolis force in a stratified atmosphere.**
2. **To compare with the diagnostics from the primitive equations.**

DATA: Infrared maps of Jovian temperatures. 4 different pressure levels.

Inviscid full and primitive equations:

Full 3D:	$\left(\frac{\partial \vec{v}}{\partial t} + \vec{v}\cdot\vec{\nabla}\vec{v} + CT\right) + 2\Omega\vec{k}\wedge\vec{v}$	$=$	$-\frac{\vec{\nabla}p}{\rho} + \vec{F}$
Primitive:	$\left(\frac{\partial \vec{v}_h}{\partial t} + \vec{v}\cdot\vec{\nabla}\vec{v}_h + PECT\right) + f\vec{e}_r\wedge\vec{v}_h$	$=$	$-\frac{\vec{\nabla}p}{\rho} + \vec{F} + O(w)$

r = Radial coordinate ; λ = Latitude ; φ = Longitude ;

$\vec{k} = \vec{e}_r \sin\lambda + \vec{e}_\lambda \cos\lambda$; $f = 2\Omega\sin\lambda$; $z = r - R_{planet}$;

$\vec{v} = (u, v, w)$; $\vec{v}_h = (u, v, 0)$

$\frac{\partial}{\partial x} = \frac{1}{r\cos\lambda}\frac{\partial}{\partial\varphi}$; $\frac{\partial}{\partial y} = \frac{1}{r}\frac{\partial}{\partial\lambda}$

CT: Curvature terms ; $PECT$: Curvature terms in the Primitive equations;

[DN.004] Geostrophic winds with a full representation of the Coriolis force. (DFD-53)

Jet Propulsion Laboratory
California Institute of Technology

Inviscid full and primitive equations:

Full 3D: $$\left(\frac{\partial \vec{v}}{\partial t} + \vec{v}\cdot\vec{\nabla}\vec{v} + CT\right) + 2\Omega\vec{k}\wedge\vec{v} = -\frac{\vec{\nabla}p}{\rho} + \vec{F}$$

$$\frac{\partial u}{\partial t} + \left(u\frac{\partial}{\partial x} + v\frac{\partial}{\partial y} + w\frac{\partial}{\partial z}\right)u - \left(2\Omega + \frac{u}{r\cos\lambda}\right)(v\sin\lambda - w\cos\lambda) + DSCT = -\frac{1}{\rho}\frac{\partial p}{\partial x} + F_x$$

$$\frac{\partial v}{\partial t} + \left(u\frac{\partial}{\partial x} + v\frac{\partial}{\partial y} + w\frac{\partial}{\partial z}\right)v + \left(2\Omega + \frac{u}{r\cos\lambda}\right)u\sin\lambda + \frac{wv}{r} + DSCT = -\frac{1}{\rho}\frac{\partial p}{\partial y} + F_y$$

$$\frac{\partial w}{\partial t} + \left(u\frac{\partial}{\partial x} + v\frac{\partial}{\partial y} + w\frac{\partial}{\partial z}\right)w - \left(2\Omega + \frac{u}{r\cos\lambda}\right)u\cos\lambda - \frac{v^2}{r} + DSCT = -\frac{1}{\rho}\frac{\partial p}{\partial z} - g + F_z$$

$$\frac{D\rho}{Dt} + \rho\vec{\nabla}\cdot\vec{v} = \frac{D\rho}{Dt} + \rho\left(\frac{\partial u}{\partial x} + \frac{\partial(v\cos\lambda)}{\partial y} + \frac{1}{r^2}\frac{\partial(r^2 w)}{\partial z}\right)$$

Primitive: $$\left(\frac{\partial \vec{v}_h}{\partial t} + \vec{v}\cdot\vec{\nabla}\vec{v}_h + PECT\right) + f\vec{e}_r\wedge\vec{v}_h = -\frac{\vec{\nabla}p}{\rho} + \vec{F} + O(w)$$

$$\frac{\partial u}{\partial t} + \left(u\frac{\partial}{\partial x} + v\frac{\partial}{\partial y} + w\frac{\partial}{\partial z}\right)u - \left(2\Omega + \frac{u}{r\cos\lambda}\right)(v\sin\lambda) = -\frac{1}{\rho}\frac{\partial p}{\partial x} + F_x + O(w)$$

$$\frac{\partial v}{\partial t} + \left(u\frac{\partial}{\partial x} + v\frac{\partial}{\partial y} + w\frac{\partial}{\partial z}\right)v + \left(2\Omega + \frac{u}{r\cos\lambda}\right)u\sin\lambda = -\frac{1}{\rho}\frac{\partial p}{\partial y} + F_y + O(w)$$

$$0 = -\frac{1}{\rho}\frac{\partial p}{\partial z} - g + O(w, F_z)$$

$$\frac{D\rho}{Dt} + \rho\vec{\nabla}\cdot\vec{v} = \frac{D\rho}{Dt} + \rho\left(\frac{\partial u}{\partial x} + \frac{\partial(v\cos\lambda)}{\partial y} + \frac{\partial w}{\partial z}\right)$$

r = Radial coordinate ; λ = Latitude ; φ = Longitude ;

$\vec{k} = \vec{e}_r\sin\lambda + \vec{e}_\lambda\cos\lambda$; $f = 2\Omega\sin\lambda$; $z = r - R_{planet}$;

$\vec{v} = (u, v, w)$; $\vec{v}_h = (u, v, 0)$

$\frac{\partial}{\partial x} = \frac{1}{r\cos\lambda}\frac{\partial}{\partial\varphi}$; $\frac{\partial}{\partial y} = \frac{1}{r}\frac{\partial}{\partial\lambda}$

CT: Curvature terms ; $PECT$: Curvature terms in the Primitive equations;

$DSCT$: Curvature terms - Curvature terms in shallow atmospheres;

0-th order solution - Hydrostatic balance:

For an ideal gas in hydrostatic balance: $\vec{\nabla} p_h(z) = \rho_h(z)\vec{g}$; $\vec{v} = 0$.

