QUANTUM METROLOGY

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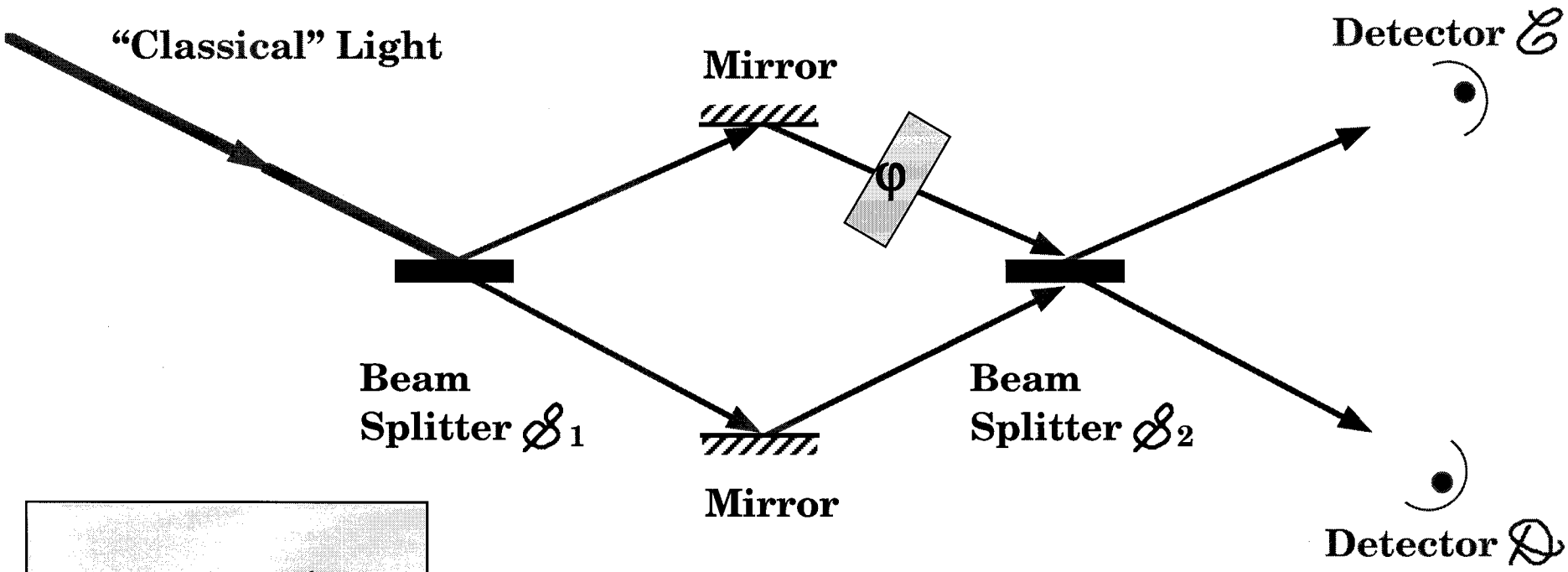
*** With help from: A. N. Boto, D. S. Abrams, S. L. Braunstein, P. Kok, G. H. Hockney H. Lee, I. K. Kulikov, U. H. Yurtsever, D. V. Strekalov, & C. P. Williams**

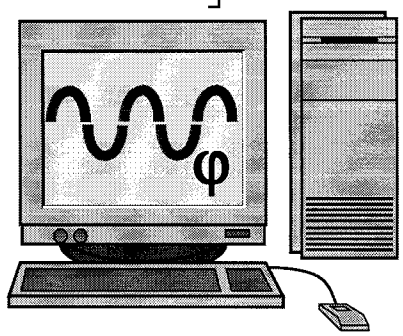
CLASSICAL OPTICAL INTERFEROMETER



$$\Delta\varphi \propto \frac{1}{\sqrt{I}}$$

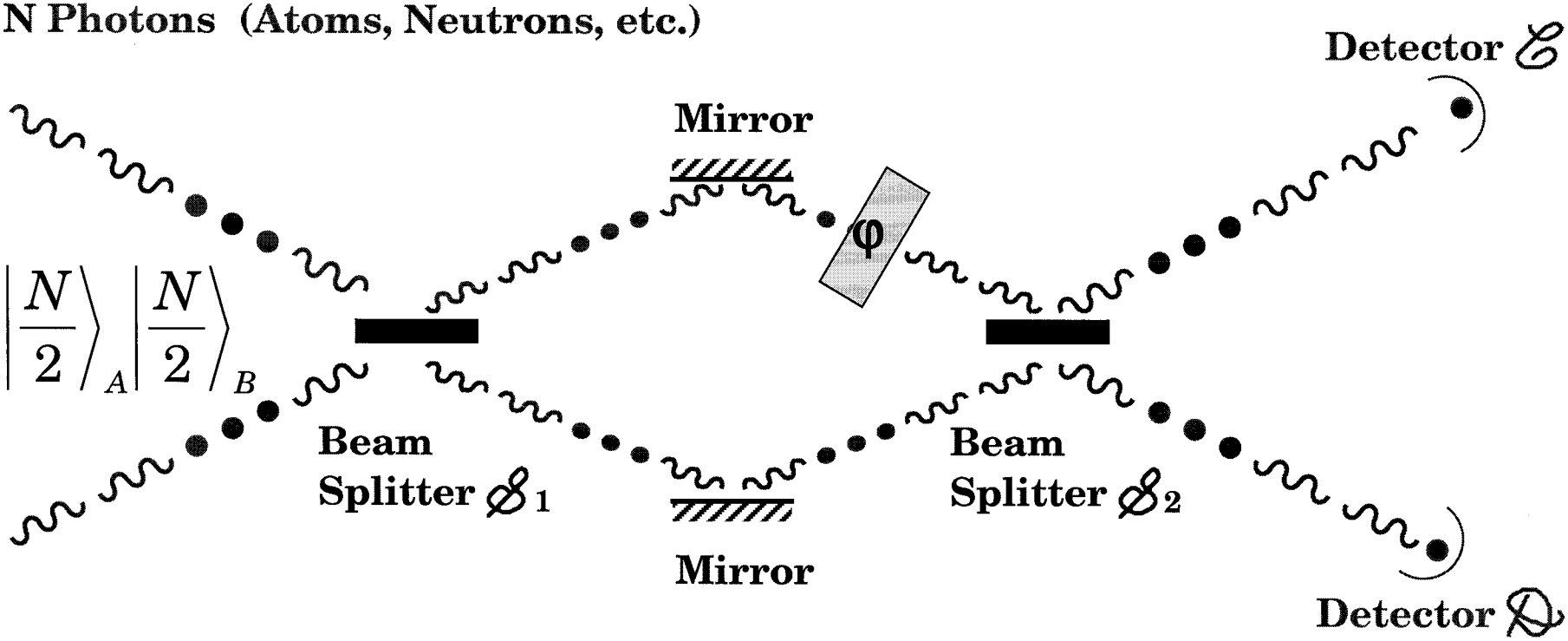
Phase Noise Scaling with Optical Intensity I .

$$\varphi = 2\pi x / \lambda$$



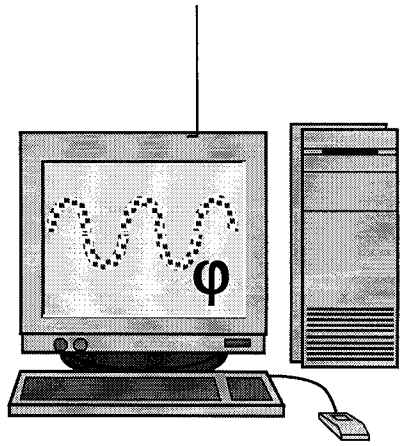
QUANTUM INTERFEROMETER

N Photons (Atoms, Neutrons, etc.)

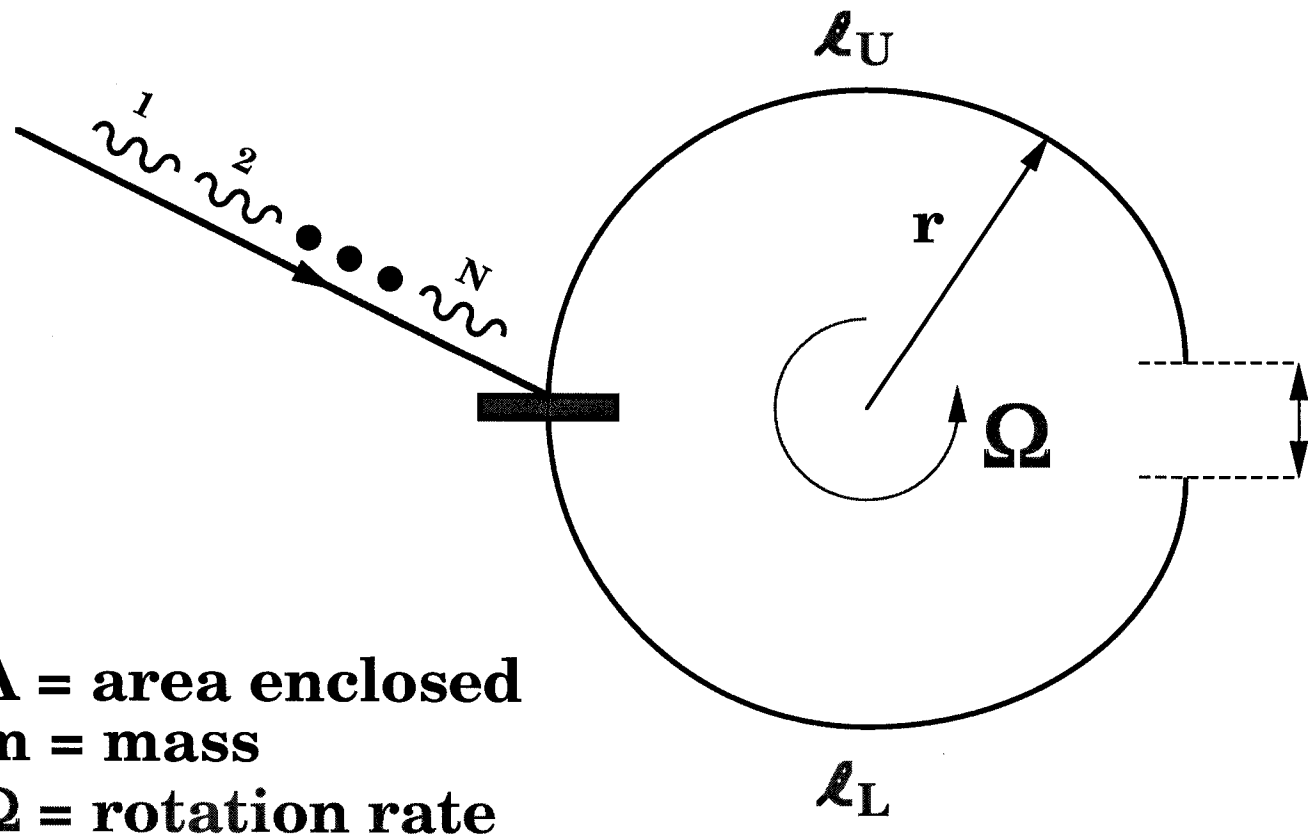


$$\Delta\varphi \propto \frac{1}{N}$$

Phase Noise Scaling with Intensity $I \propto N$.



