

# Low-Frequency Gravitational Wave Searches Using Spacecraft Doppler Tracking

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# Low-Frequency Gravitational Wave Searches Using Spacecraft Doppler Tracking

- The Doppler Method
  - Signal transfer function
  - Noises and their transfer functions
- Experiments to date: some results from GLL/MO/MGS/Pioneer/ULS
- Cassini Experiment
  - Status
  - Expected sensitivity
- Can we do better than Cassini with earth-spacecraft Doppler tracking?

**J. Anderson  
B. Bertotti  
F. Estabrook  
R. Hellings  
L. Iess  
M. Tinto  
H. Wahlquist**

**and many engineers and analysts from the JPL technical divisions, the  
Deep Space Network, and the flight projects**

## SOME JARGON

DSN	Deep Space Network, the NASA/JPL tracking system with antennas in California, Spain, and Australia
S-band	radio frequency $\approx 2.3$ GHz (e.g., Galileo)
X-band	radio frequency $\approx 8.4$ GHz (e.g., Mars Observer)
Ka-band	radio frequency $\approx 32$ GHz (e.g., Cassini)
$y(t)$	time series of $\Delta f/f$
$S_y(f)$	power spectrum of $y(t)$
$S_\phi(f)$	power spectrum of phase

## MORE JARGON

Allan variance

$\sigma_y(\tau)$ , a measure of fractional frequency stability,  $\Delta f/f$ , as a function of integration time

$$\sigma_y^2(\tau) = \frac{1}{2} \left\langle \left| \bar{y}(t) - \bar{y}(t + \tau) \right|^2 \right\rangle$$

$$\bar{y}(t) = \frac{1}{\tau} \int_t^{t+\tau} y(t') dt'$$

$$\sigma_y^2(\tau) = 4 \int_0^{\infty} S_y(f) \frac{\sin^4(\pi f \tau)}{(\pi f \tau)^2} df$$

$$S_y(f) = S_\phi(f) \cdot f^2 f_0^{-2}$$

scintillation

variation of phase of radio signals due to refractive index variations by a medium (solar wind, ionosphere, troposphere) between the source and the receiver

## **JARGON (CONCLUDED)**

**clock**

**precision frequency standard**

**uplink**

**radio beam transmitted from the earth to a distant spacecraft**

**downlink**

**radio beam transmitted from a distant spacecraft to the earth**

**DSS**

**Deep Space Station. Followed by a number it designates antennas within the Deep Space Network, as in "DSS 25"**

# REPRESENTATIVE REFERENCES

Regarding the method:

Estabrook and Wahlquist, *GRG*, 6, 439 (1975)

Wahlquist *GRG*, 19, 1101 (1987)

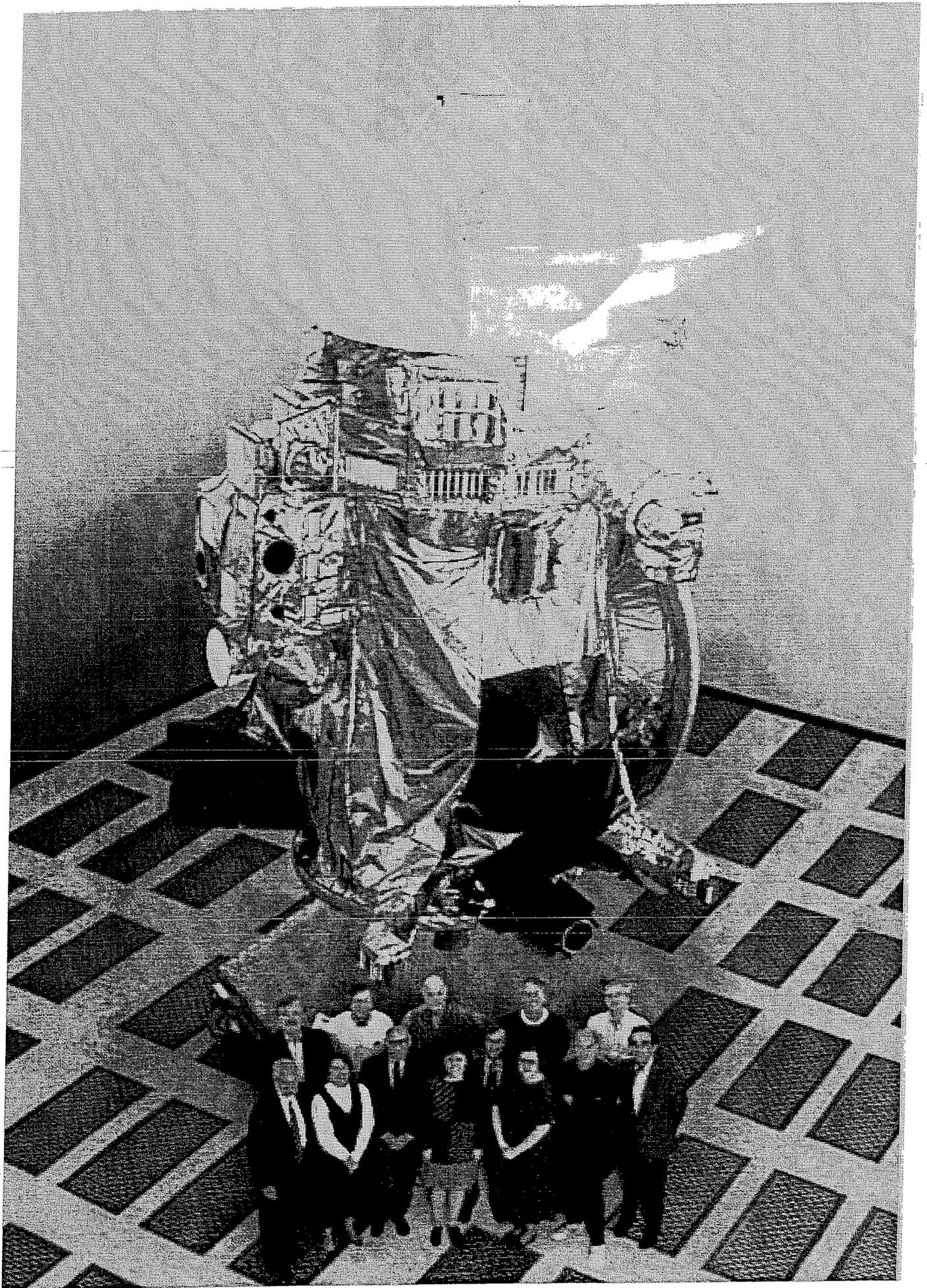
Tinto *Phys. Rev. D.* 53, 5354 (1996)

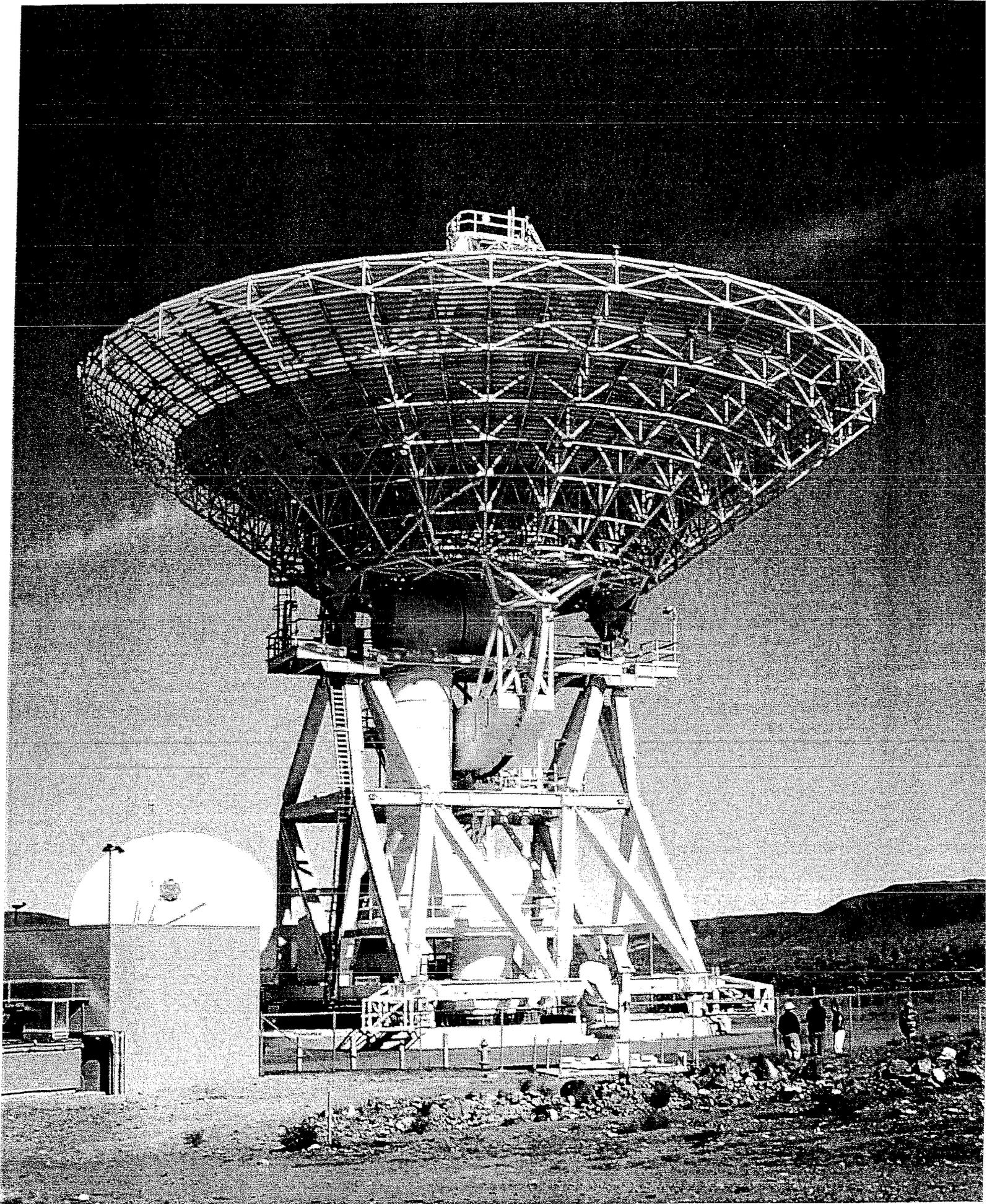
Regarding the noises:

Armstrong, Woo, and Estabrook *Ap. J.* 230, 570 (1979)

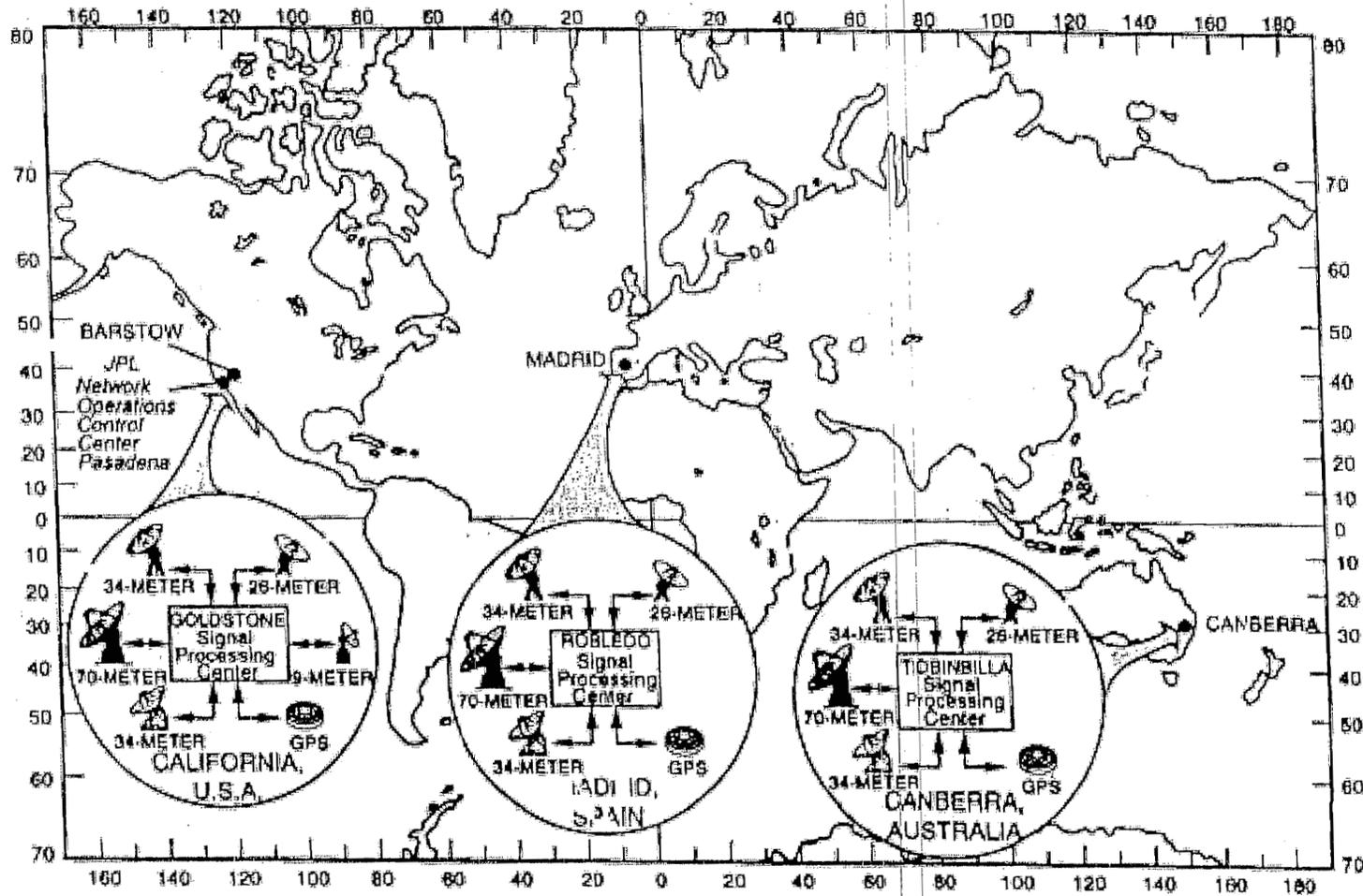
Armstrong *Radio Sci.* 33, 1727 (1998)

less et al. *CQG* 16, 1487 (1999)