1-st order solution - Geostrophic balance:

$\vec{v} \sim \epsilon \vec{v}_g \sim \epsilon \vec{v}_{gp}$ (velocity in geostrophic balance).
$Ro \sim (\vec{\nabla} \wedge \vec{v})/2\Omega$ (Rossby number).

Introducing: $\vec{\nabla}\pi = \vec{\nabla} p_h(z) + \epsilon \vec{\nabla}\pi_1(x,y,z)$
$\vec{\nabla}\pi \equiv \vec{\nabla} p + \rho \vec{\nabla}\left(v^2/2\right)$ (dynamic pressure)

Full 3D $O(Ro)$	
$2\Omega\vec{k} \wedge \vec{v}_g = \frac{1}{\epsilon}\left(-\frac{1}{\rho}\vec{\nabla}\pi + \vec{g}\right)$	$\vec{v}_g = \frac{\vec{k}}{\rho\epsilon 2\Omega} \wedge \left(\vec{\nabla}\pi - \rho\vec{g}\right) + V_g^0 \vec{k}$
Primitive $O(w, Ro)$	
$f\vec{e}_r \wedge \vec{v}_{gp} = \frac{1}{\epsilon}\left(-\frac{1}{\rho}\vec{\nabla}\pi + \vec{g}\right)$	$\vec{v}_{gp} = \frac{\vec{e}_r}{\rho\epsilon f} \wedge \left(\vec{\nabla}\pi - \rho\vec{g}\right) + V_{gp}^0 \vec{e}_r$

Geostrophic balance requires:
- $\vec{k} \cdot \left(\vec{\nabla}\pi - \rho\vec{g}\right) = 0 \rightarrow \frac{1}{\rho}\frac{\partial \pi}{\partial Z} = \frac{1}{\rho}\left(\sin\lambda \frac{\partial \pi}{\partial z} + \cos\lambda \frac{\partial \pi}{\partial y}\right) = -g\sin\lambda$
- $\vec{e}_r \cdot \left(\vec{\nabla}\pi - \rho\vec{g}\right) = 0 \rightarrow \frac{1}{\rho}\frac{\partial \pi}{\partial z} = -g$ (hydrostatic approximation)

Note that: $\vec{v}_g \perp \vec{k}$, while $\vec{v}_{gp} \perp \vec{e}_r$ (contained in the horizontal plane)

Full 3D (non-singular at $\lambda = 0$)	**Primitive (singular)**
$\vec{v}_g = \frac{1}{\rho 2\Omega}\begin{pmatrix} -\sin\lambda\frac{\partial \pi_1}{\partial y} + \cos\lambda\left(\rho_1 g + \frac{\partial \pi_1}{\partial z}\right) \\ \sin\lambda\frac{\partial \pi_1}{\partial x} \\ -\cos\lambda\frac{\partial \pi_1}{\partial x} \end{pmatrix}$	$\vec{v}_{gp} = \frac{1}{\rho 2\Omega\sin\lambda}\begin{pmatrix} -\frac{\partial \pi_1}{\partial y} \\ \frac{\partial \pi_1}{\partial x} \\ 0 \end{pmatrix}$

Thermal winds:

Taking the curl of the geostrophic balance condition and assuming static compressibility (anelastic approximation): $\epsilon(2\vec{\Omega}\cdot\vec{\nabla})(\rho\vec{v_g}) = -\vec{\nabla}\rho\wedge\vec{g}$.

$\epsilon(f\vec{e_r}\cdot\vec{\nabla})(\rho\vec{v_g}) = -\vec{\nabla}\rho\wedge\vec{g}$.

If $Z^* \equiv Z/H$, and $z^* \equiv z/H$, where $1/H = -\partial(lnp_h)/\partial z$, leads to:

Full 3D $O(Ro,\epsilon)$

$$\begin{cases} \frac{1}{\rho}\frac{\partial\rho u_g}{\partial Z^*} \equiv \frac{\partial u_g}{\partial Z_p^*} = -\frac{R}{2\Omega M_r}\left(\frac{\partial T}{\partial y}\right)_{p_h} \\ \frac{1}{\rho}\frac{\partial\rho v_g}{\partial Z^*} \equiv \frac{\partial v_g}{\partial Z_p^*} = \frac{R}{2\Omega M_r}\left(\frac{\partial T}{\partial x}\right)_{p_h} \\ \frac{1}{\rho}\frac{\partial\rho w_g}{\partial Z^*} \equiv \frac{\partial w_g}{\partial Z_p^*} = O(\epsilon) \end{cases}$$