INTERFEROMETER AS A GYRO



$$\delta\ell = r\Omega \bullet \frac{\pi r}{v}$$

A = area enclosed
m = mass
Ω = rotation rate

$m_{photon} = \hbar\omega/c^2$

$\lambda_{atoms} = h / mv$

$$\varphi_{signal} = 2\,A\,m\,\Omega / \hbar$$

SENSITIVITY ENHANCEMENT

One-Port Matter over One-Port Optical Gyro = 10^4
Two-Port Optical over One-Port Optical Gyro = 10^8
Two-Port Matter over One-Port Optical Gyro = 10^{10}

- Inertial Guidance
- Accelerometry
- Gravimetry / Relativity
- Geological Surveying

Dowling JP, PHYS REV A 57: (6) 4736-4746 JUN 1998.

INTERACTION-FREE IMAGING

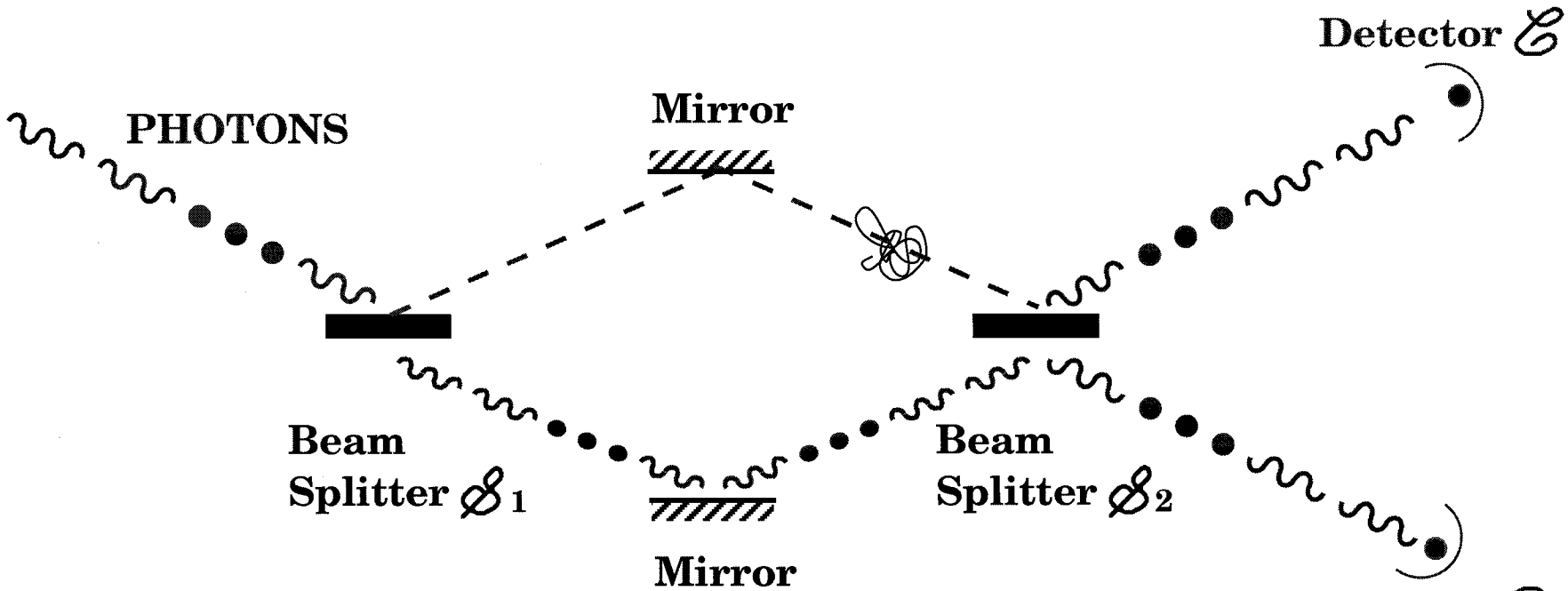
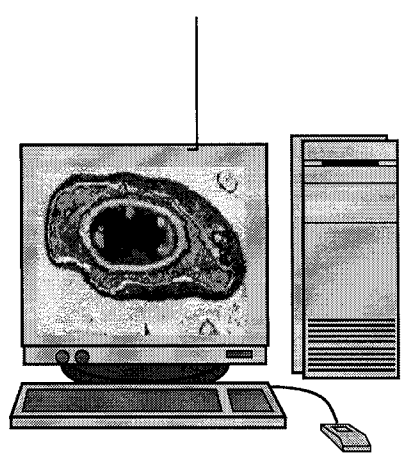
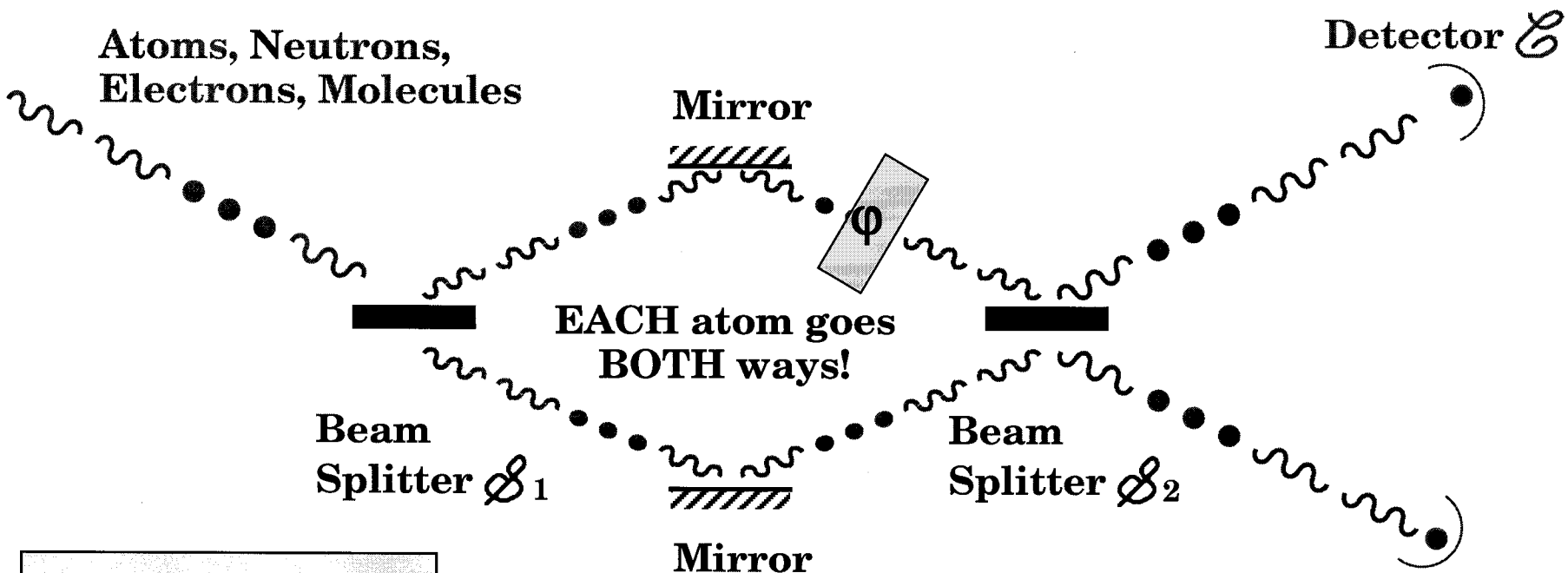


Image of object formed even though NO photons actually encounter object. "Quantum seeing in the dark."



White AG, Mitchell JR, Nairz O, Kwiat PG
PHYSICAL REVIEW A, 58: (1) 605-613 JUL 1998.

INCOHERENT MATTER-WAVE INTERFEROMETER

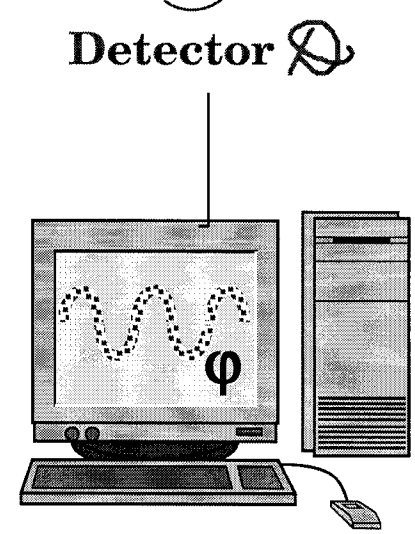


$$\lambda_{matter} = h / mv$$

v = velocity
m = mass
h = Planck's

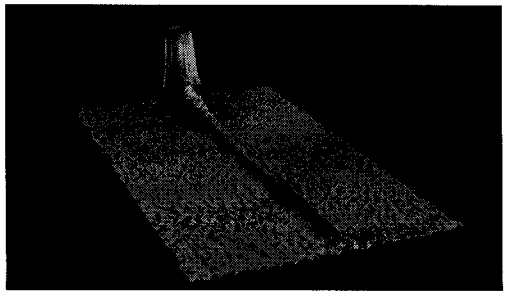
$$\Delta\varphi \propto \frac{1}{\sqrt{N}}$$