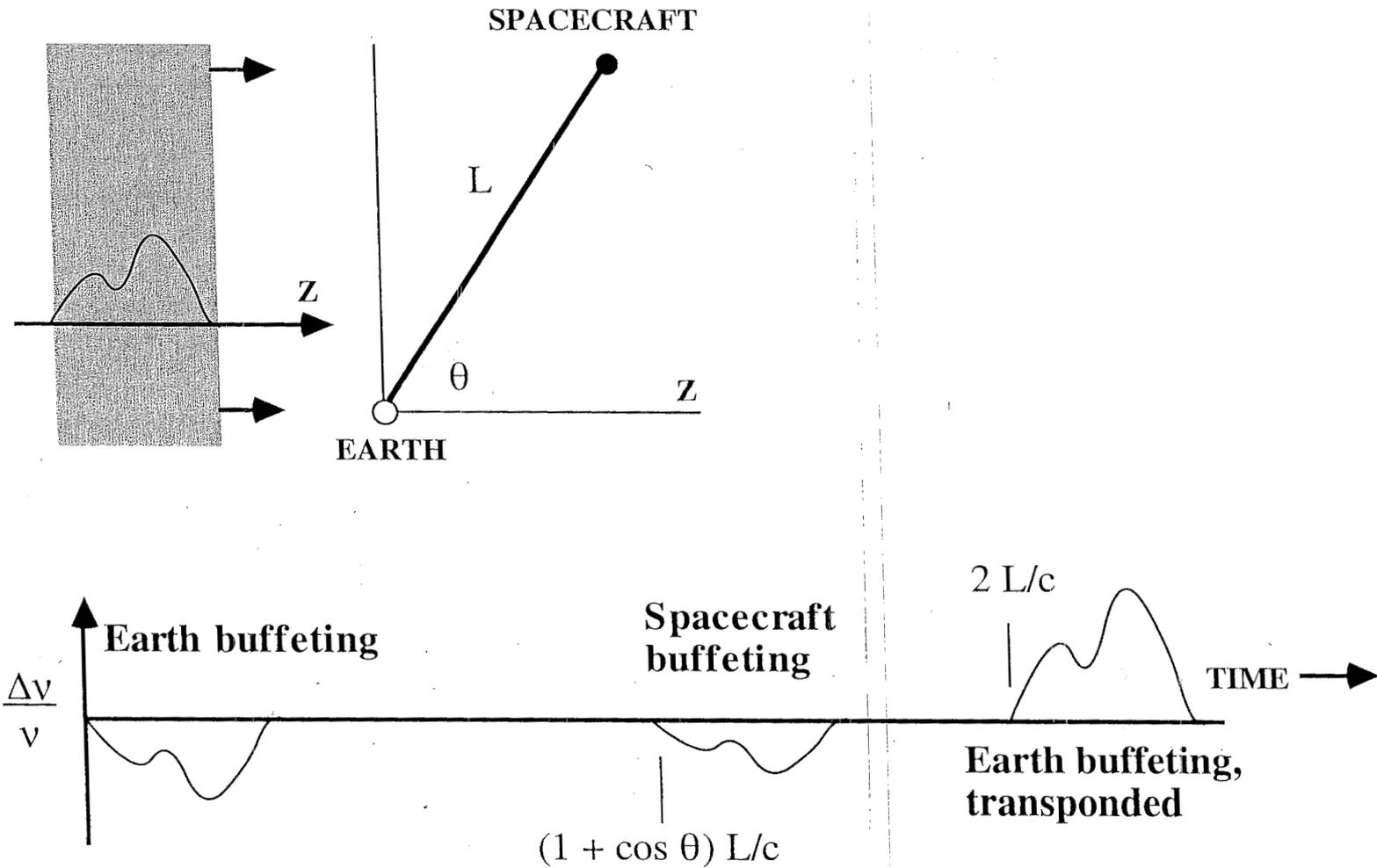


# THE DEEP SPACE NETWORK 1992 CONFIGURATION



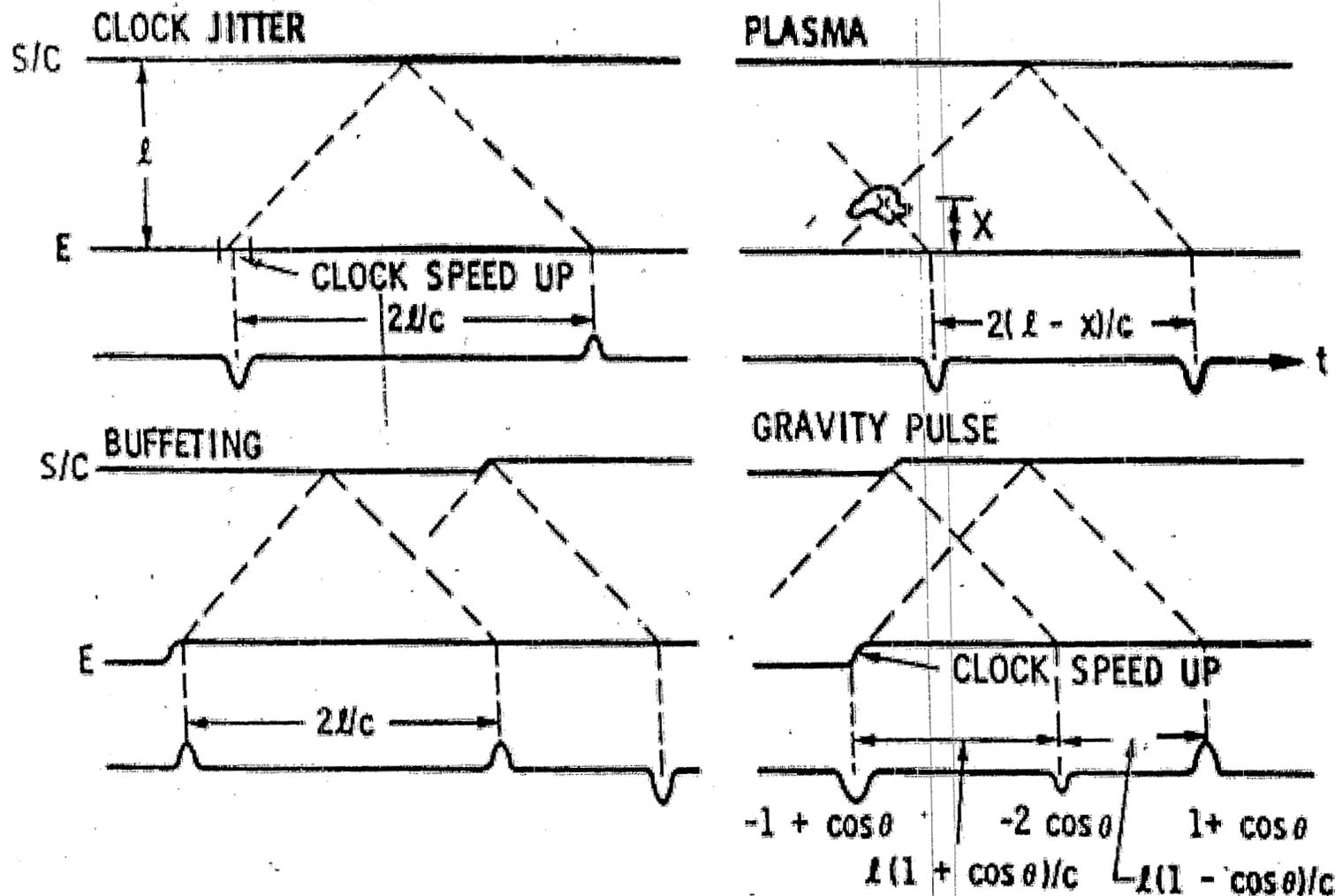
CaJAGWR-10  
11/3/00

# Response of Spacecraft Doppler Tracking to Gravitational Wave



Estabrook and Wahlquist, *Gen. Rel. Grav.* 6, 439 (1975)

# DOPPLER SIGNALS CORRESPONDING TO DIFFERENT TYPES OF DISTURBANCE IN THE COMMUNICATION LINK



## Estimated Value of S/C's $\sigma_Z^{GWE}$

- **Results:**

- 40 hours of data beginning 2001-DOY-152/T02:00:03.558 (sampling time is 4 s)
- CAPS articulation motion was active over this time span (articulation frequency  $\approx 0.0025$  Hz)
- Estimated value of  $\sigma_Z^{GWE}$  is  $\approx 0.071$   $\mu\text{m}/\text{sec}$ . It is significantly better than the requirement (0.3  $\mu\text{m}/\text{sec}$ )

- **Corresponding Allan deviation**

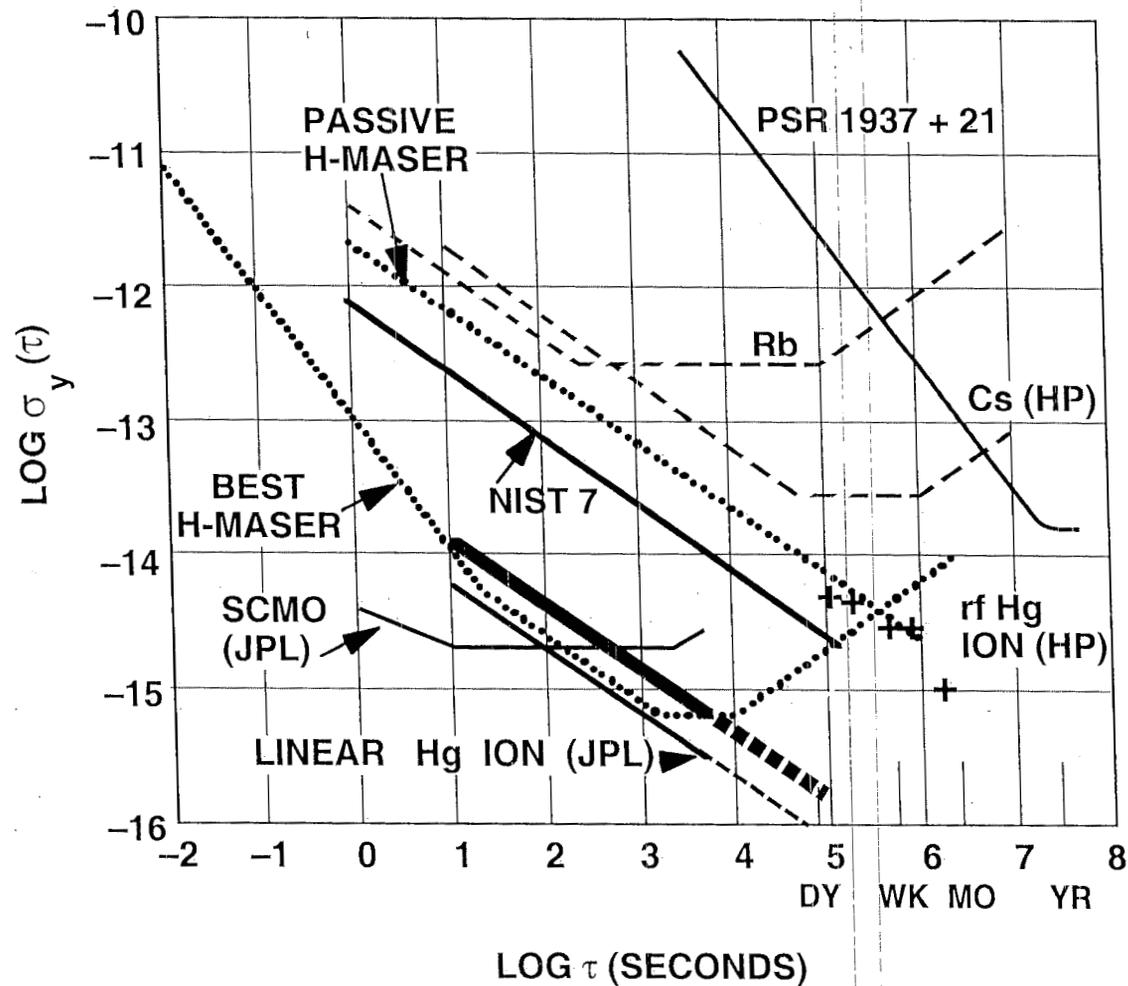
$$\approx 2.3 \times 10^{-16} < 10^{-15} \text{ (Requirement)}$$

from: Wan, Hanover, Beletky & Lee 10/18/01

## MAIN NOISES: FREQUENCY STANDARD STABILITY

- Spacecraft Doppler tracking is not interferometric; coherence is maintained through the frequency and timing system. Thus FTS is fundamental
- Transfer function in two-way Doppler:  $\delta(t) - \delta(t - T_2)$
- Cassini era LITS/SCO has excellent stability on integration times 1–10,000 seconds (see Allan deviation plot, due to L. Maleki)