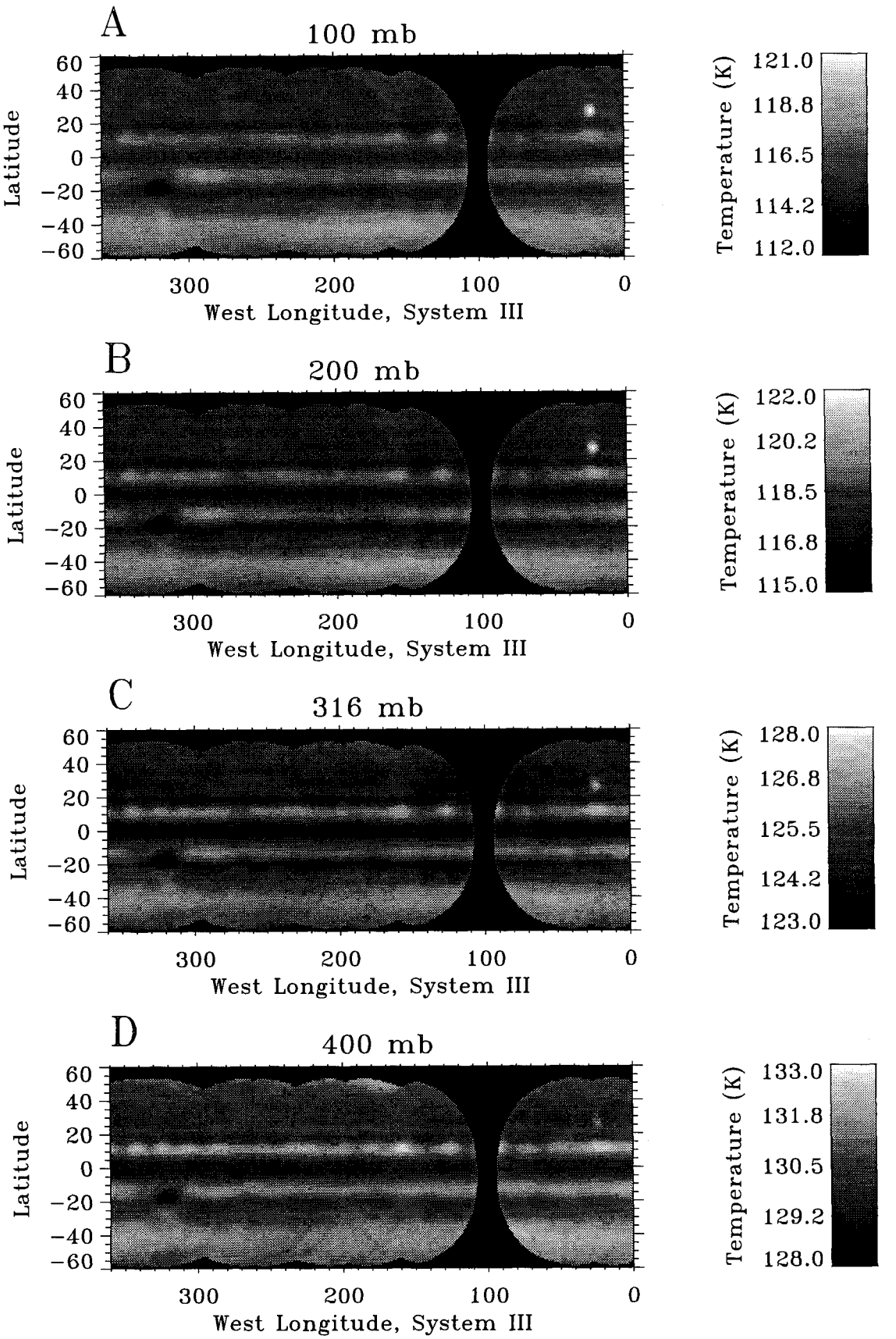
Primitive $O(w,Ro,\epsilon)$

$$\begin{cases} \frac{\partial u_{gp}}{\partial z^*} = -\frac{R}{fM_r}\left(\frac{\partial T}{\partial y}\right)_p \\ \frac{\partial v_{gp}}{\partial z^*} = \frac{R}{fM_r}\left(\frac{\partial T}{\partial x}\right)_p \end{cases}$$