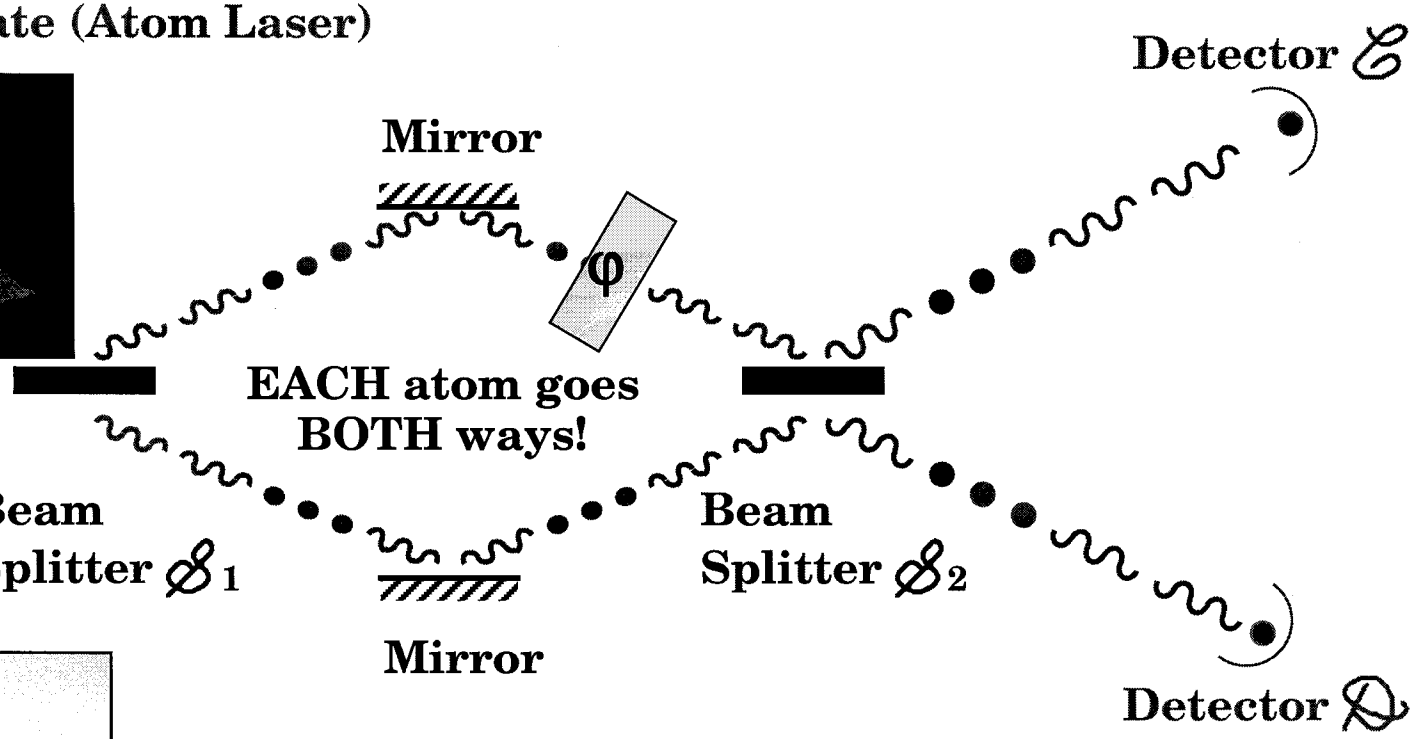
Massive quantum particles are interferometrically much more sensitive to gravitational and inertial effects than photons.



COHERENT ATOM-LASER INTERFEROMETER

Bose Condensate (Atom Laser)



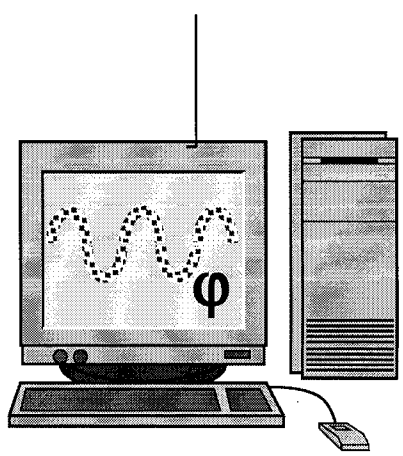


$$\lambda_{matter} = h / mv$$

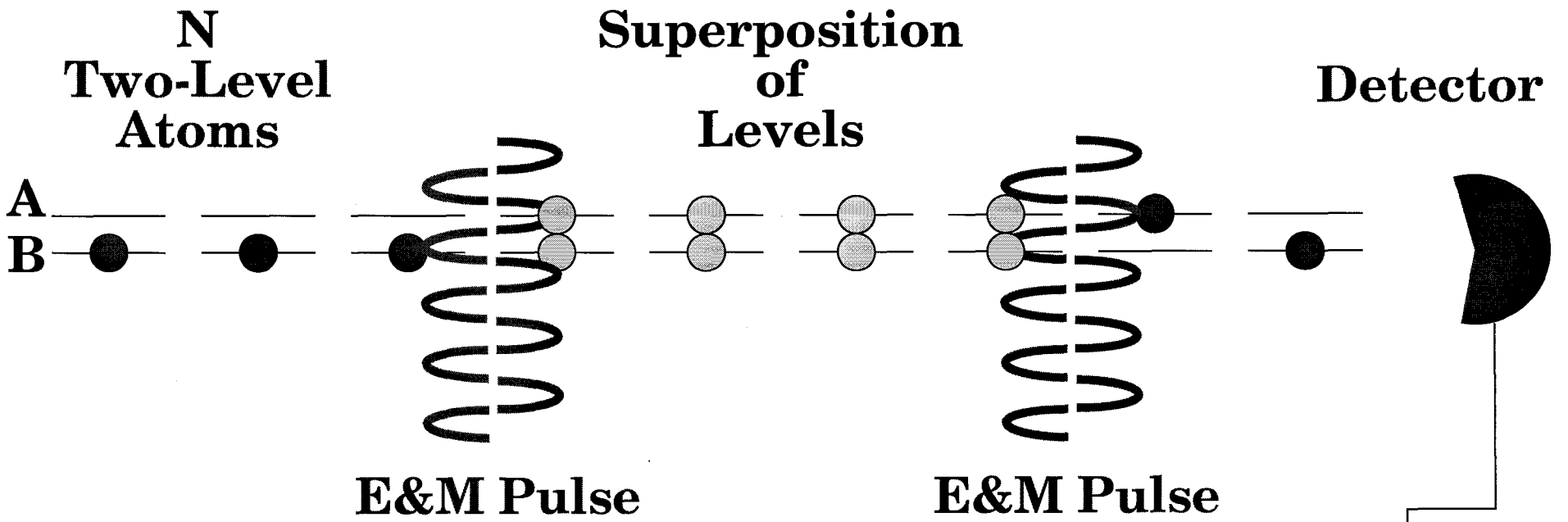
v = velocity
m = mass
h = Planck's

$$\Delta\varphi \propto \frac{1}{N}$$

Coherence gives rise to entanglement that can be uses to boost signal to noise by quadratic factor.



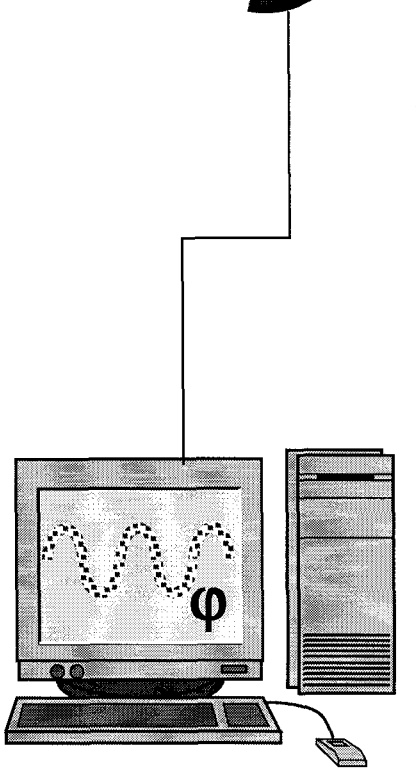
ATOMIC CLOCK INTERFEROMETER



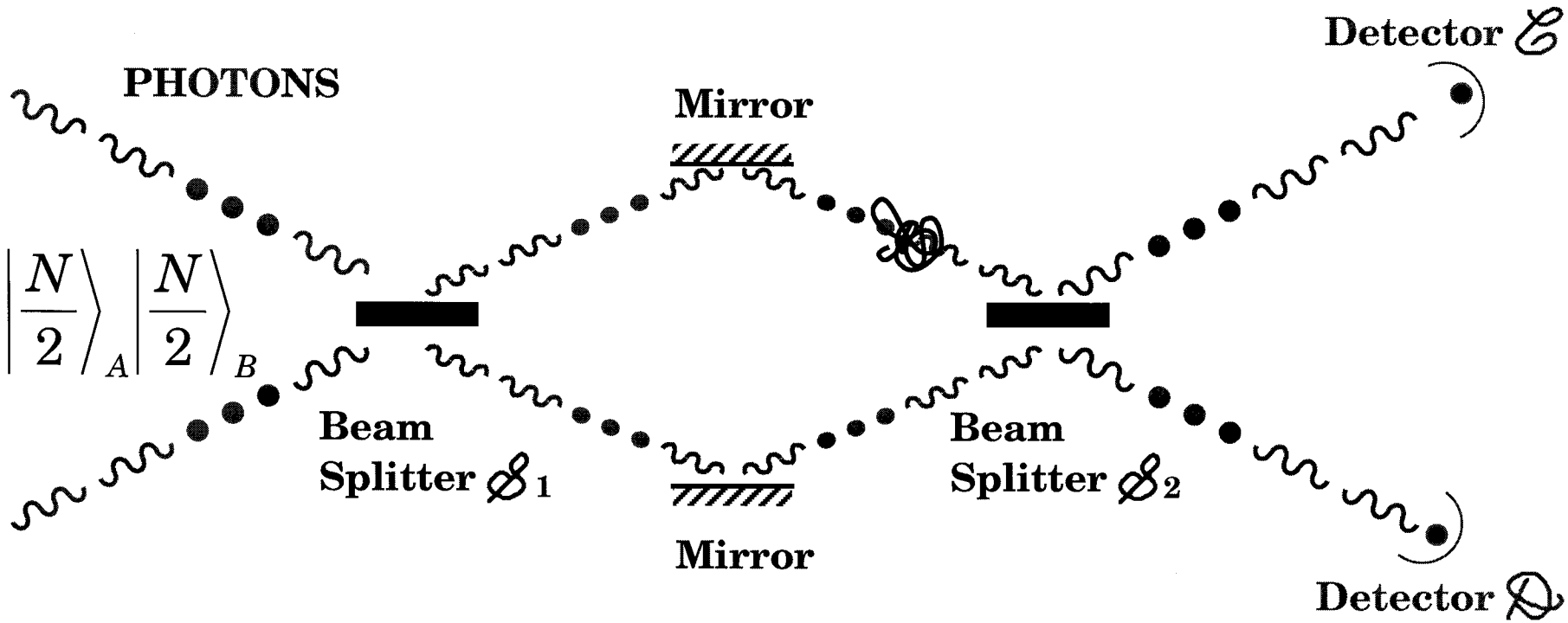
Interference in frequency and time, rather than in wavelength and space. Use same tricks to improve signal to noise quadratically.