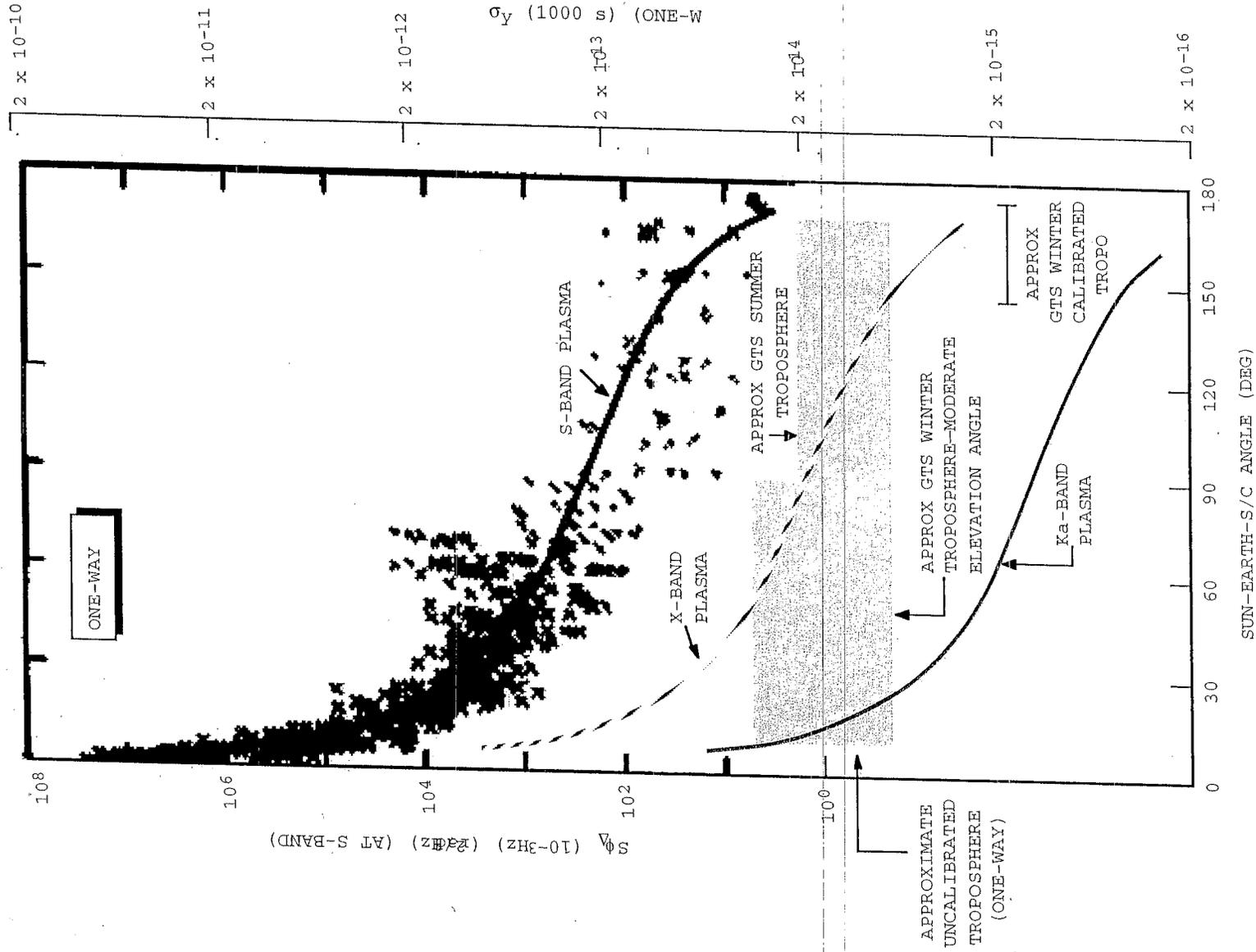
# LINEAR ION TRAP STANDARD (LITS) FRACTIONAL FREQUENCY STABILITY



# MAIN NOISES: PLASMA SCINTILLATION

- Dispersive, refractive index fluctuations proportional to  $\lambda^2$
- Transfer function in two-way Doppler:  $\delta(t) + \delta(t - T_2 + 2x/c)$
- Plasma scintillation is dominant noise in S-band observations (even at opposition), but a secondary noise source for Ka-band observations at opposition