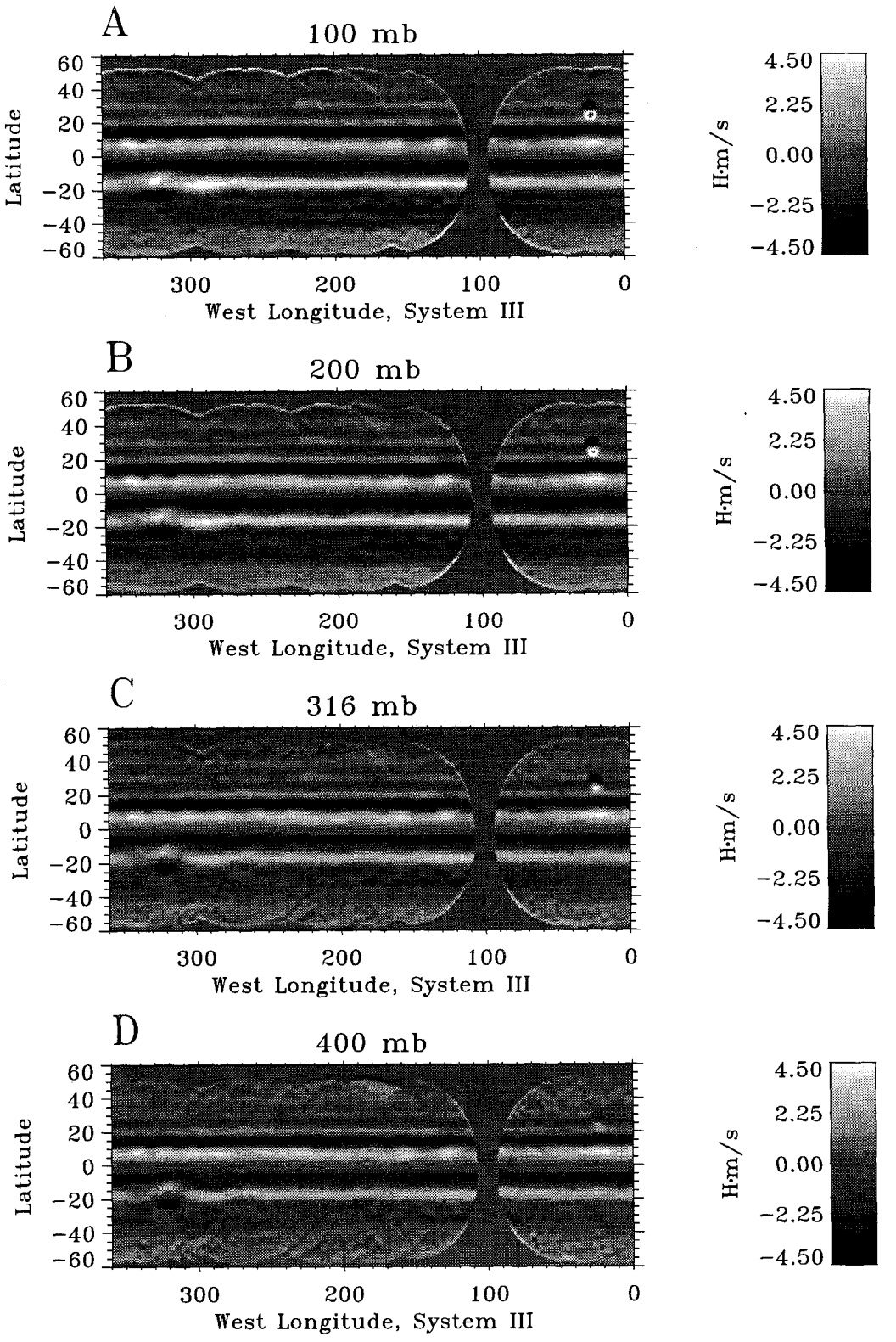
← The hydrostatic approximation has been used.

Comparisons:

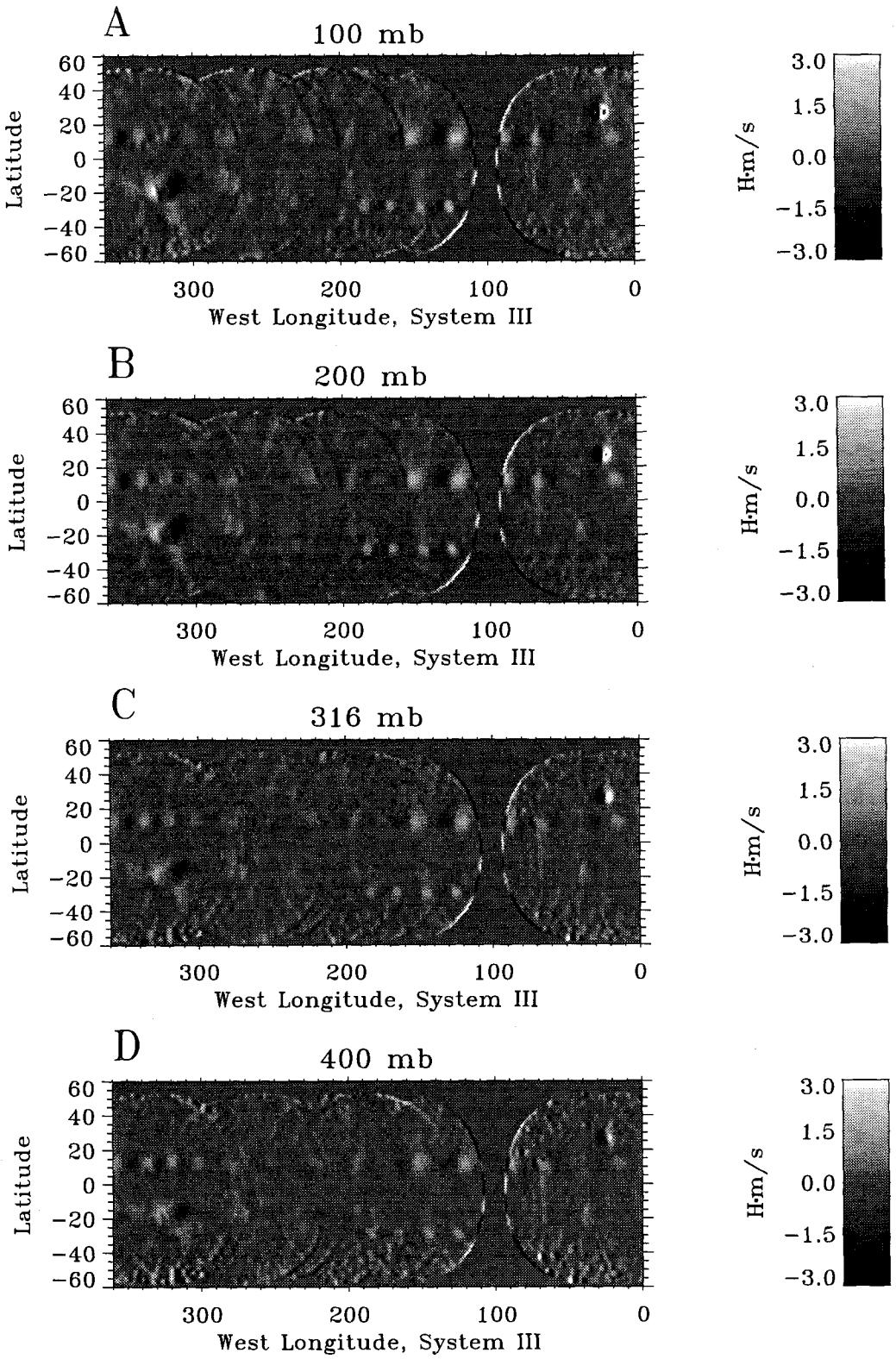
Non-hydrostatic $O(\epsilon)$	Hydrostatic $O(1)$
$\begin{cases} \frac{\partial u_g}{\partial z^*} = \frac{\partial u_{gp}}{\partial z^*} - H\cot\lambda\frac{\partial u_g}{\partial y} \\ \frac{\partial v_g}{\partial z^*} = \frac{\partial v_{gp}}{\partial z^*} - H\cot\lambda\frac{\partial v_g}{\partial y} \\ \frac{\partial\rho w_g}{\partial z^*} = -H\cot\lambda\frac{\partial\rho w_g}{\partial y} + O(\epsilon) \end{cases}$	$\begin{cases} \frac{\partial u_g}{\partial z^*} = -\frac{R\sin\lambda}{2\Omega M_r}\left(\frac{\partial T}{\partial y}\right)_p \\ \frac{\partial v_g}{\partial z^*} = \frac{R\sin\lambda}{2\Omega M_r}\left(\frac{\partial T}{\partial x}\right)_p \\ \frac{\partial w_g}{\partial z^*} = -\frac{R\cos\lambda}{2\Omega M_r}\left(\frac{\partial T}{\partial x}\right)_p \end{cases}$