$$\varphi = \omega T$$

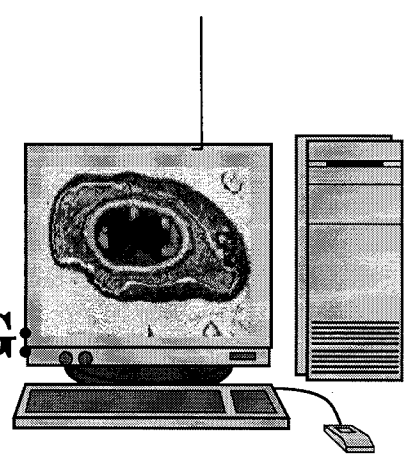
$$\Delta\varphi: 1/\sqrt{N} \rightarrow 1/N$$



SUB WAVELENGTH MICROSCOPY

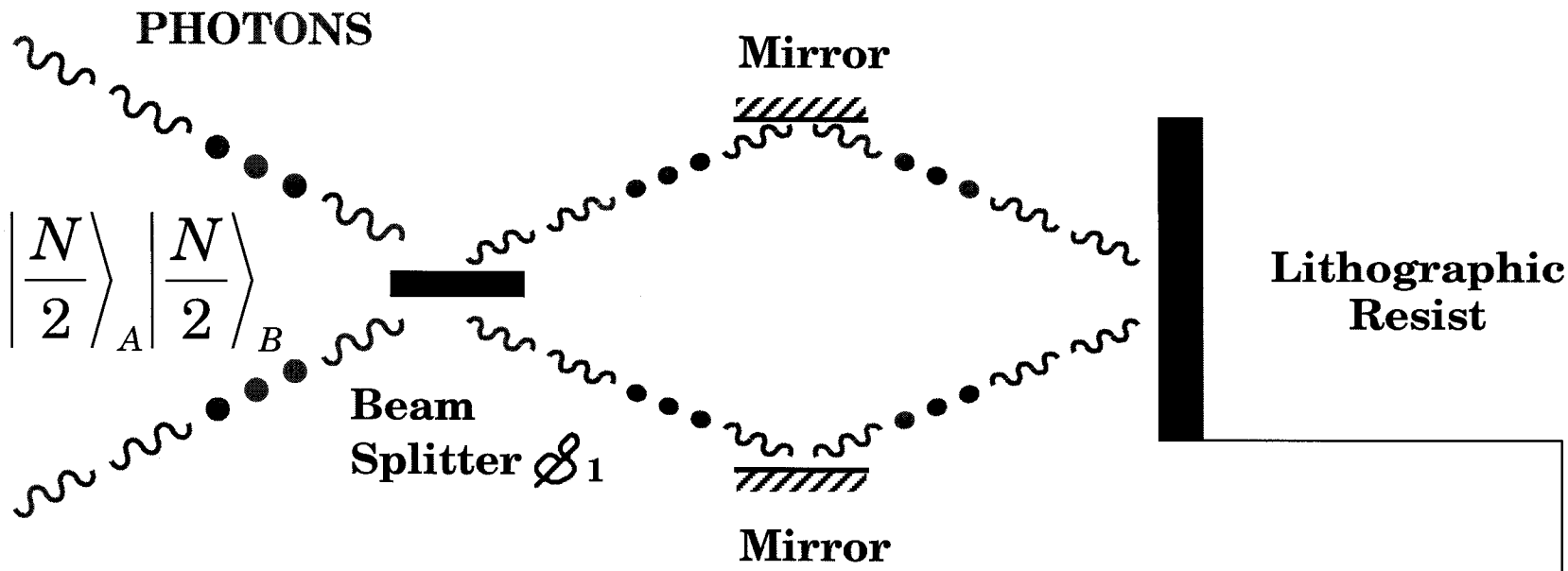


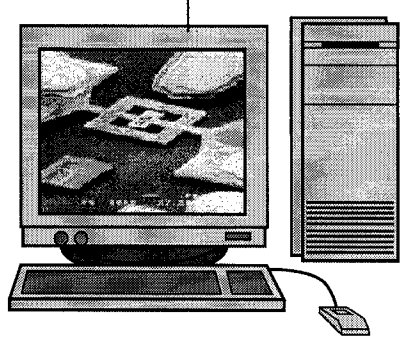
Classical microscope resolves features $\Delta x > \lambda/2$.
Quantum device has resolution $\Delta x > \lambda/2N$.



COMBINE WITH NON-INTERACTIVE IMAGING:
Subwavelength resolution with NO PHOTONS!

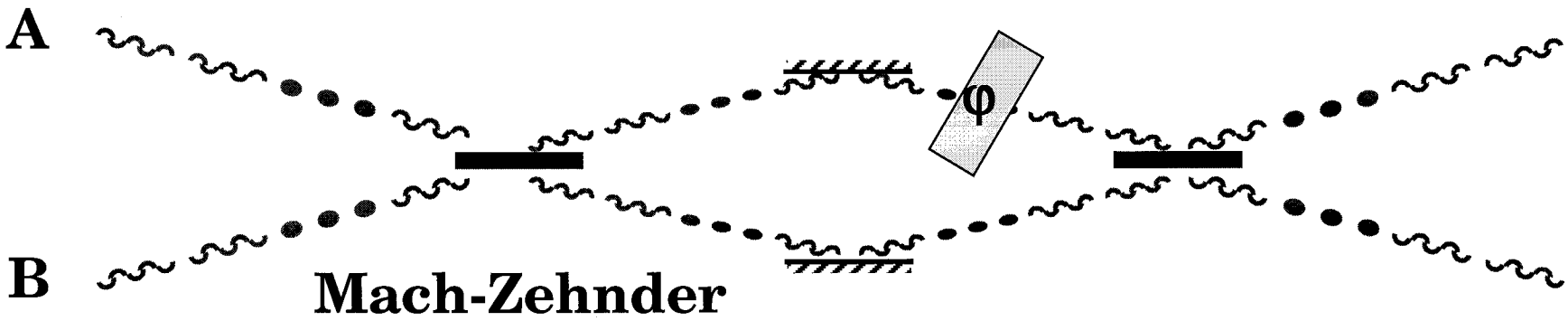
QUANTUM OPTICAL LITHOGRAPHY



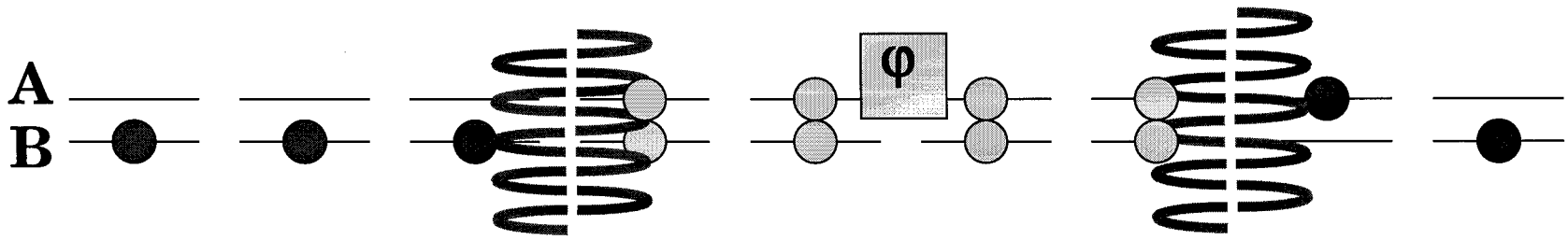


Classical lithography resolves features $\Delta x > \lambda/2$.
Quantum device has resolution $\Delta x > \lambda/2N$.