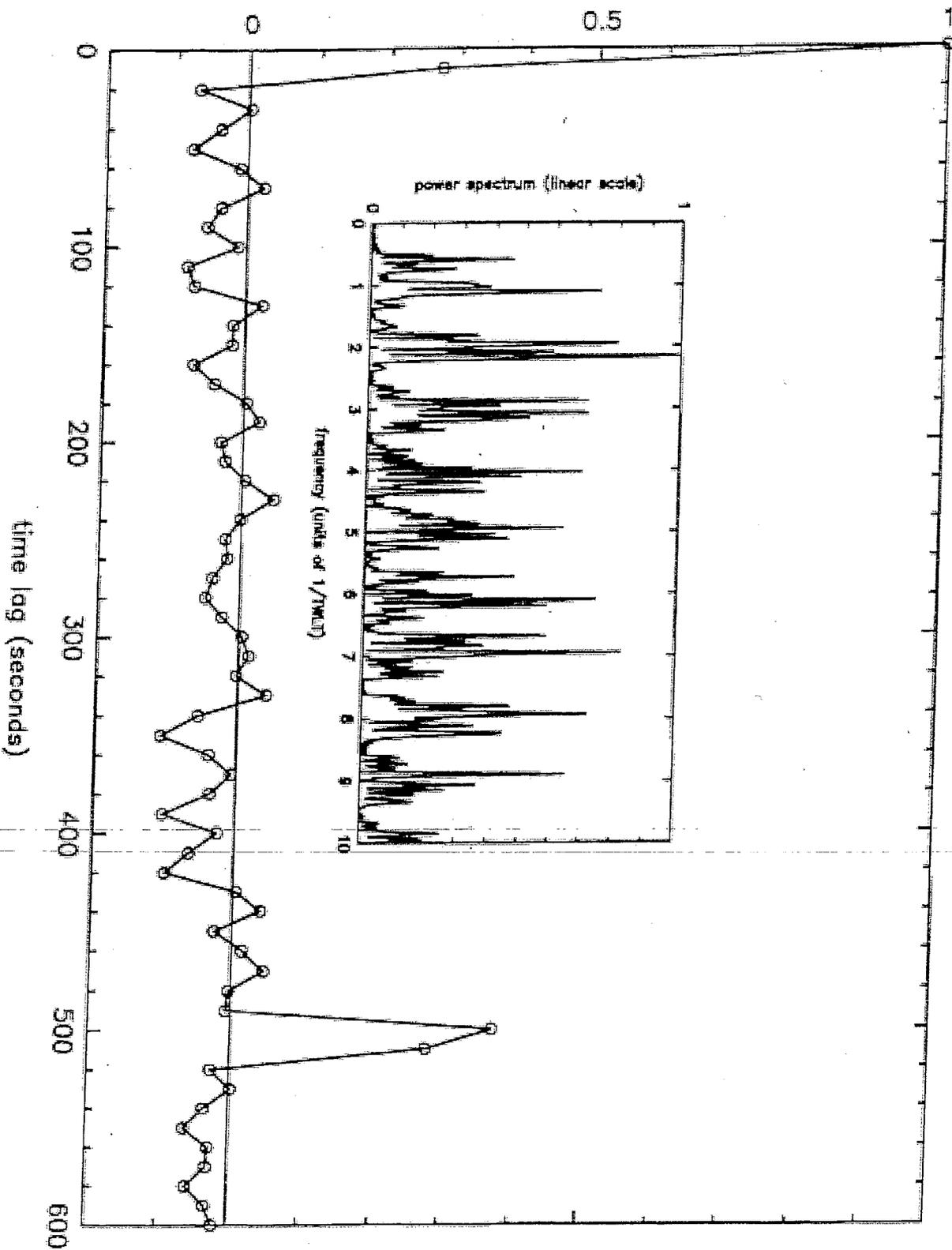
INTERPLANETARY PHASE SCINTILLATION

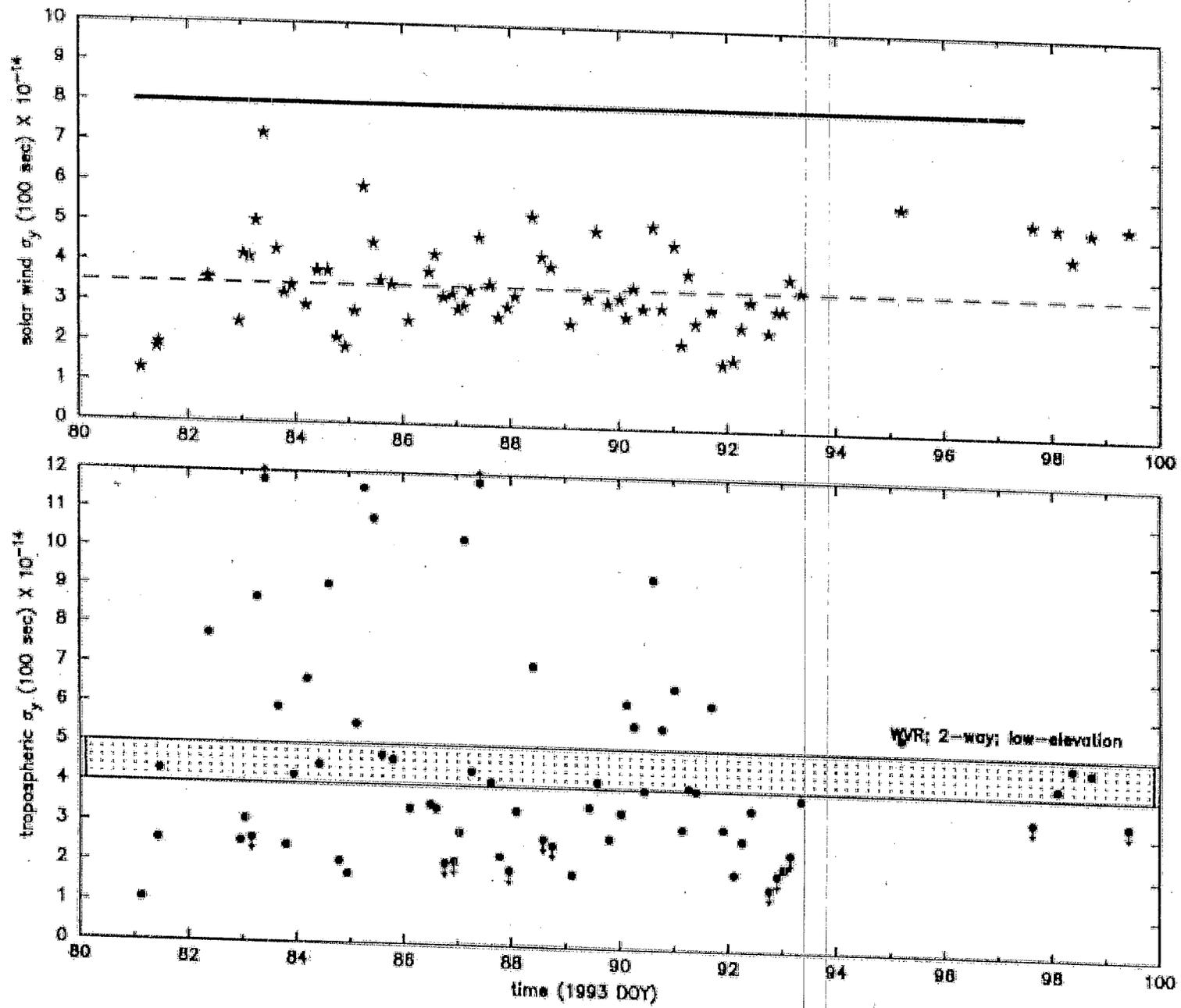


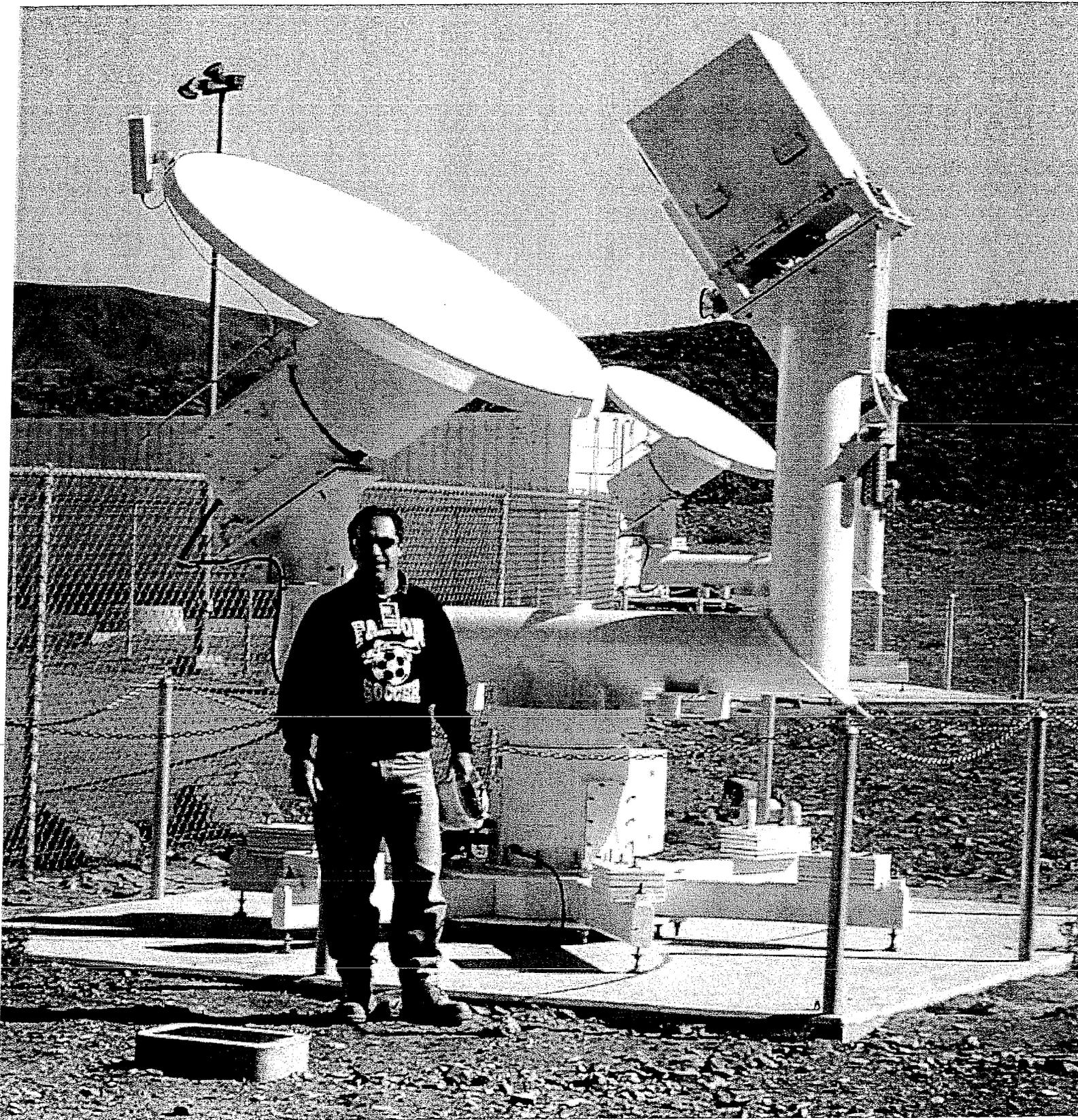
# MAIN NOISES: TROPOSPHERIC SCINTILLATION

- Refractive index fluctuations at microwave frequencies are dominated by fluctuations of the water vapor along the LOS.
- Transfer function in two-way Doppler:  $\delta(t) + \delta(t - T_2)$
- Independent measures of the effect available using WVRs (e.g., Keihm *TDA Prog. Rep. 42-122, 1* (1995))
- Operational X-band data can be approximately decomposed into tropospheric and plasma scintillation; results consistent with Keihm's observations (*Armstrong Radio Sci. 33, 1727* (1998))
- Cassini-era Advanced Media Calibrations System will calibrate and allow removal of  $\approx 80\%$  of the wet component; dry component + residual wet component will have transfer function  $\delta(t) + \delta(t - T_2)$