A
100 mb
Latitude
60
40
20
0
−20
−40
−60
300
200
100
0
West Longitude, System III
Temperature (K)
121.0
118.8
116.5
114.2
112.0
B
200 mb
Latitude
West Longitude, System III
Temperature (K)
122.0
120.2
118.5
116.8
115.0
C
316 mb
Latitude
West Longitude, System III
Temperature (K)
128.0
126.8
125.5
124.2
123.0
D
400 mb
Latitude
West Longitude, System III
Temperature (K)
133.0
131.8
130.5
129.2
128.0

Figures 1 a-d


A
100 mb
B
200 mb
C
316 mb
D
400 mb
Latitude
60
40
20
0
−20
−40
−60
300
200
100
0
West Longitude, System III
H·m/s
4.50
2.25
0.00
−2.25
−4.50

Figures 2a-d


A
100 mb
B
200 mb
C
316 mb
D
400 mb
Latitude
60
40
20
0
−20
−40
−60
300
200
100
0
West Longitude, System III
H·m/s
3.0
1.5
0.0
−1.5
−3.0

Figures 3a-d

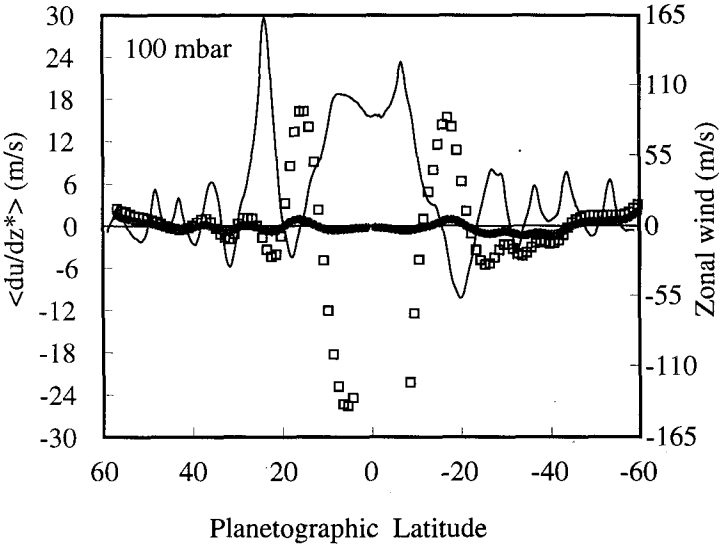


Figure 4a

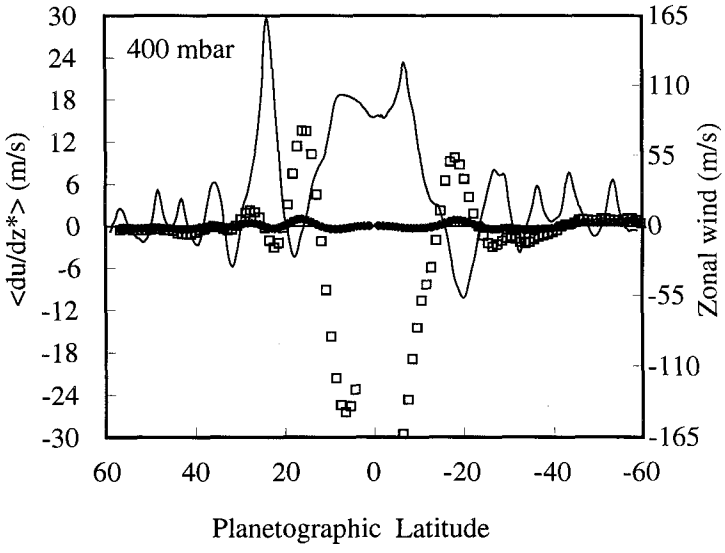


Figure 4b

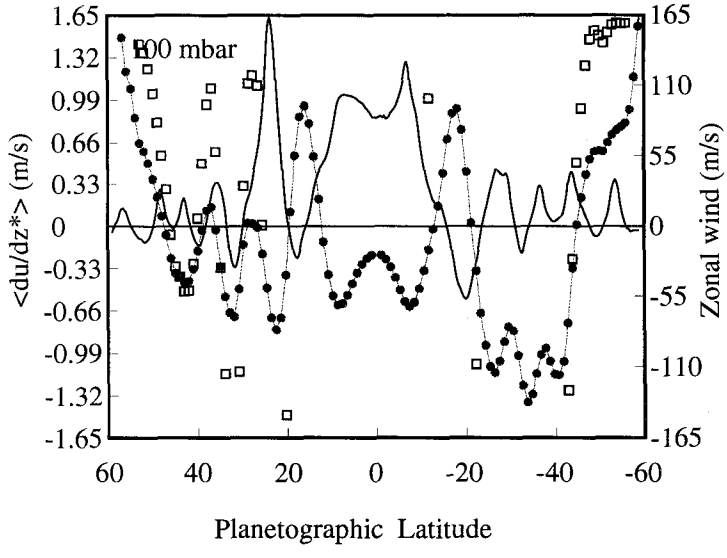


Figure 4c

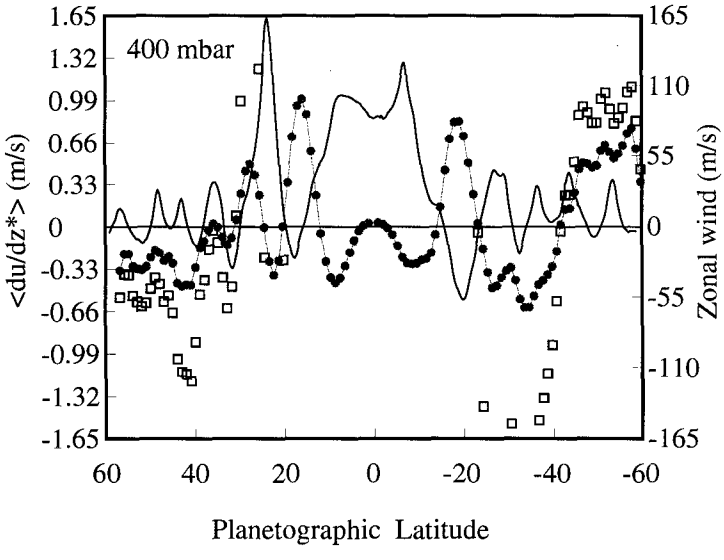
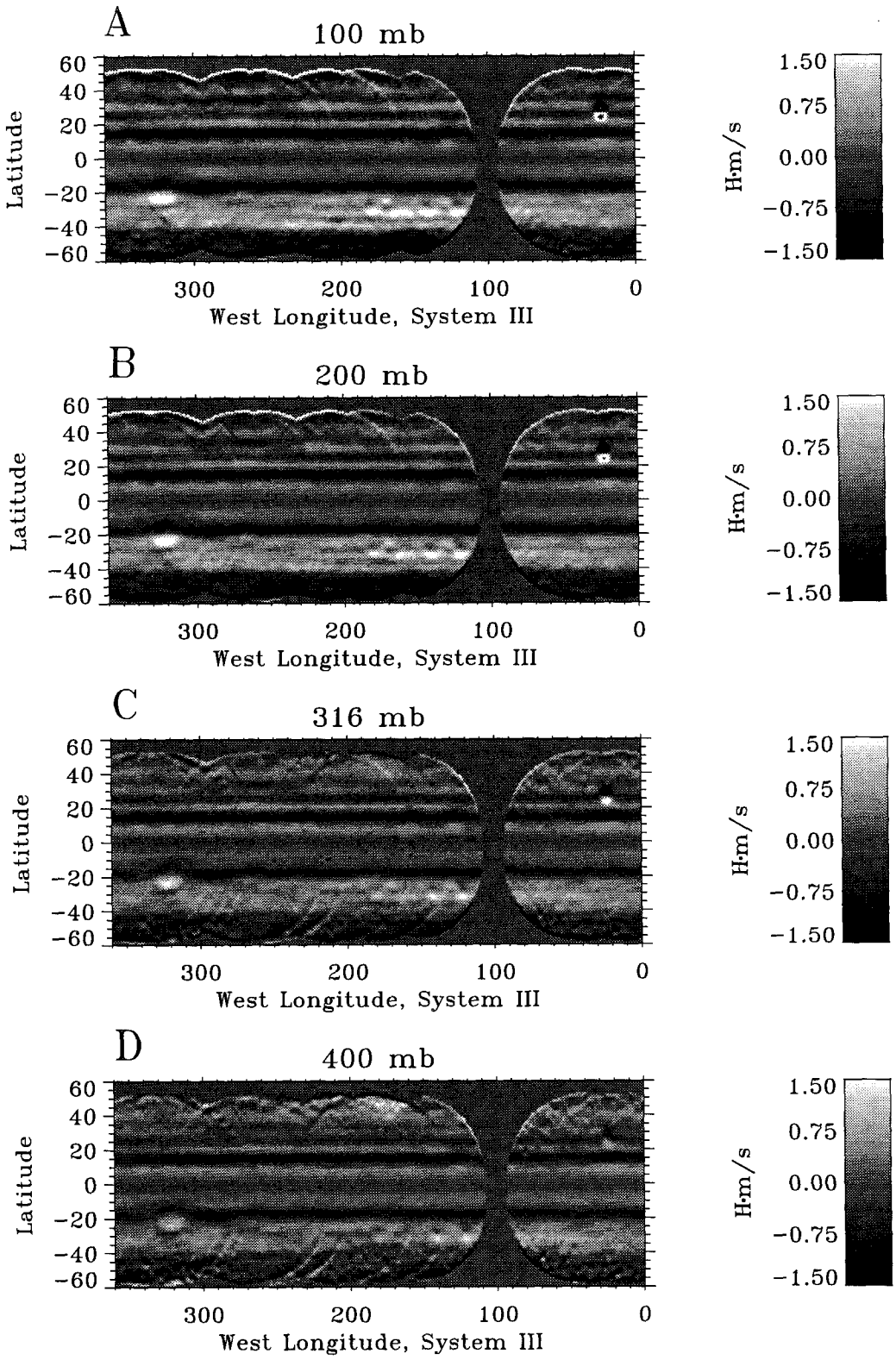
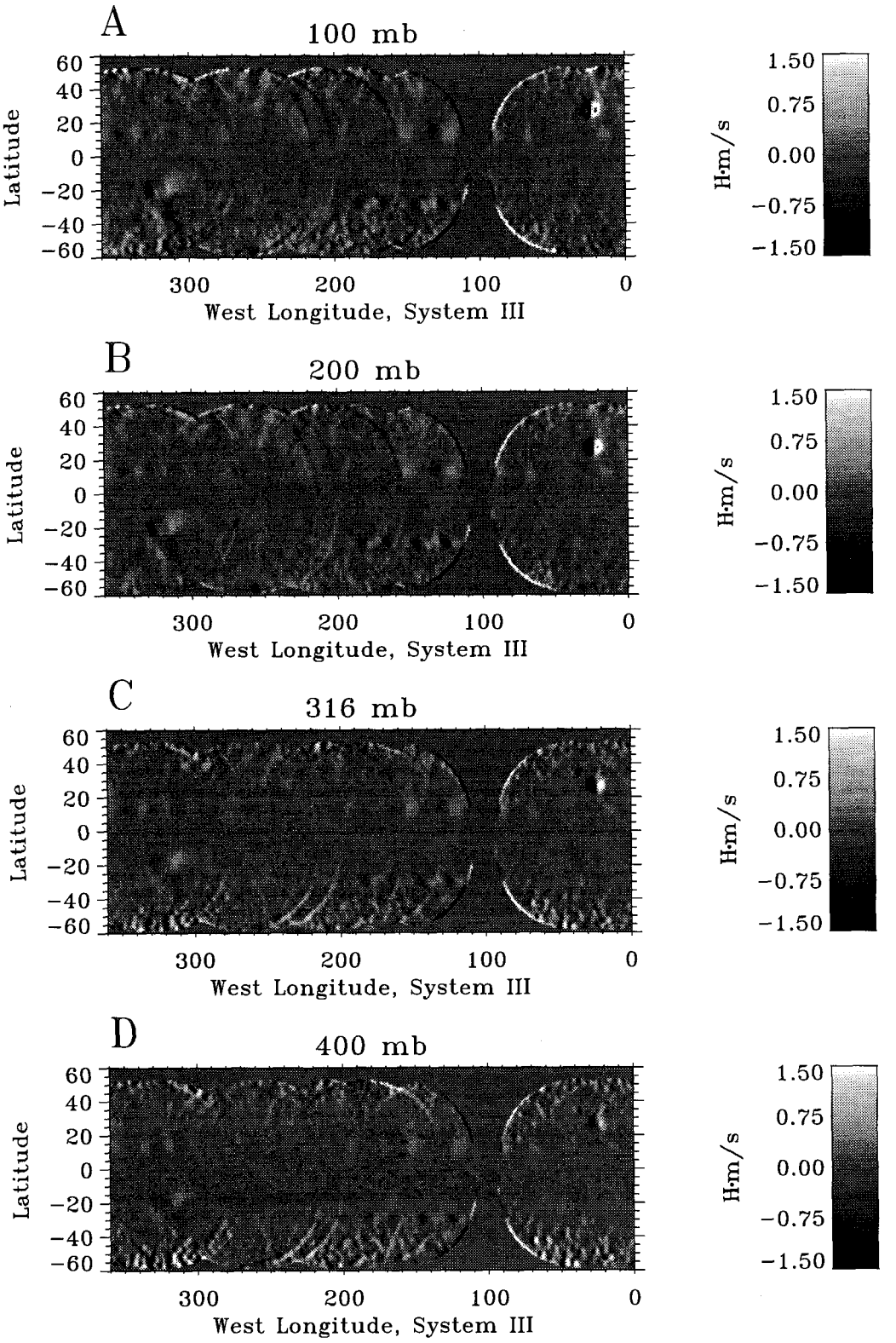