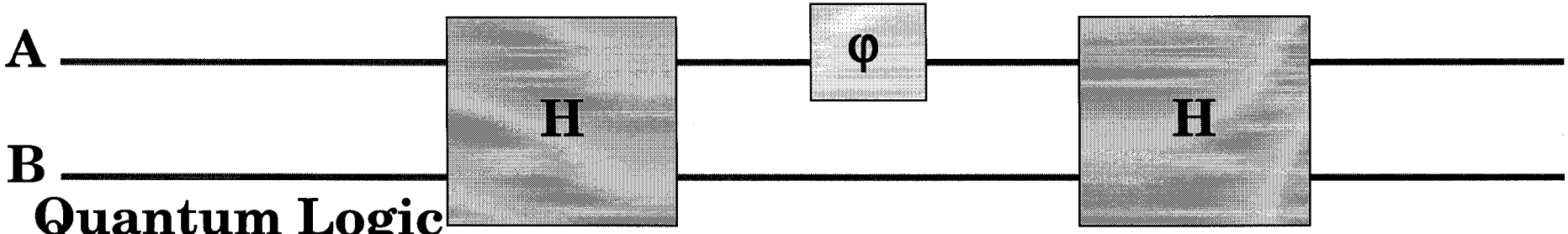
QUANTUM COMPUTER VS INTERFEROMETER



Mach-Zehnder



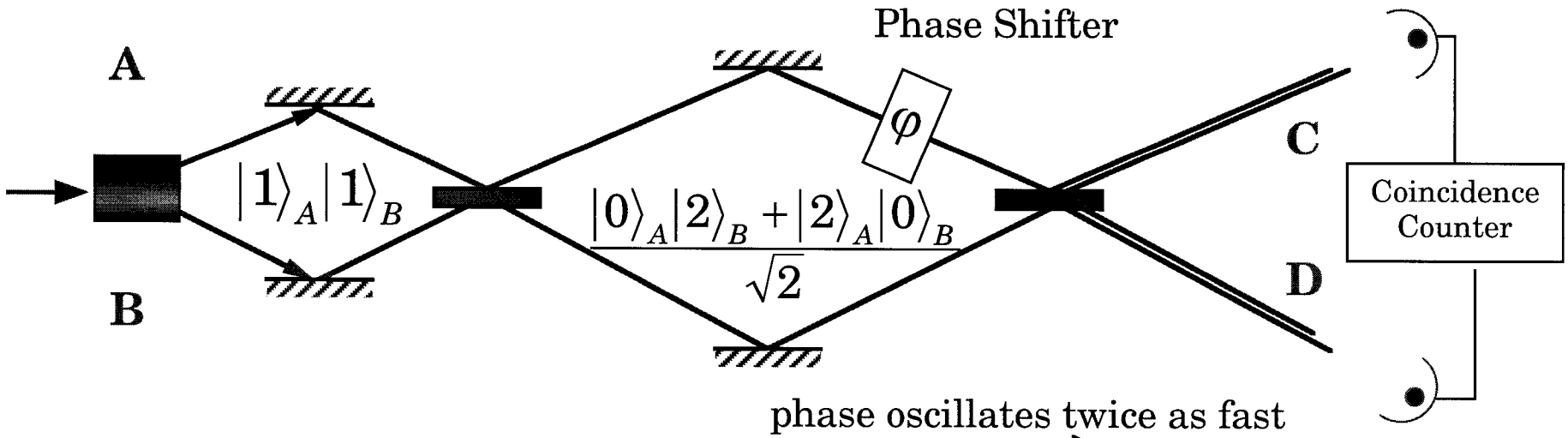
Ramsey Atomic Clock



Quantum Logic Circuit

ORIGIN OF THE LITHO EFFECT

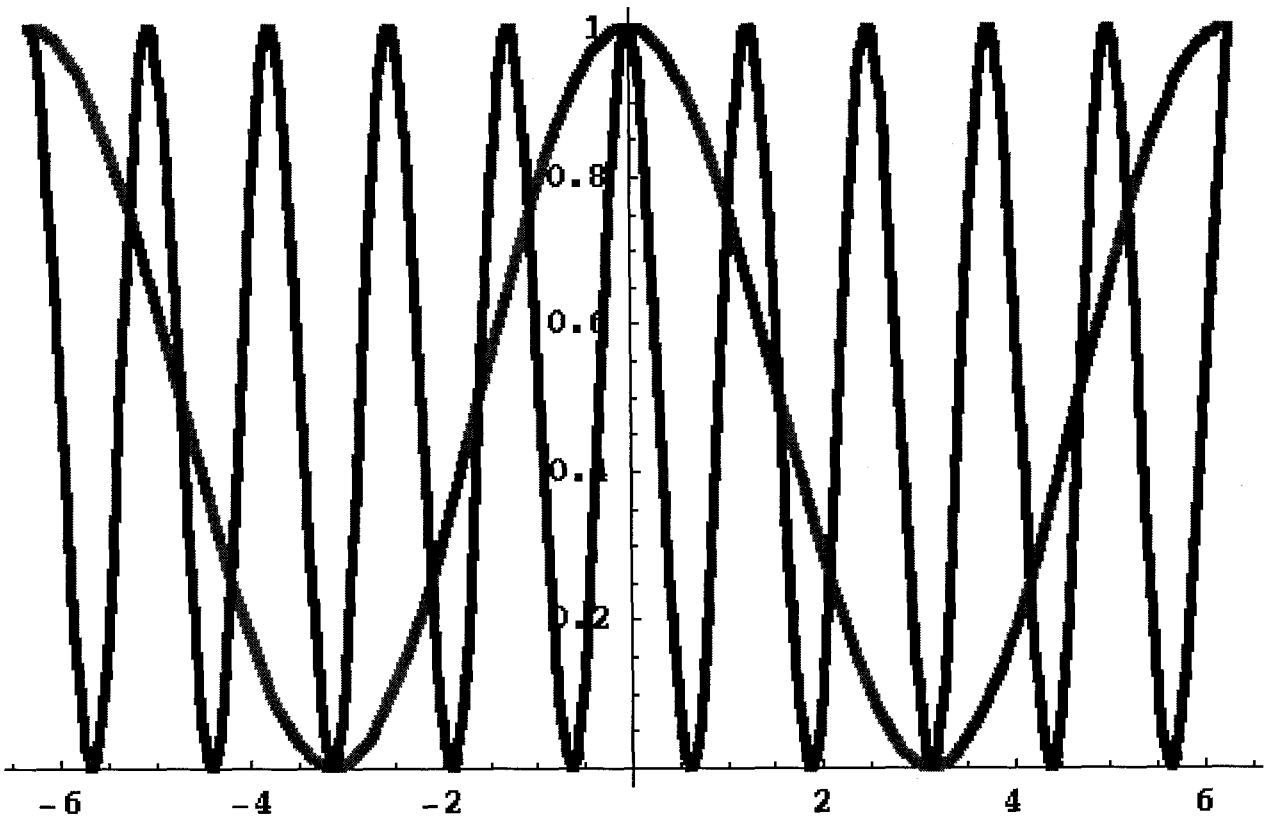
Parametric Downcoversion



phase oscillates twice as fast

$$\frac{|0\rangle_A|2\rangle_B + |2\rangle_A|0\rangle_B}{\sqrt{2}} \rightarrow \frac{|0\rangle_A|2\rangle_B + e^{2i\varphi}|2\rangle_A|0\rangle_B}{\sqrt{2}}$$