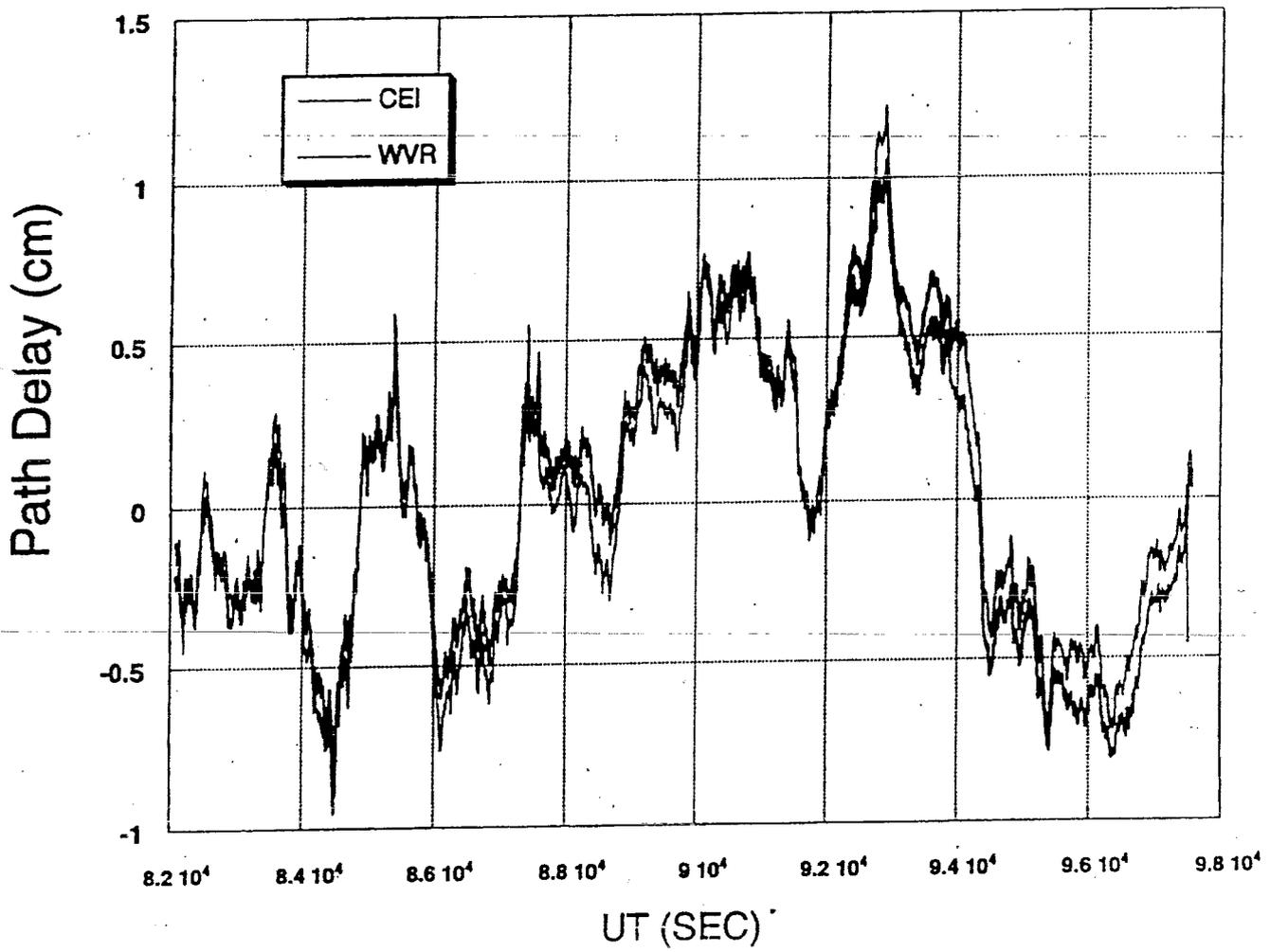
autocorrelation of Doppler residuals

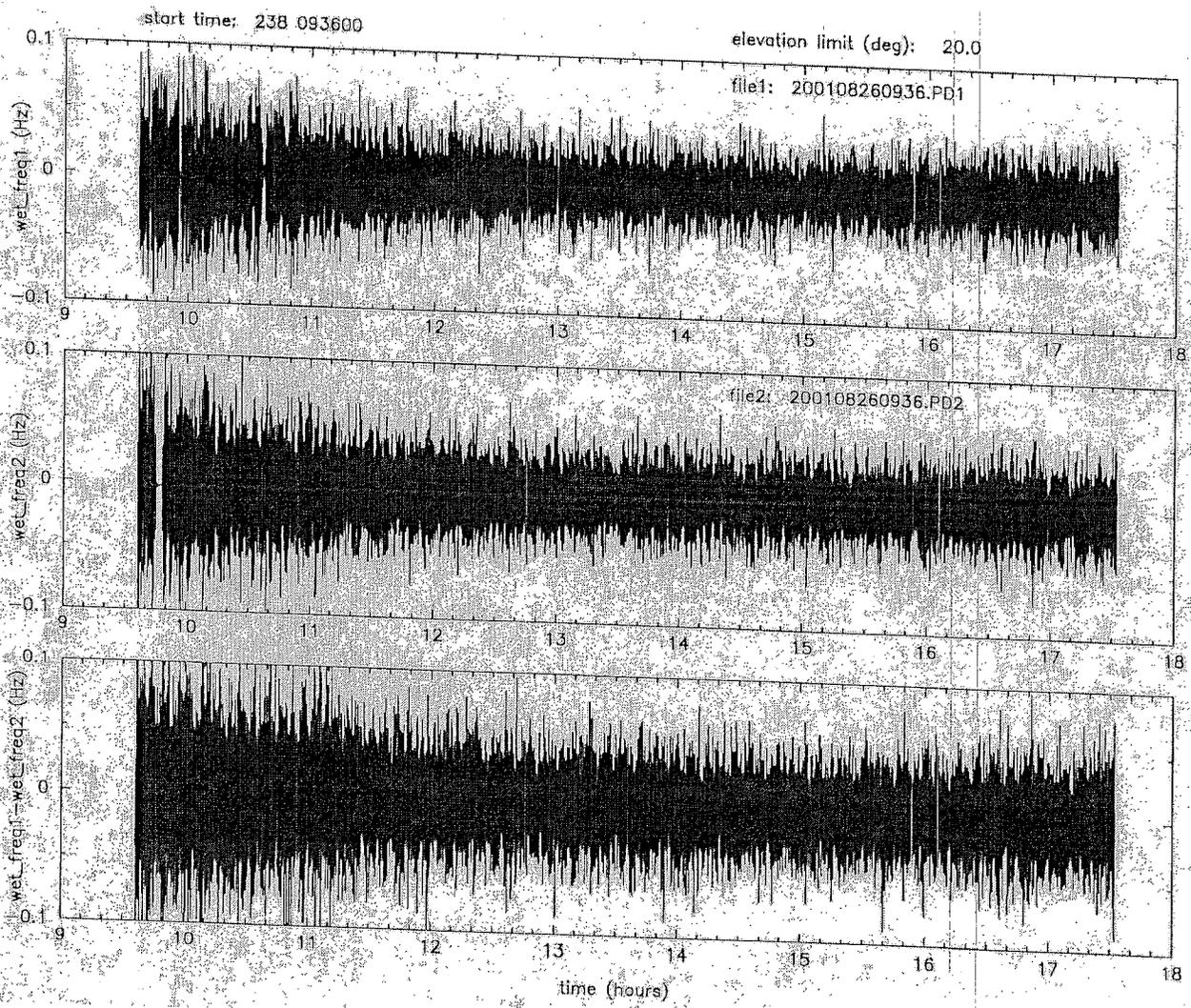






# WVR-CEI Comparison DOY 138, 2000

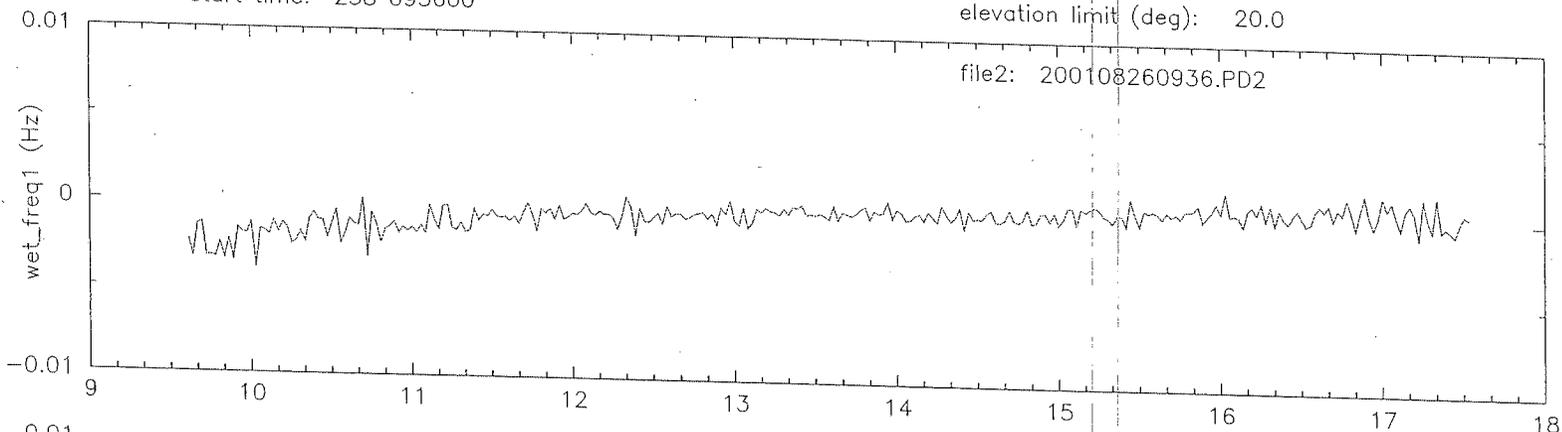




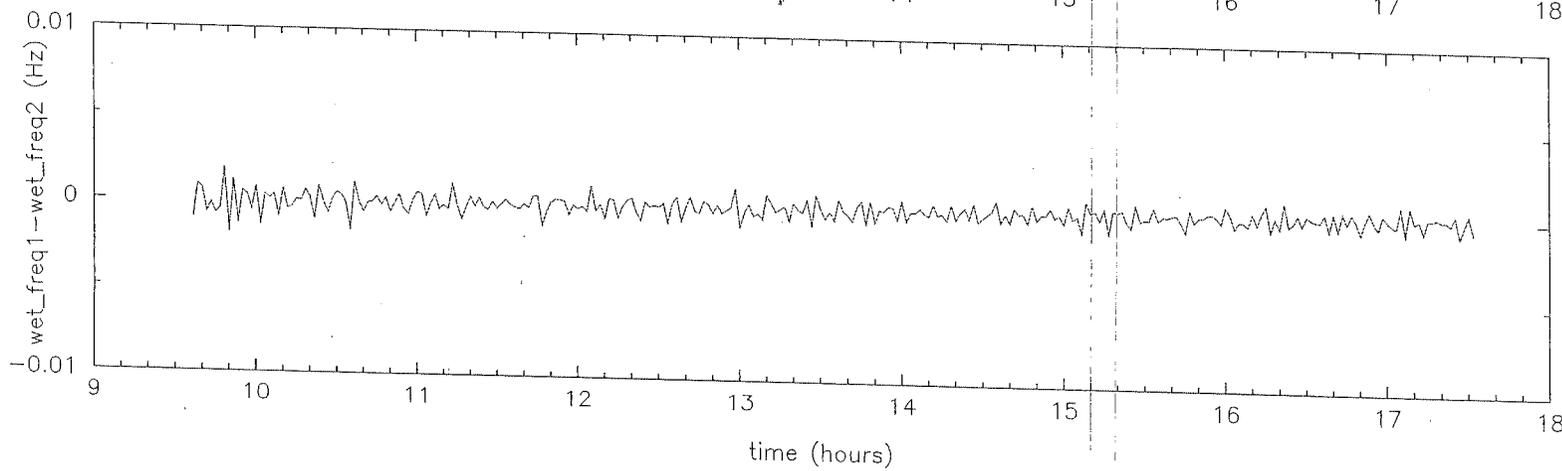
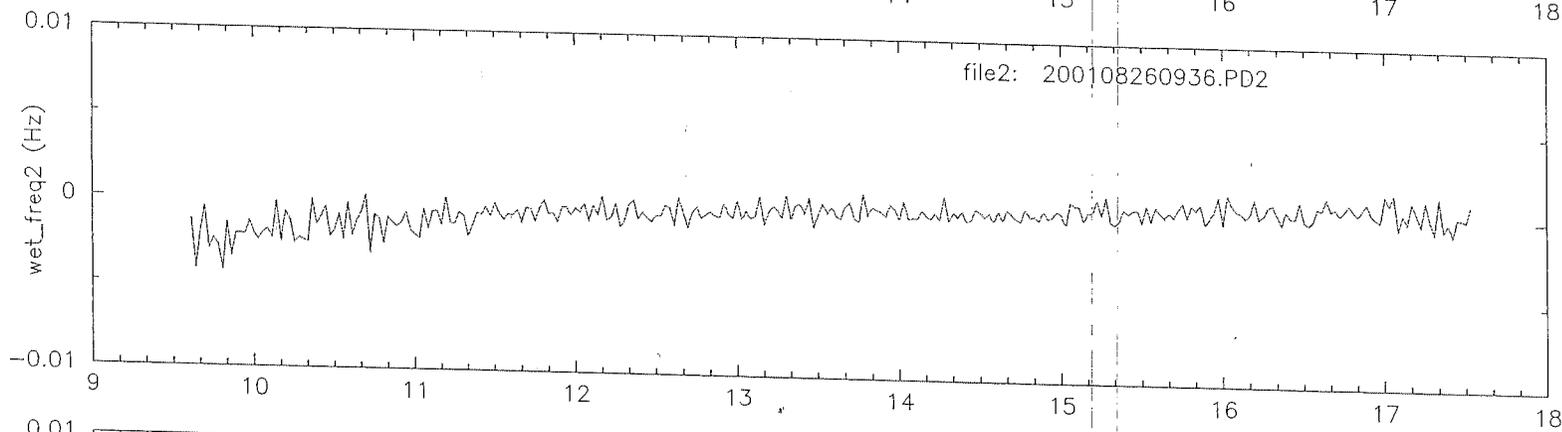
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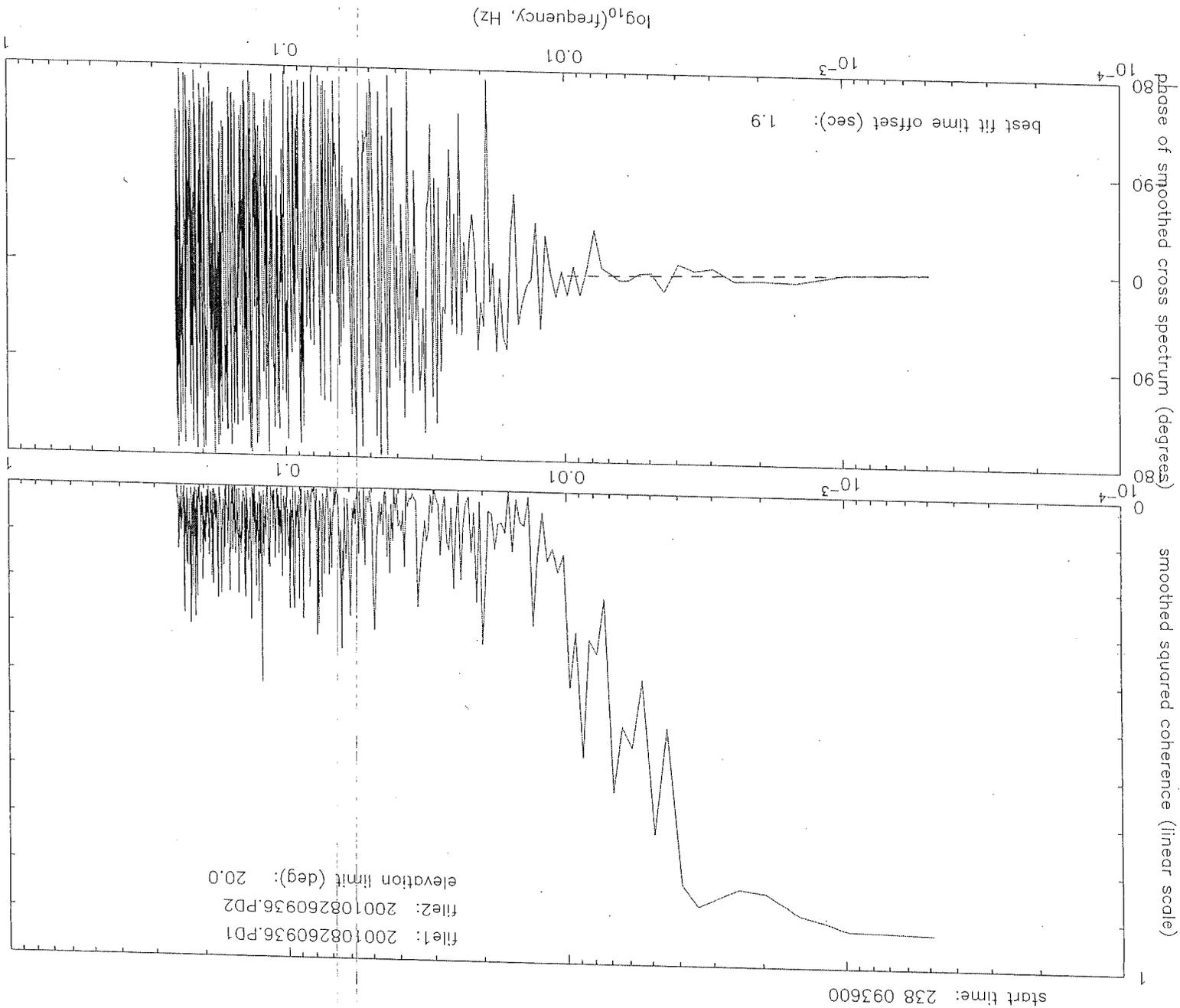
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file2: 200108260936.PD2