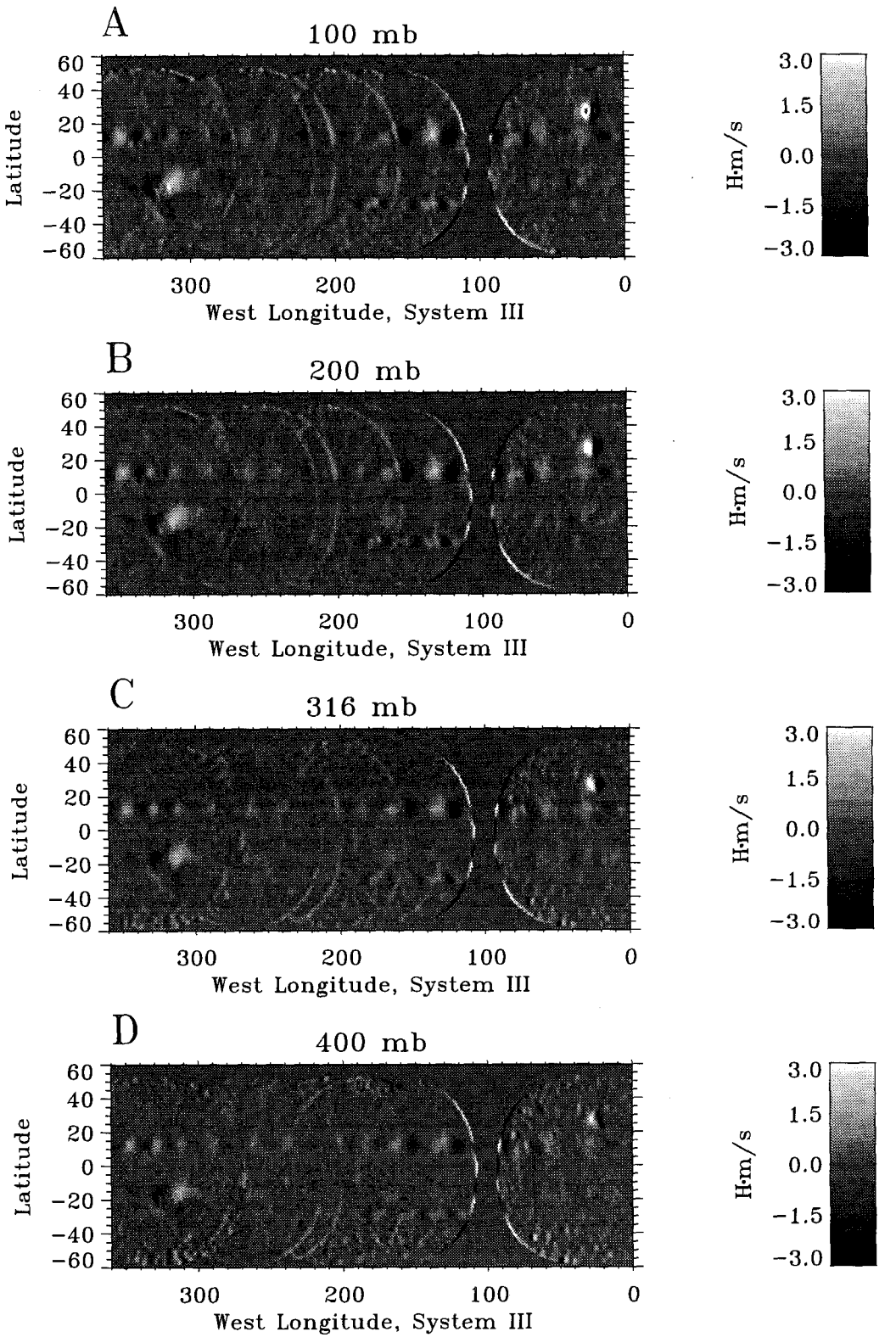
Figure 4d


A
100 mb
B
200 mb
C
316 mb
D
400 mb
Latitude
60
40
20
0
−20
−40
−60
300
200
100
0
West Longitude, System III
Hm/s
1.50
0.75
0.00
−0.75
−1.50

Figures 5a-d


A
100 mb
B
200 mb
C
316 mb
D
400 mb
Latitude
60
40
20
0
−20
−40
−60
300
200
100
0
West Longitude, System III
H·m/s
1.50
0.75
0.00
−0.75
−1.50

Figures 6a-d


A
100 mb
B
200 mb
C
316 mb
D
400 mb
Latitude
60
40
20
0
−20
−40
−60
300
200
100
0
West Longitude, System III
H·m/s
3.0
1.5
0.0
−1.5
−3.0

Figures 7a-d