COINCIDENCE COUNTS

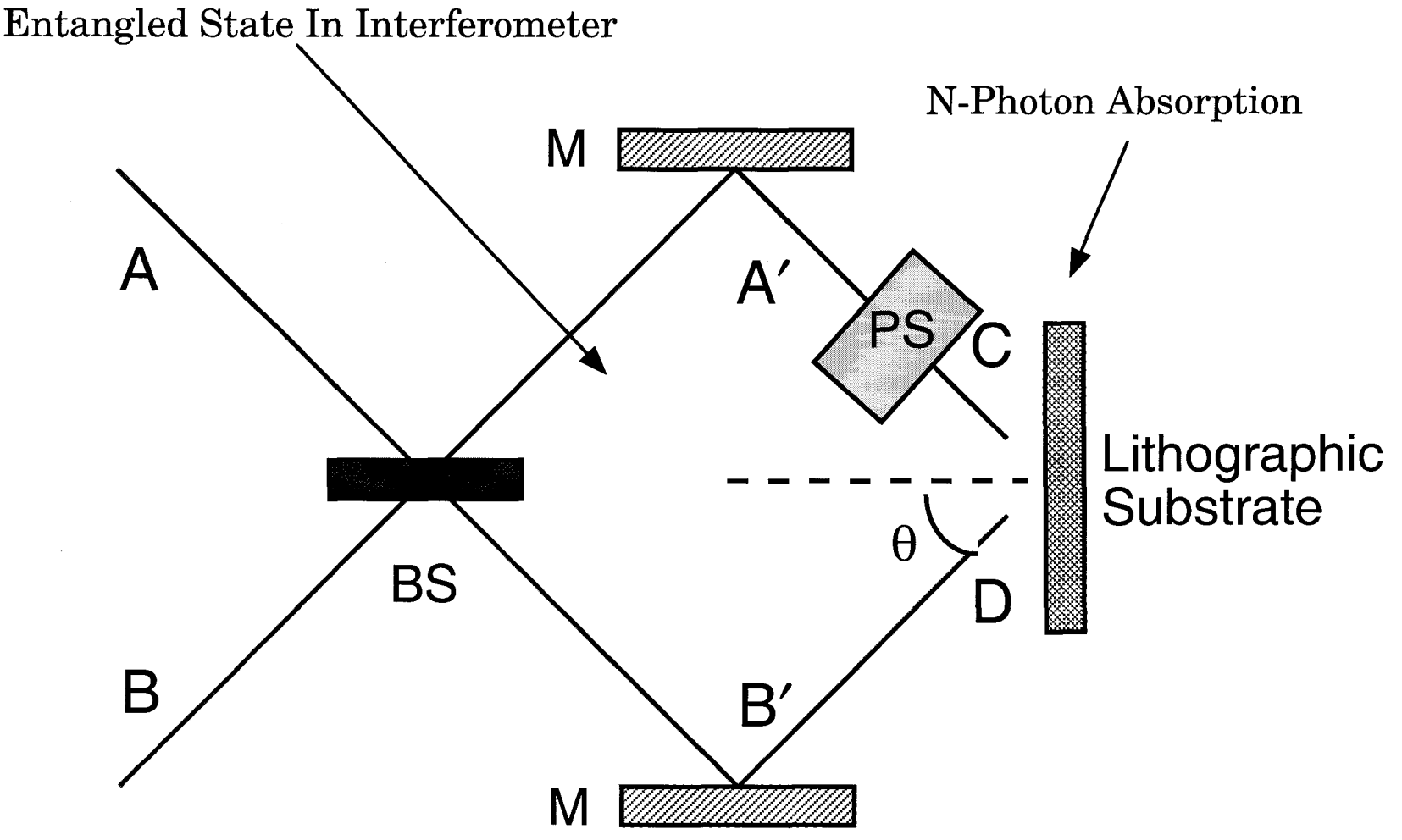


$$\frac{1+\cos\varphi}{2}$$

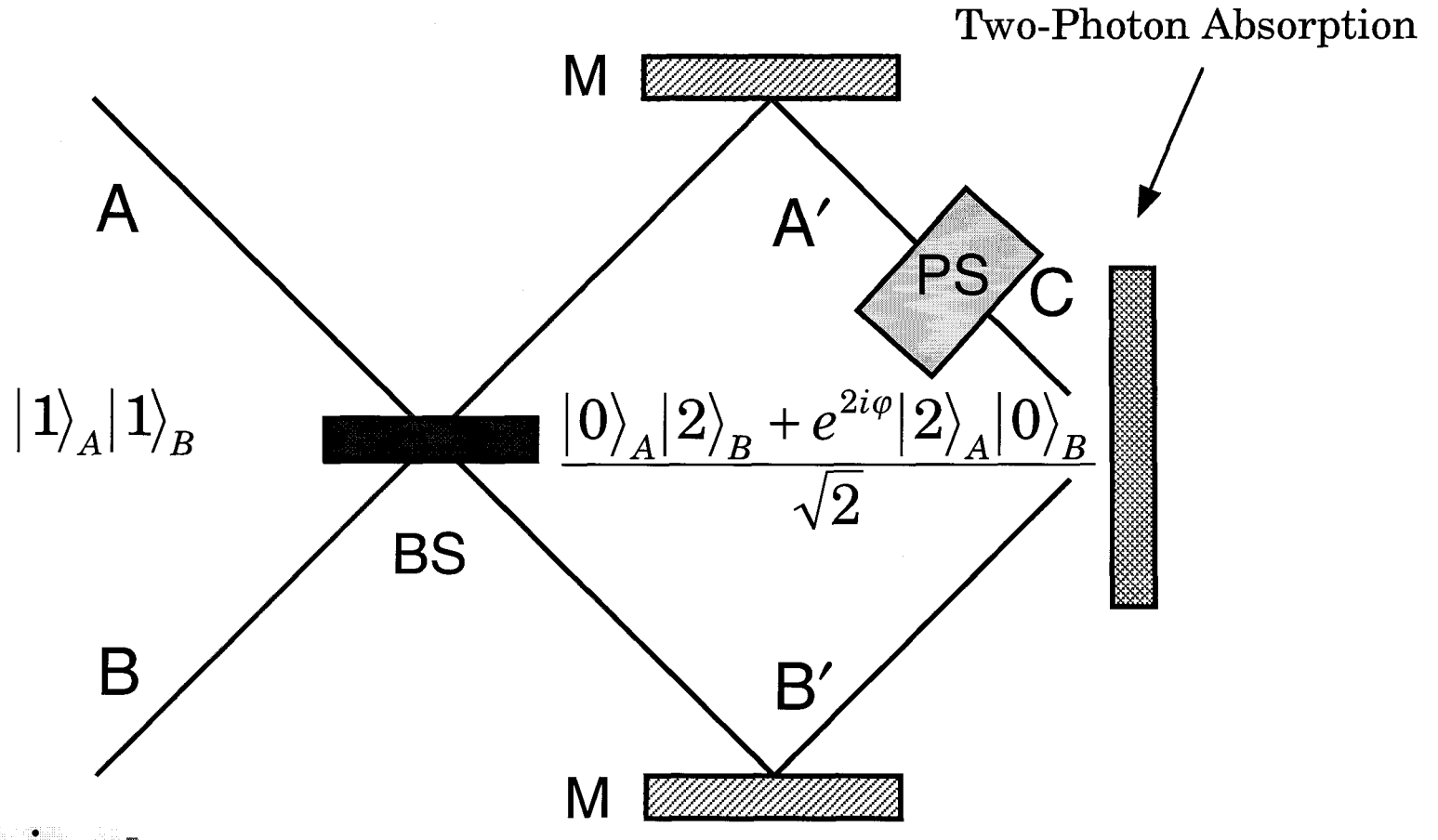
$$\frac{1+\cos N\varphi}{2}$$

Coincidence pattern with N correlated photons oscillates N times as fast as with uncorrelated photons. Phase shift is N times easier to spot, but signal goes down.

THIS PATTERN EXISTS IN REAL SPACE!



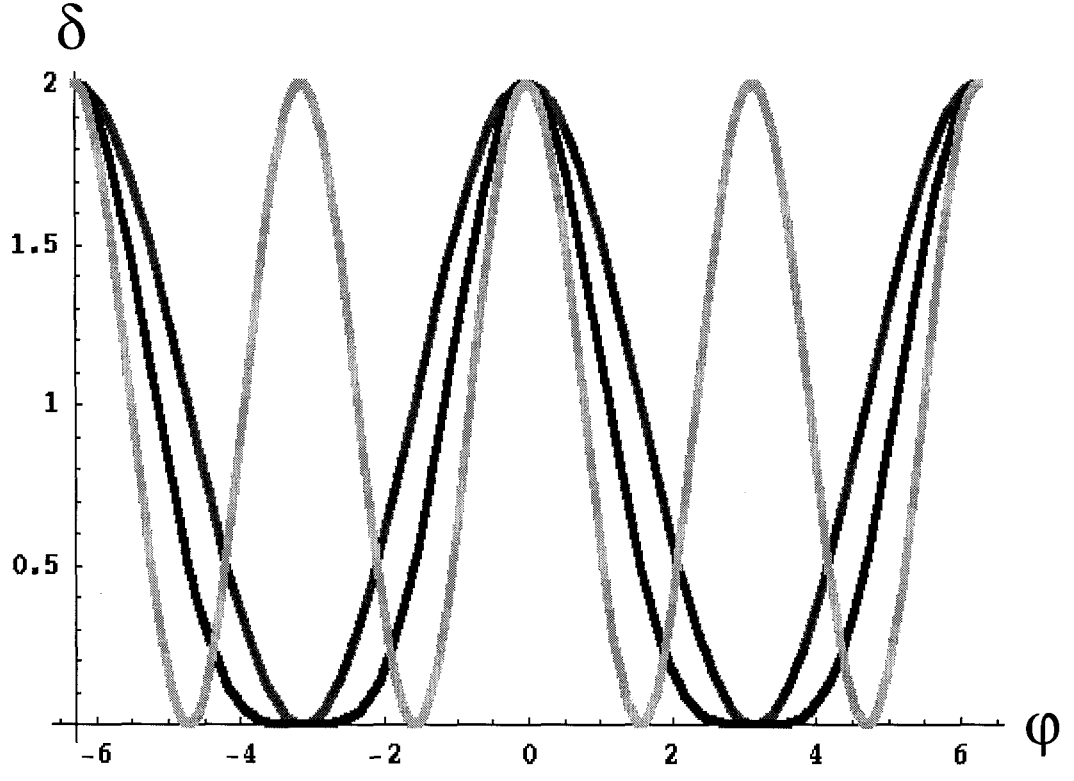
THE CASE FOR N=2



Parametric Downcoversion

TWO-PHOTON ABSORPTION RATE

$$\hat{\delta}_N = \left(\hat{e}^\dagger\right)^N \left(\hat{e}\right)^N / N!$$ deposition operator



1-photon

$$1+\cos\varphi$$

uncorrelated 2-photon

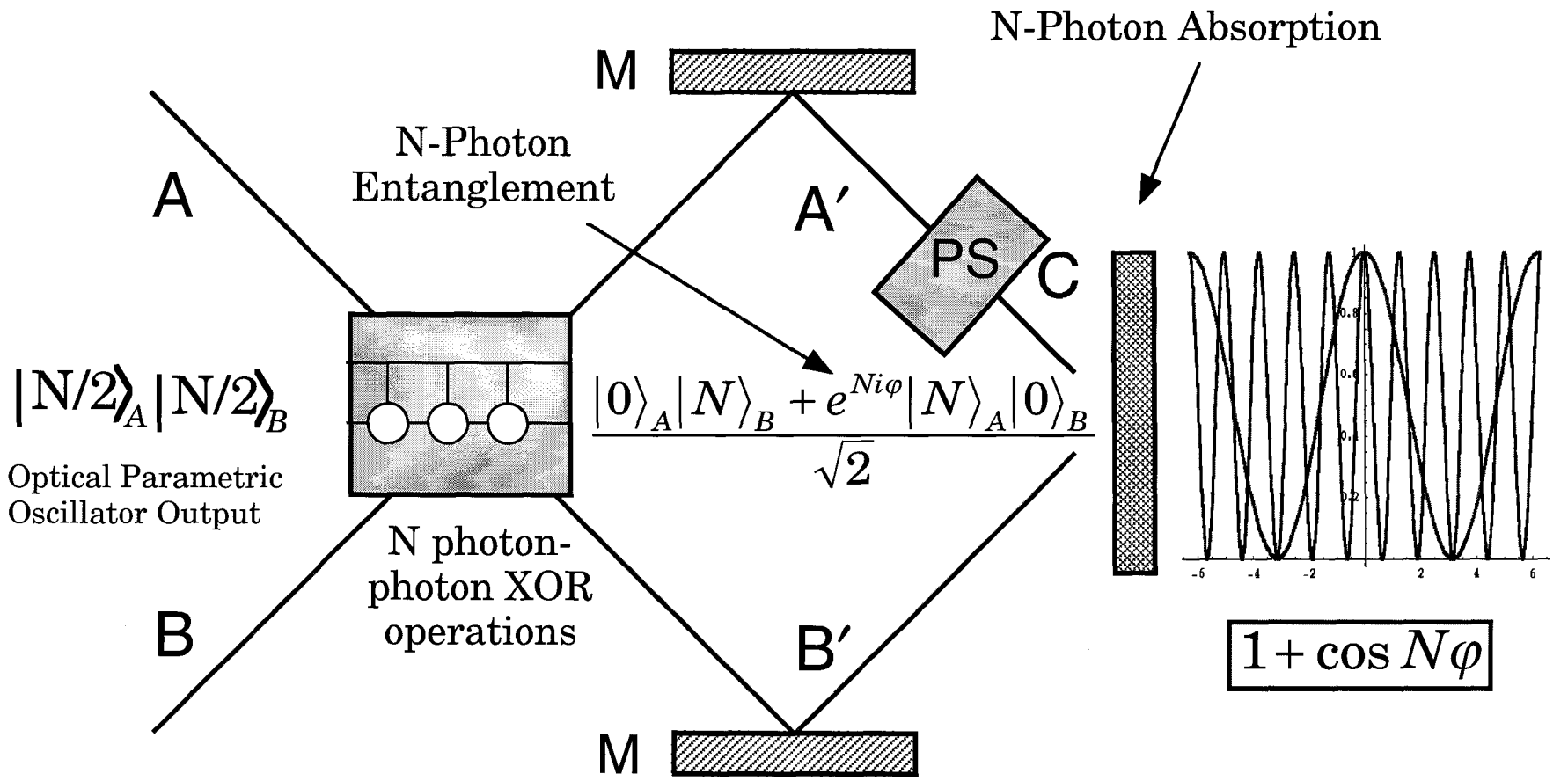
$$\left(1+\cos\varphi\right)^2$$

correlated 2-photon

$$1+\cos 2\varphi$$

Beats Rayleigh Diffraction Limit by Factor of Two!

THE CASE FOR GENERAL N



Beats Rayleigh Diffraction Limit by Factor of N (!)

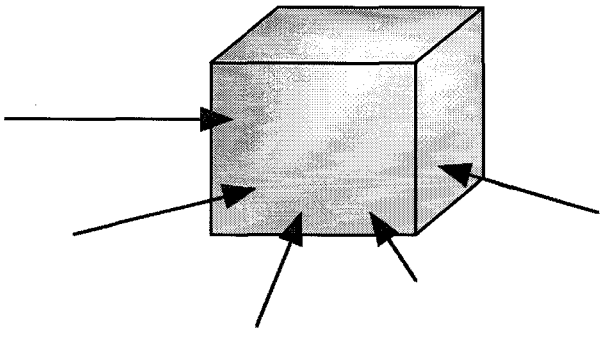
SCALING LAWS

Uncorrelated N-Photon Absorption Probability

$$P \propto I^N$$

Correlated N-Photon Absorption Probability

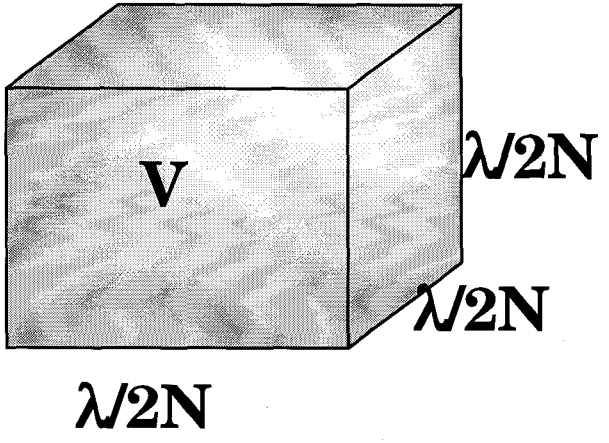
$$P \propto I$$



P is the probability of finding N photons in a unit volume per unit time. Hence low intensities for entangled photons will do.