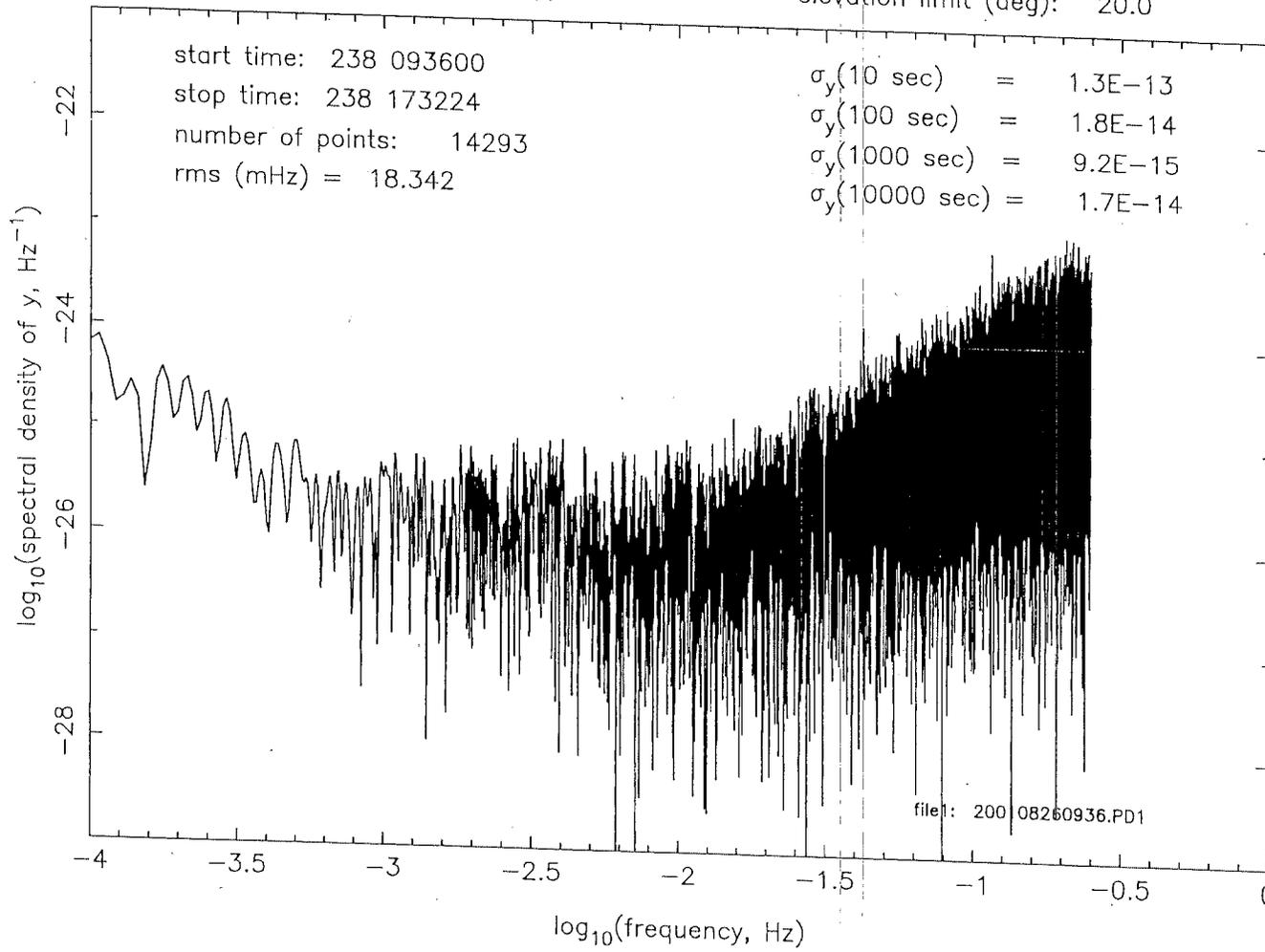
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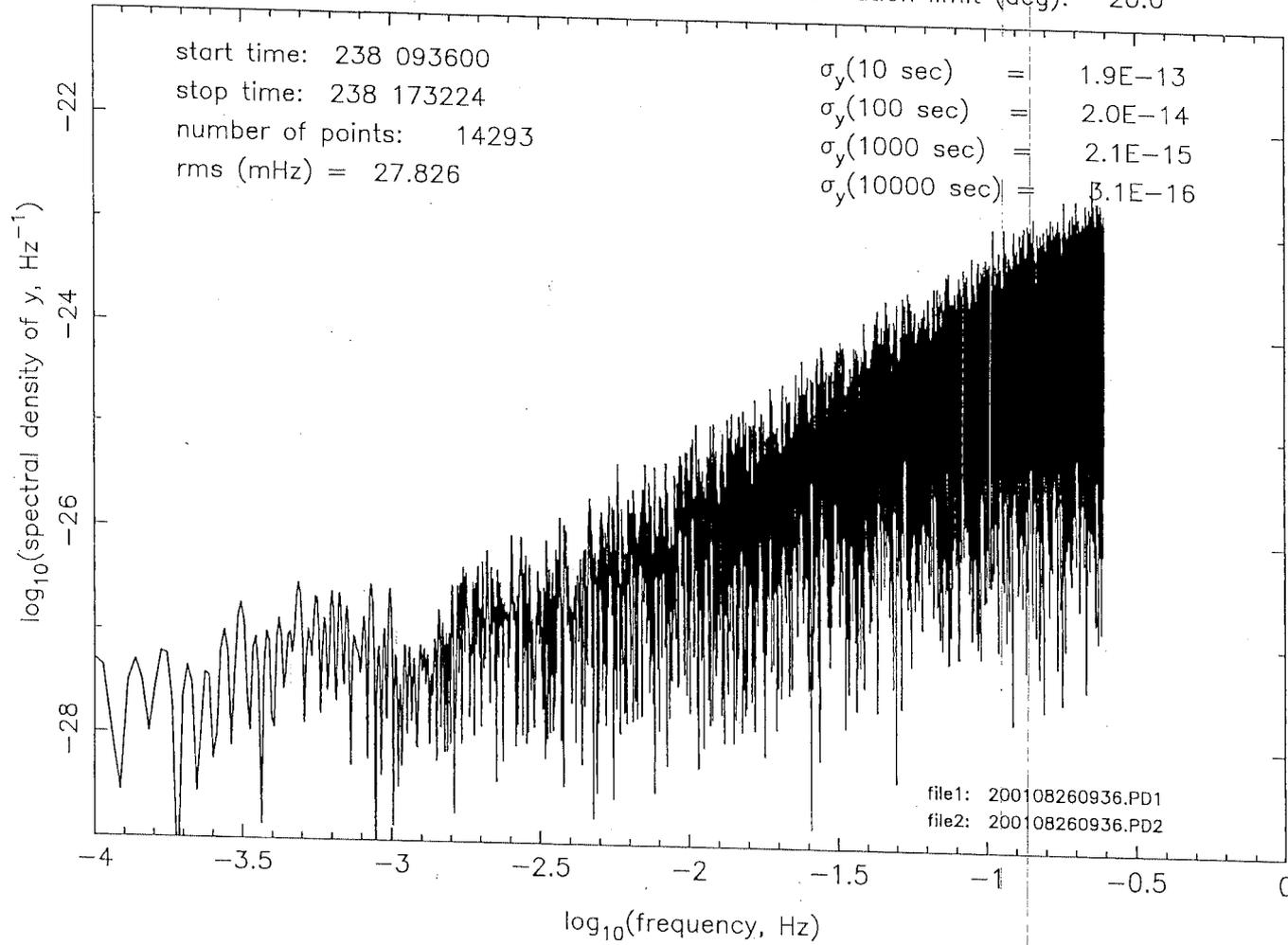
(Ka-band normalization for y)

elevation limit (deg): 20.0



(Ka-band normalization for y)

elevation limit (deg): 20.0

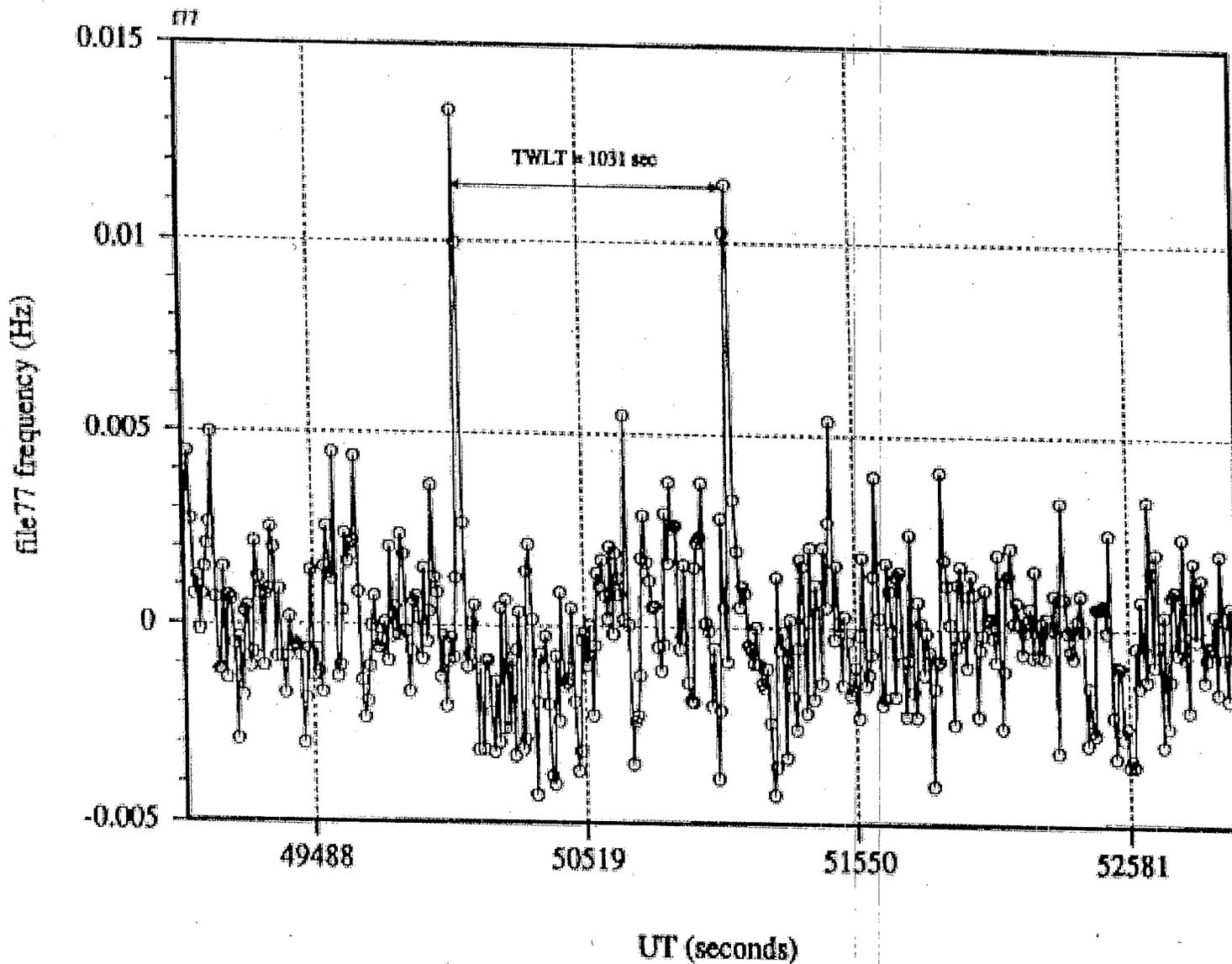


john 15-Oct-200

# MAIN NOISES: ANTENNA MECHANICAL STABILITY

- Differential measurements (under controlled conditions) indicate  $\sigma_y(1000\text{sec}) \approx 1 \times 10^{-15}$  (Otoshi and Franco, *TDA Prog. Report 42-10*, 151 (1992))
- Transfer function in two-way Doppler is  $\delta(t) + \delta(t - T_2)$
- Measurements at X-band under operational conditions are confused with tropospheric scintillation and produce only poor limits ( $< 1 \times 10^{-14}$  at  $\tau = 1000$  sec) (Armstrong *Radio Sci.* 33, 1727 (1998))
- Infrequent large events—almost certainly antenna mechanical—are observed, however (see example)

oj052\_mo\_089\_1254\_65\_a (file77)



# MODEL OF THE DOPPLER TIME SERIES

$y(t) = \Delta f(t)/f_0 =$  gravity waves

- + propagation noise + antenna mechanical noise
- + clock noise + thermal noise
- + systematic effects

$$\begin{aligned}
 &= g(t) * \left\{ \left[ \frac{(\mu - 1)}{2} \right] \delta(t) - \mu \delta \left[ t - (1 + \mu)L/c \right] + \left[ \frac{(1 + \mu)}{2} \right] \delta(t - 2L/c) \right\} \\
 &+ \text{propagation } (t) * \left\{ \delta(t) + \delta(t - 2L/c) \right\} \\
 &+ \text{antenna mechanical } (t) * \left\{ \delta(t) + \delta(t - 2L/c) \right\} \\
 &+ \text{frequency standard } (t) * \left\{ \delta(t) + \delta(t - 2L/c) \right\} \\
 &+ \text{thermal } (t) \\
 &+ \text{systematic effects}
 \end{aligned}$$

where:  $g(t) = (1 - \mu^2)^{-1} \left\{ n \cdot [h_+(t)e_t + h_x(t)e_x] \cdot n \right\}$

$L =$  earth-s/c distance

$\mu = k \cdot n$

$*$  = convolution

# SUMMARY OF SIGNALS AND NOISE

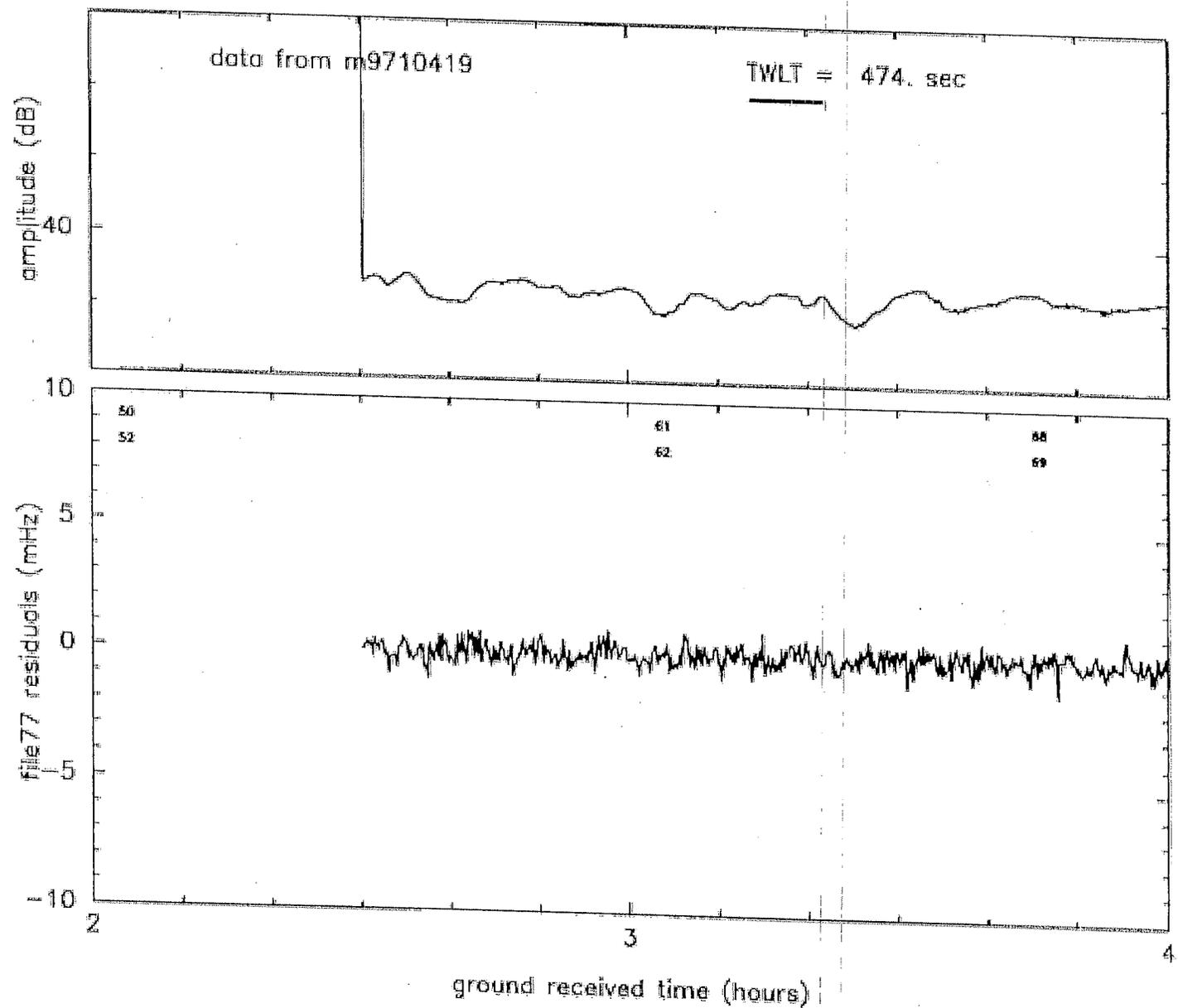
- Signals: "three-pulse" response in the Doppler of the GW excitation
  - Depends on direction to source and s/c two-way light time
  - Not shift-invariant if direction or distance depend on time-of-observation
  - Bandpass: low-frequency signals attenuated due to pulse cancellation; high frequencies cut off by thermal and clock noise. Typical wave duration for best sensitivity depends on  $T_2$  but ~10-10,000 seconds
  - Unlike LISA and other detectors: antenna size/wavelength ~1 to 100
- Noise sources: various "2-pulse" transfer functions for clock instability, propagation noise (solar wind, ionosphere, troposphere), thermal noise
  - Noise nonstationarity
  - Systematic errors