SCHMUELIAN DEATH RAY

(Romulan Disrupter?)

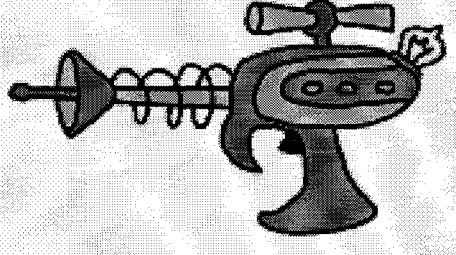


Volume: $V = (\lambda/2N)^3$

Energy: $E=Nh\omega$

Energy Density: $u=E/V = 64\pi hcN^4/\lambda^4$

N=4	Atom Ionization
$N=10^3$	Thermonuclear Fusion
$N=10^6$	Nuclear Disruption into Quark-Gluon Plasma



Entangled state behaves like a single photon of wavelength $\lambda eff = \lambda/N$ (gamma ray laser).

TABLE OF CONTENTS

- Interferometer theory.
- Correlated Input-Ports
- Atom Laser Interferometer and Gyro
- Optical Interferometry
- Quantum Lithography
- Schmuelian Death Ray

ADVANTAGES OF QUANTUM METROLOGY

- Atom Lasers Provide Coherent Beams of Matter-Waves.

- Simple Matter-Waves Gyros can be 10^4 Times More Sensitive to Inertial Effects than Light.

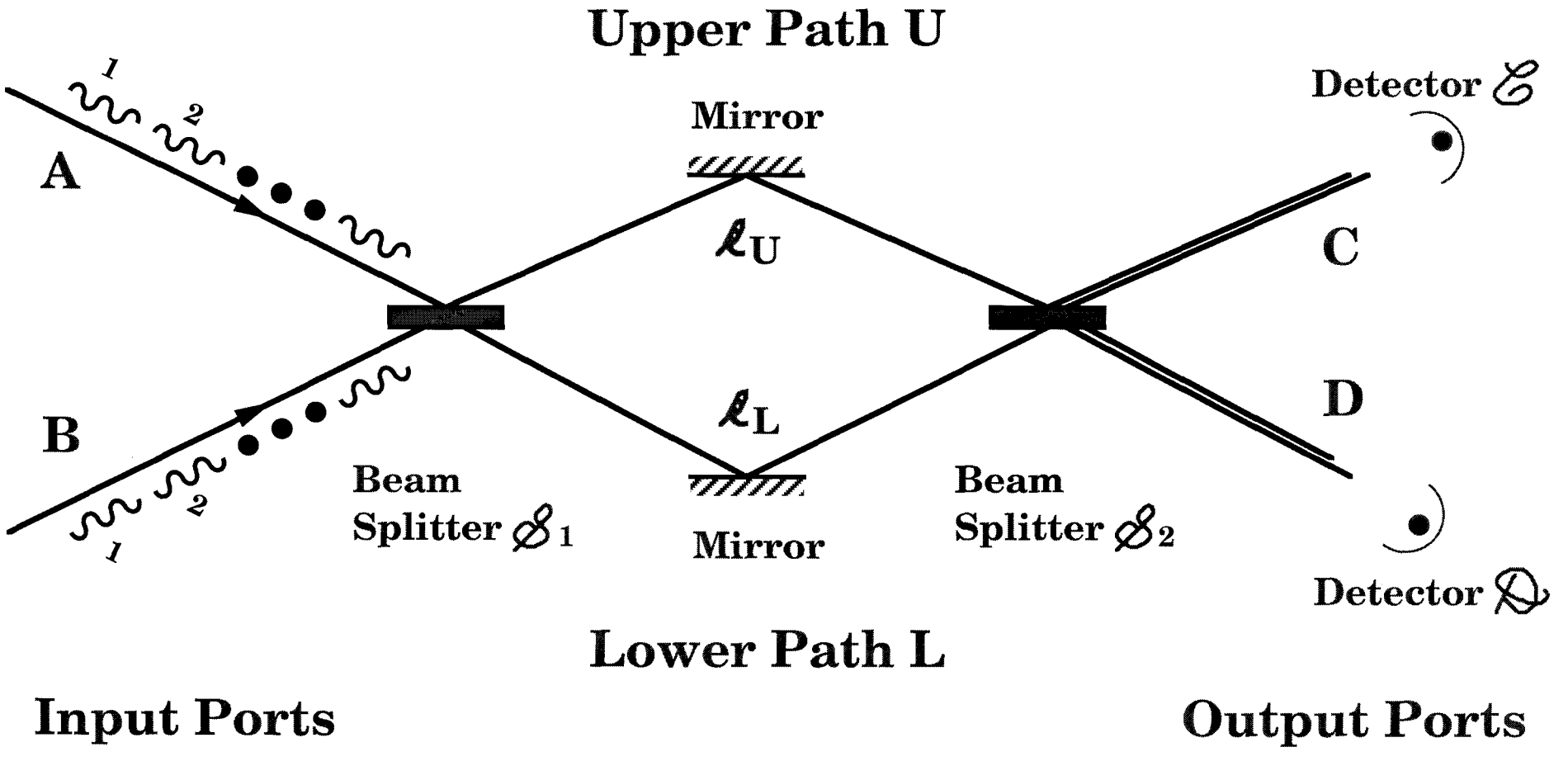
- Optical Gyroscopes Have Reached The Quantum Limits to Improved Sensitivity.

- Atom Laser Gyros that Utilized Quantum Superposition Effects Could be 10^{10} Times More Sensitive to Rotation than Ordinary Laser Gyros.

- Applications are to Inertial Navigation, Accelerometers, and Gravimeters.

TWO-PORT ATOM INTERFEROMETER

Atomic Matter Waves



INTERFEROMETER THEORY

Out **Scattering Matrix** **In**

$$\begin{bmatrix} \hat{c} \\ \hat{d} \end{bmatrix} = \frac{1}{2} \begin{bmatrix} \left(e^{i\mu} + e^{i\nu}\right) & i\left(e^{i\mu} - e^{i\nu}\right) \\ -i\left(e^{i\mu} - e^{i\nu}\right) & \left(e^{i\mu} + e^{i\nu}\right) \end{bmatrix} \begin{bmatrix} \hat{a} \\ \hat{b} \end{bmatrix}$$

$$\hat{a}\,\hat{a}^{\dagger} \pm \hat{a}^{\dagger}a = 1 \qquad \mu = k\ell_U \qquad \hat{c}\hat{c}^{\dagger} \pm \hat{c}^{\dagger}\hat{c} = 1$$

$$\hat{b}\hat{b}^{\dagger} \pm \hat{b}^{\dagger}b = 1 \qquad \nu = k\ell_L \qquad \hat{d}\hat{d}^{\dagger} \pm \hat{d}^{\dagger}\hat{d} = 1$$

Creation & Annihilation Operators

Bosons = (+)
Fermions = (–)

QUANTUM PHASE NOISE LIMITS

$$\Delta\varphi^{min} \cong \frac{1}{\sqrt{N}}$$

One-Port Gyro

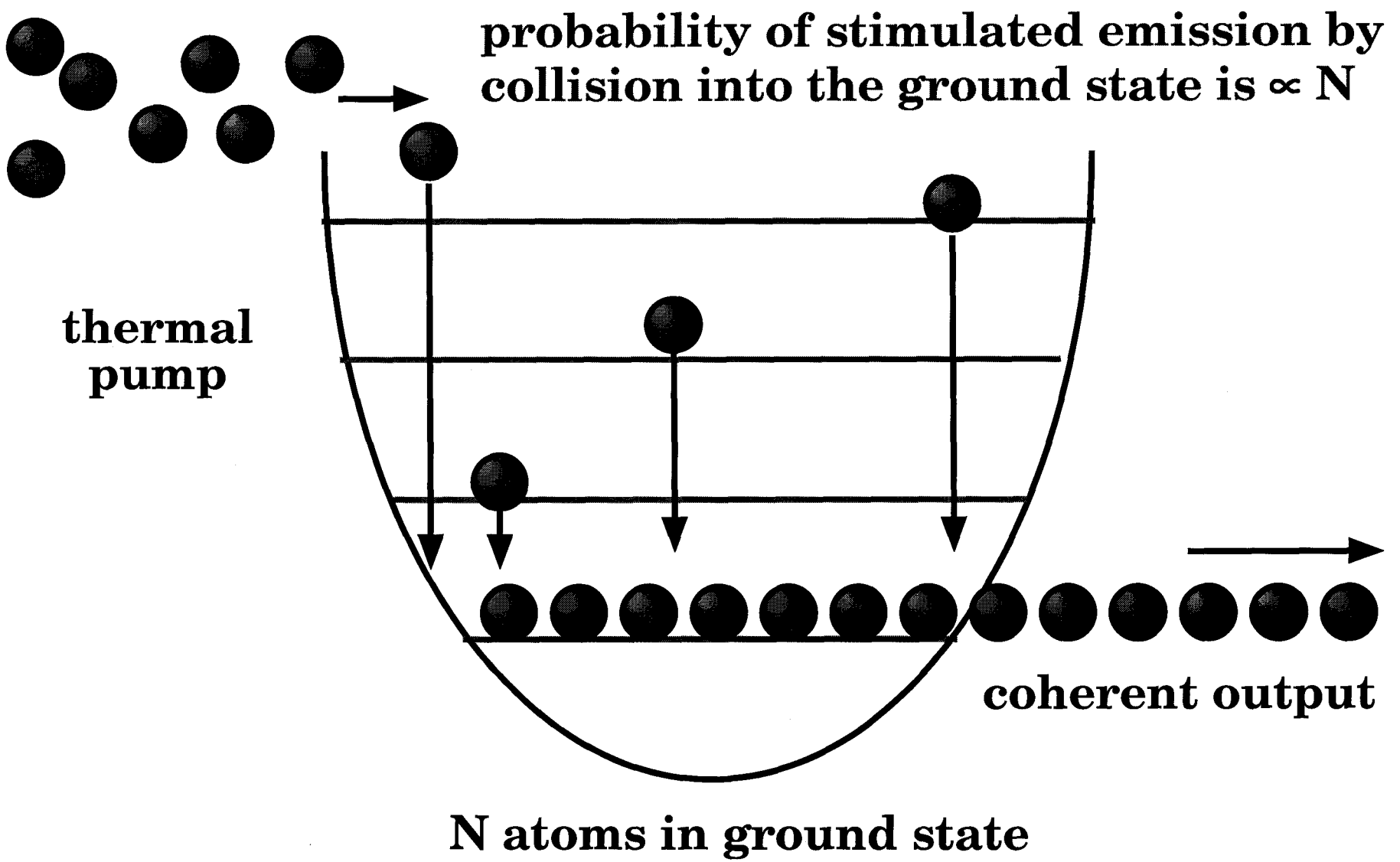
$$|\psi\rangle_I \equiv |N\rangle_A |0\rangle_B$$

$$\Delta\varphi^{min} \cong \frac{2}{N}$$

Correlated Two Input-Port Gyro

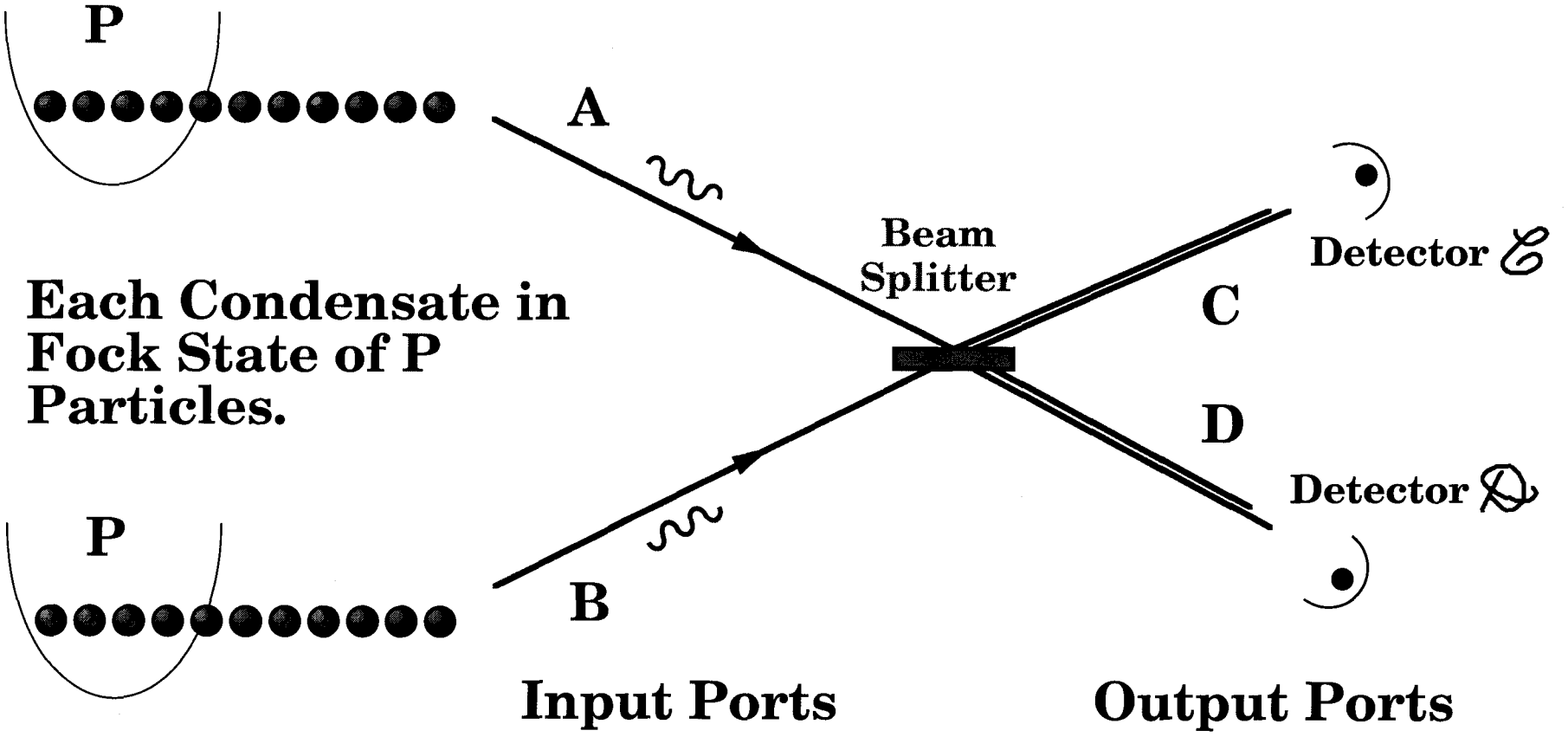
$$|\psi\rangle_{II} \equiv \frac{1}{\sqrt{2}}\left\{ \left|\frac{N+1}{2}\right\rangle_A \left|\frac{N-1}{2}\right\rangle_B + \left|\frac{N-1}{2}\right\rangle_A \left|\frac{N+1}{2}\right\rangle_B \right\}$$

ATOM LASER


probability of stimulated emission by collision into the ground state is ∝ N
thermal pump
coherent output
N atoms in ground state

DUAL-CONDENSATE ENTANGLED STATE PREPARATION

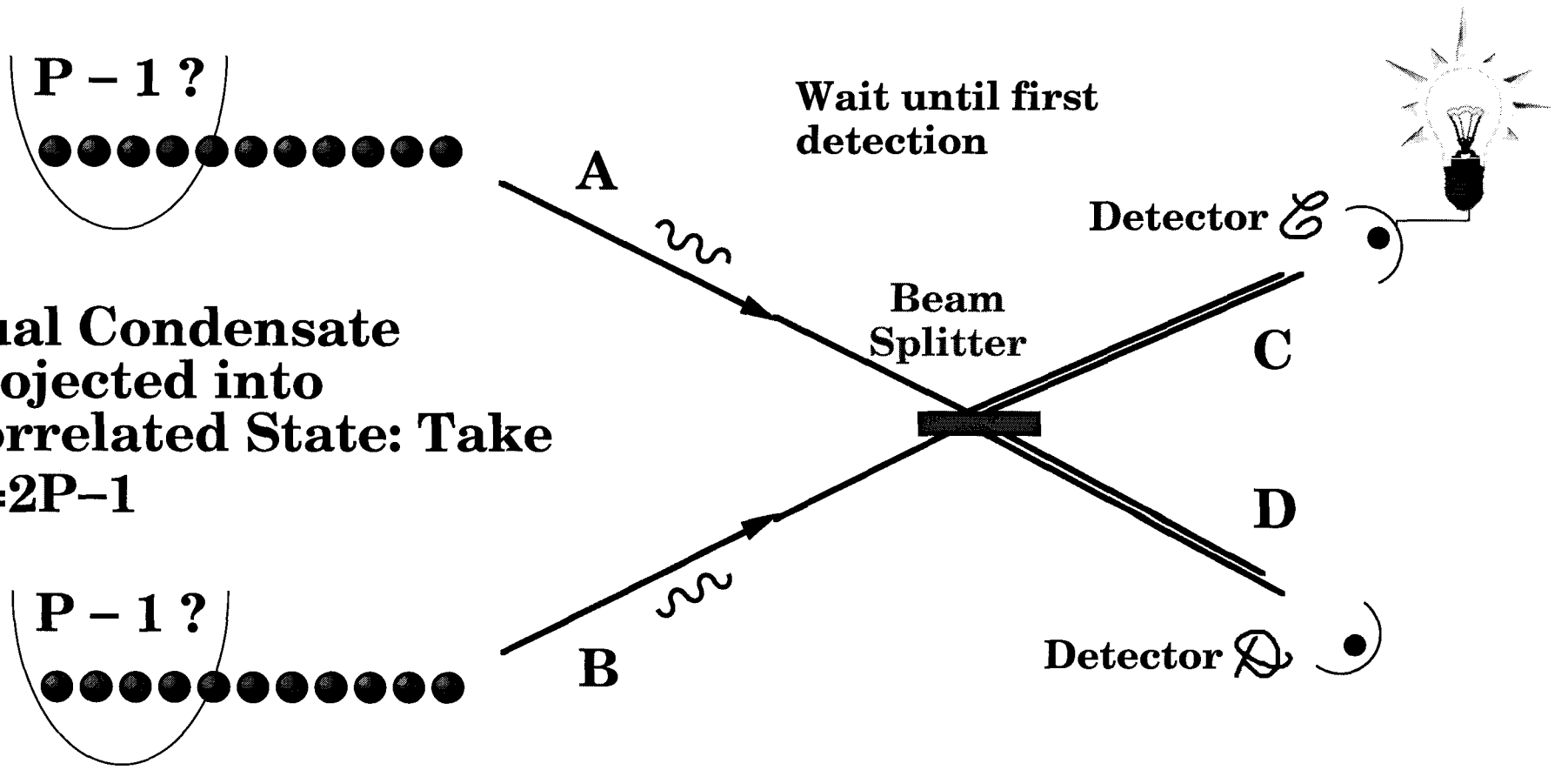
$$|\psi\rangle \equiv |P\rangle_A |P\rangle_B$$



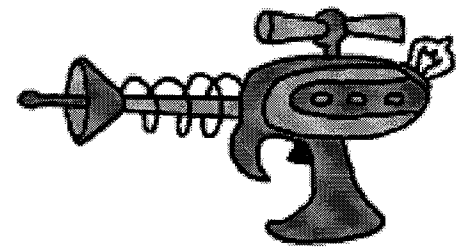
Each Condensate in Fock State of P Particles.

DUAL CONDENSATE AFTER DETECTION

$$|\psi\rangle = \frac{1}{\sqrt{2}}\left[\hat{a} + \hat{b}\right]|P\rangle_A\,|P\rangle_B = \frac{1}{\sqrt{2}}\left[\,|P-1\rangle_A\,|P\rangle_B + |P\rangle_A\,|P-1\rangle_B\right]$$



Dual Condensate Projected into Correlated State: Take N=2P–1

SUMMARY

- Entangled Interferometer theory
- Applications to Atom Laser Interferometer & Gyro
- Gyro Sensitivity Comparison.
- Applications to Atomic Clocks
- Applications to Optical Microscopy
- Applications to Optical Lithography
- Schmuelian Death Ray