# HOW TO DO A DOPPLER TRACKING EXPERIMENT

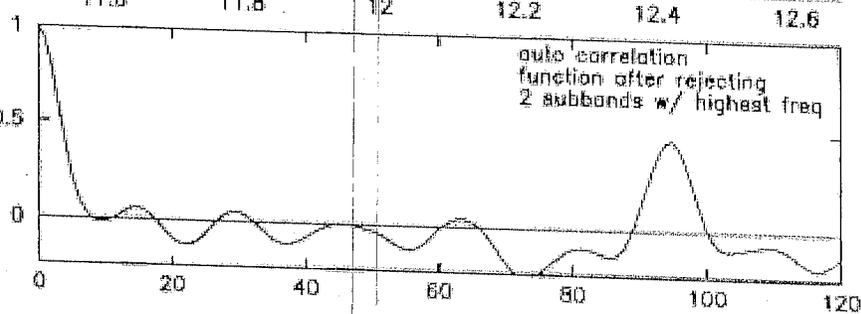
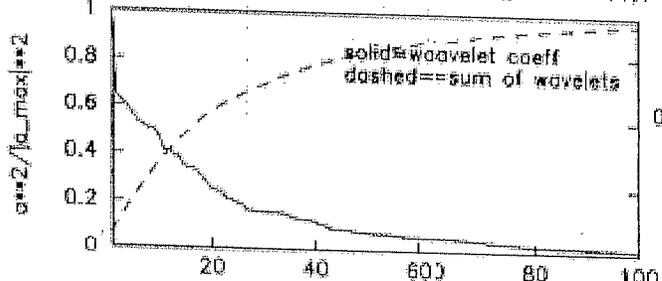
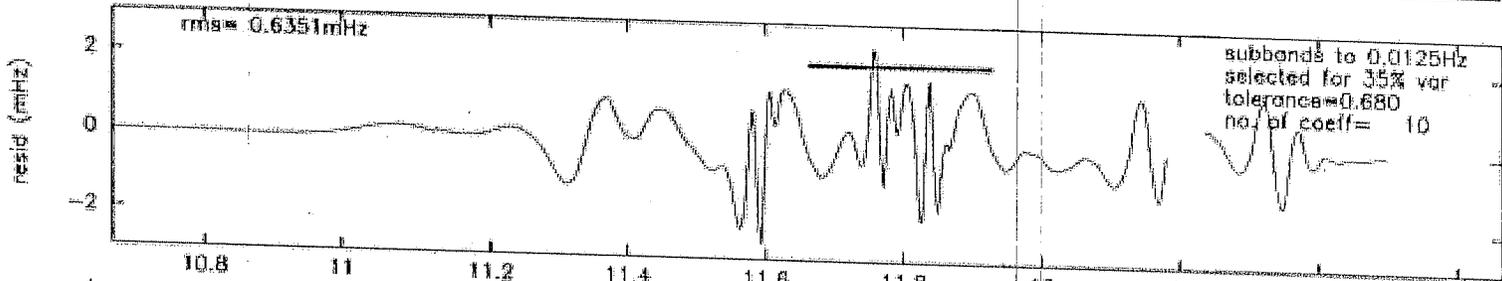
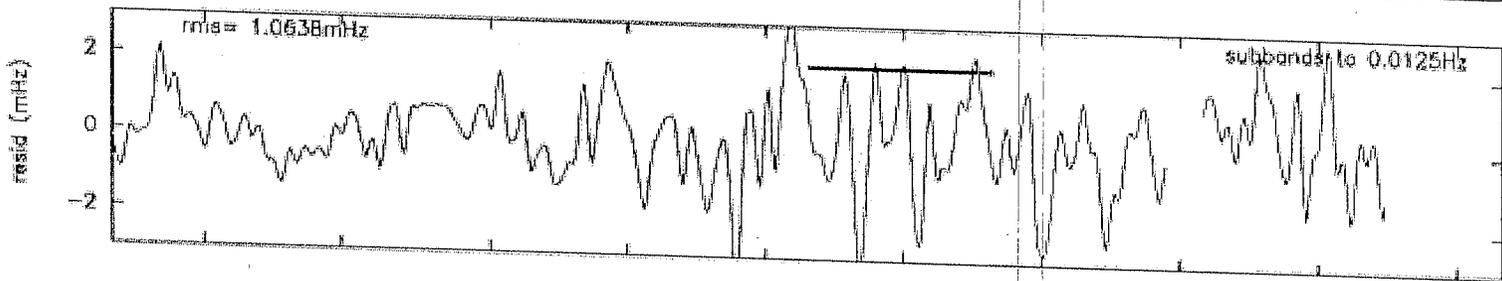
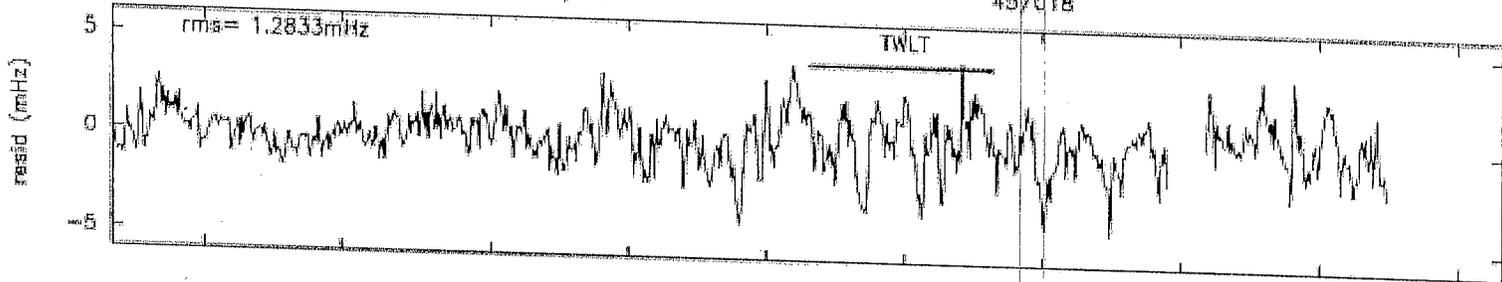
- Need two separated test masses—the earth and a spacecraft in cruise as operationally quiet as possible (need to be far from perturbing masses and need to minimize unmodeled motion of the spacecraft)
- Spacecraft should be as close to anti-solar direction as practical (minimize charged particle scintillation due to solar wind)
- Spacecraft-earth separation should be large (maximize band of Fourier frequencies to which the experiment is sensitive)
- Highly-stable Doppler system to measure relative velocity of the earth and spacecraft (excellent frequency standard; careful signal distribution, etc.)
- Ground system and spacecraft telemetry (correct for or veto data based on known systematics of the apparatus)
- Good weather and media calibration data

## OBSERVATIONS TO DATE

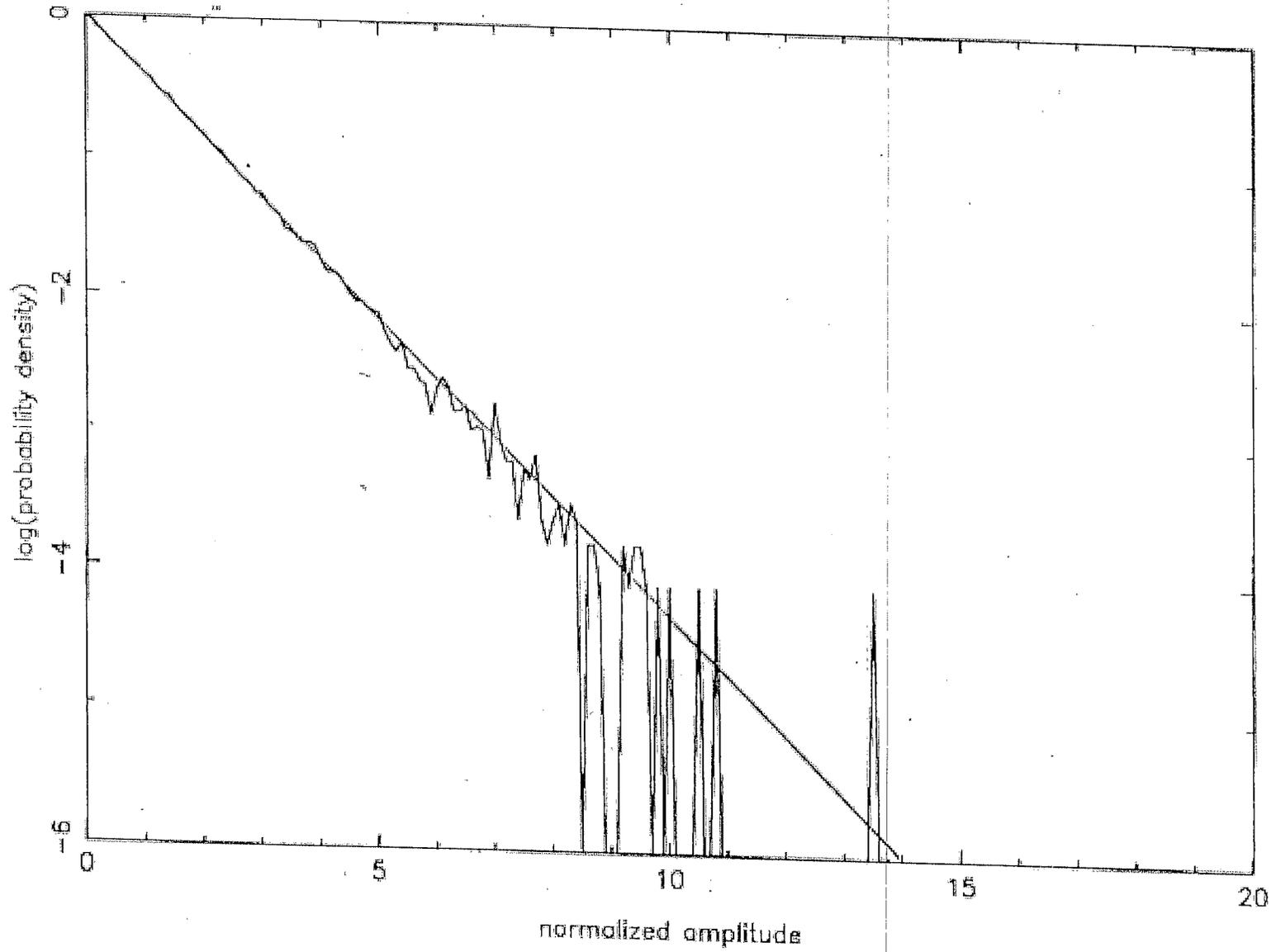
1980	Voyager	Hellings et al. (1981) (few passes; bursts)
1981	Pioneer 10	Anderson et al. (1984) (3 passes, long $T_2$ ; no GW from Geminga)
1983	Pioneer 11	Armstrong, Estabrook, Wahlquist (1987) (broadband search for periodic waves)
1988	Pioneer 10	Anderson et al. (1993) (10 days; chirps and coalescing binaries)
1992	Ulysses	Bertotti et al. (1995) (1 month; sinusoids and chirps)
1993	MO/GLL/ULS	jGWE collaboration (19 days; X-band on MO; only LF coincidence experiment)
1994-5	Galileo	Estabrook et al. (40 days; long $T_2$ )
1997	Mars Global Surveyor	Armstrong et al. (3 weeks; X-band)



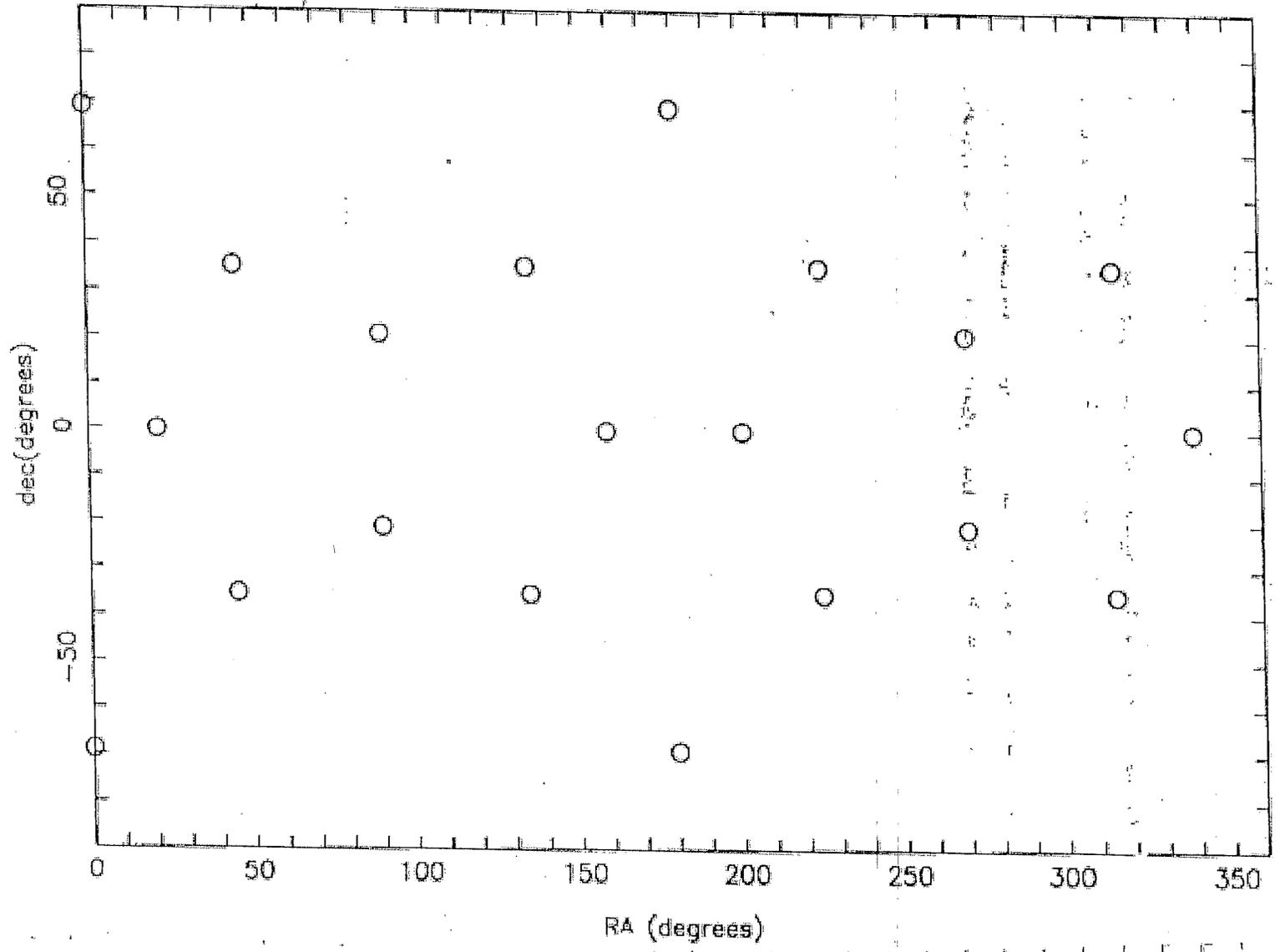
MO Start Time=1993 81 1044000, Stop Time=1993 81 123950, TWLT= 945.9sec  
 45/028 45/023 45/018

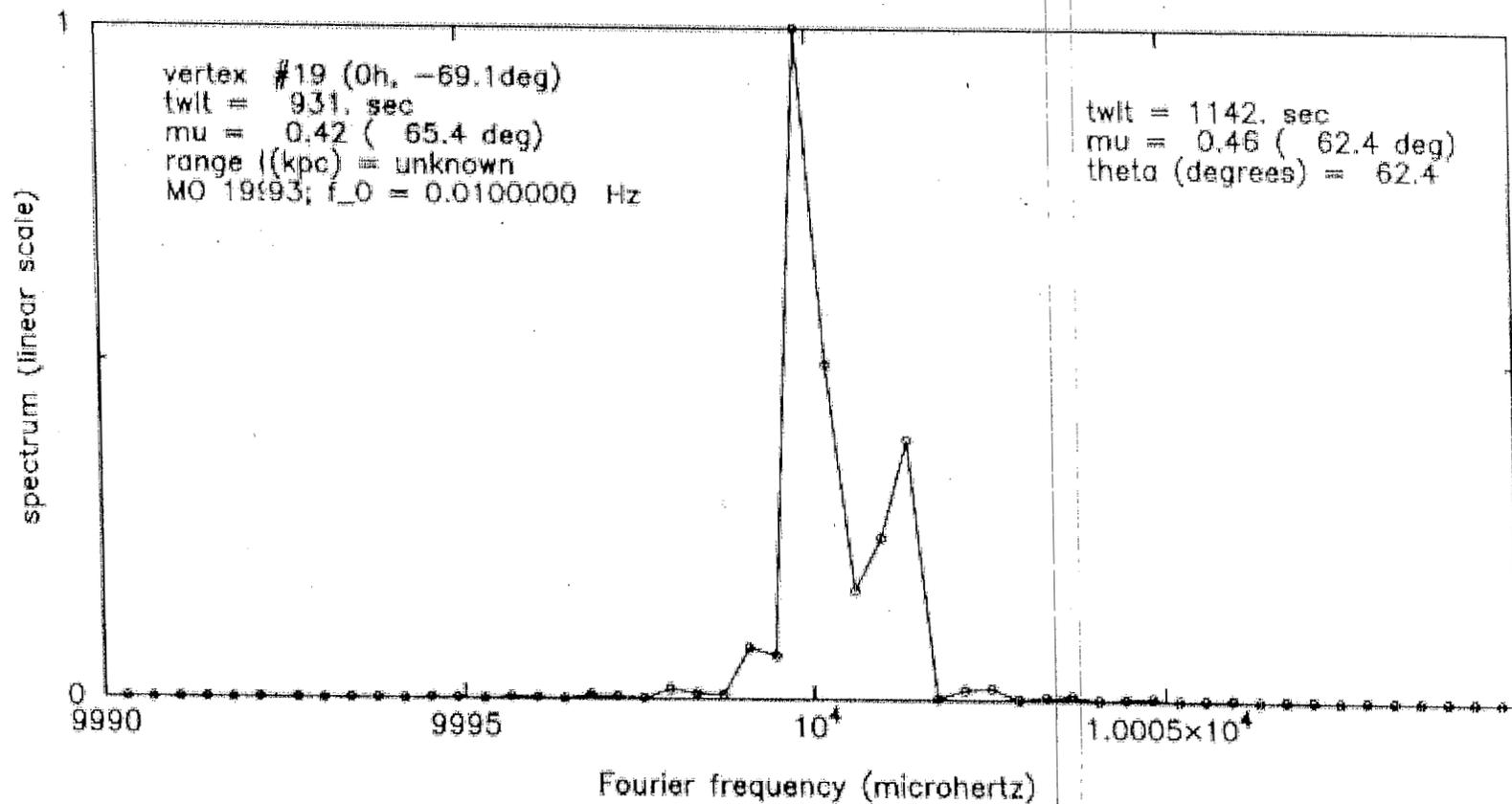
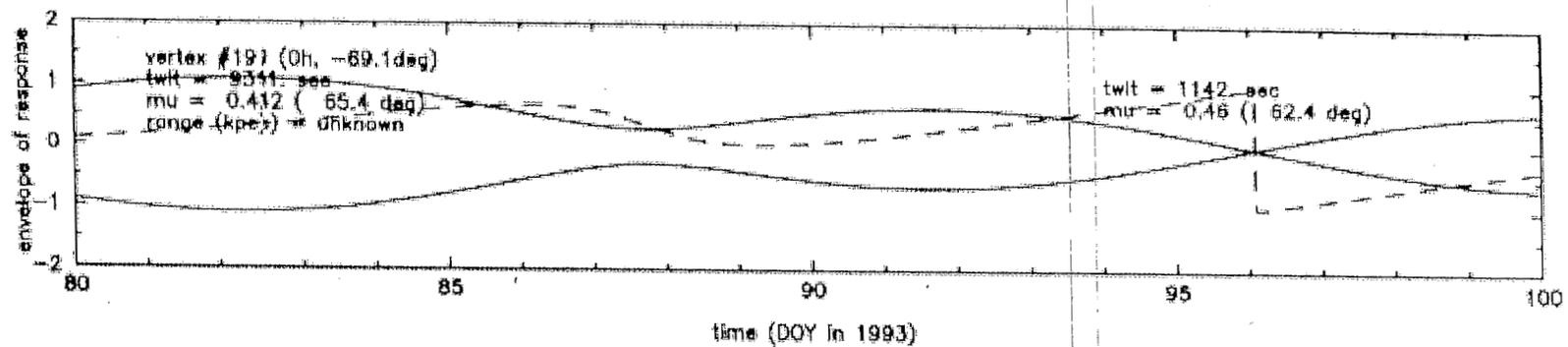


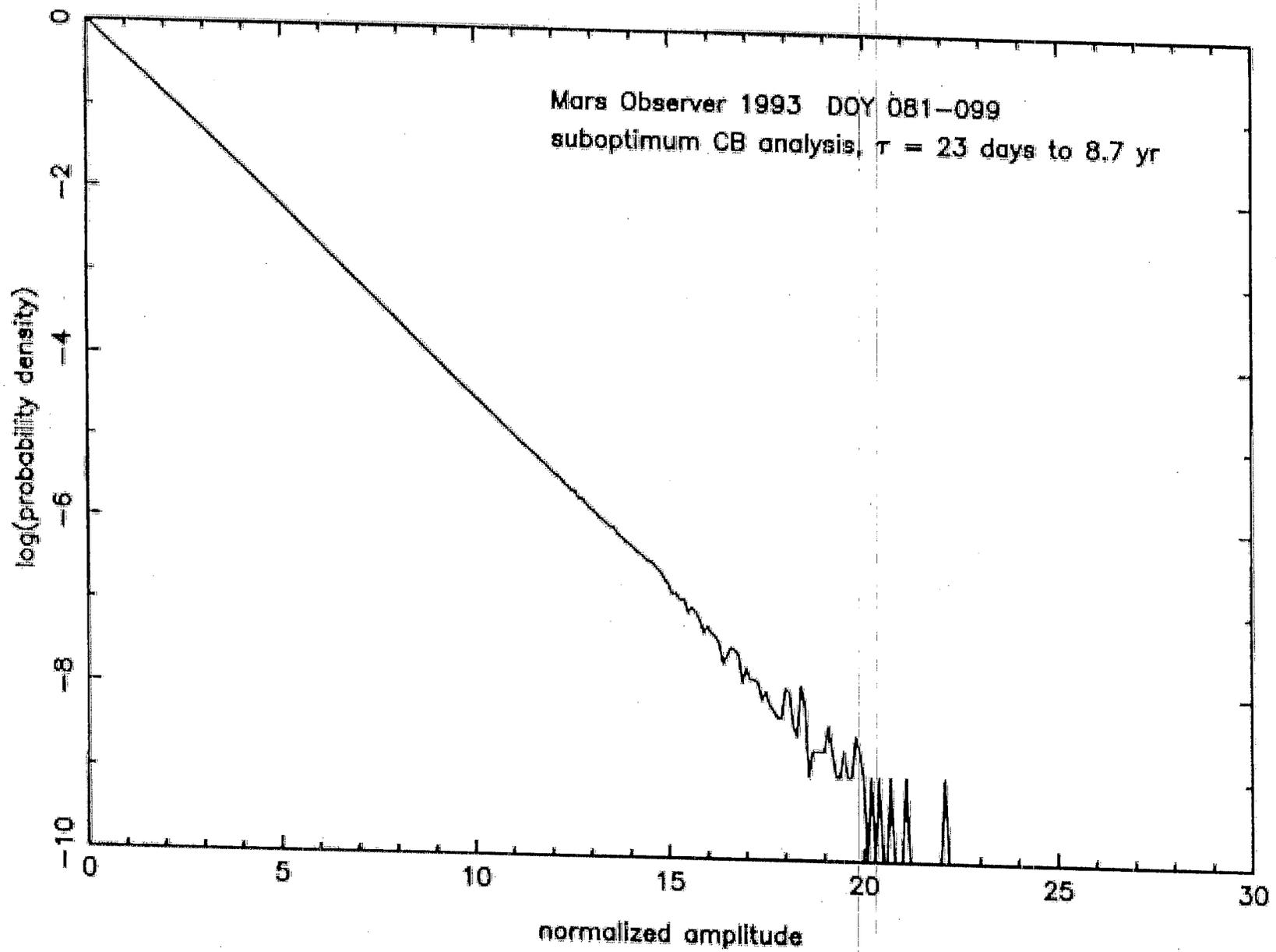
1993 MO sine candidates; edited\_superfile\_mo



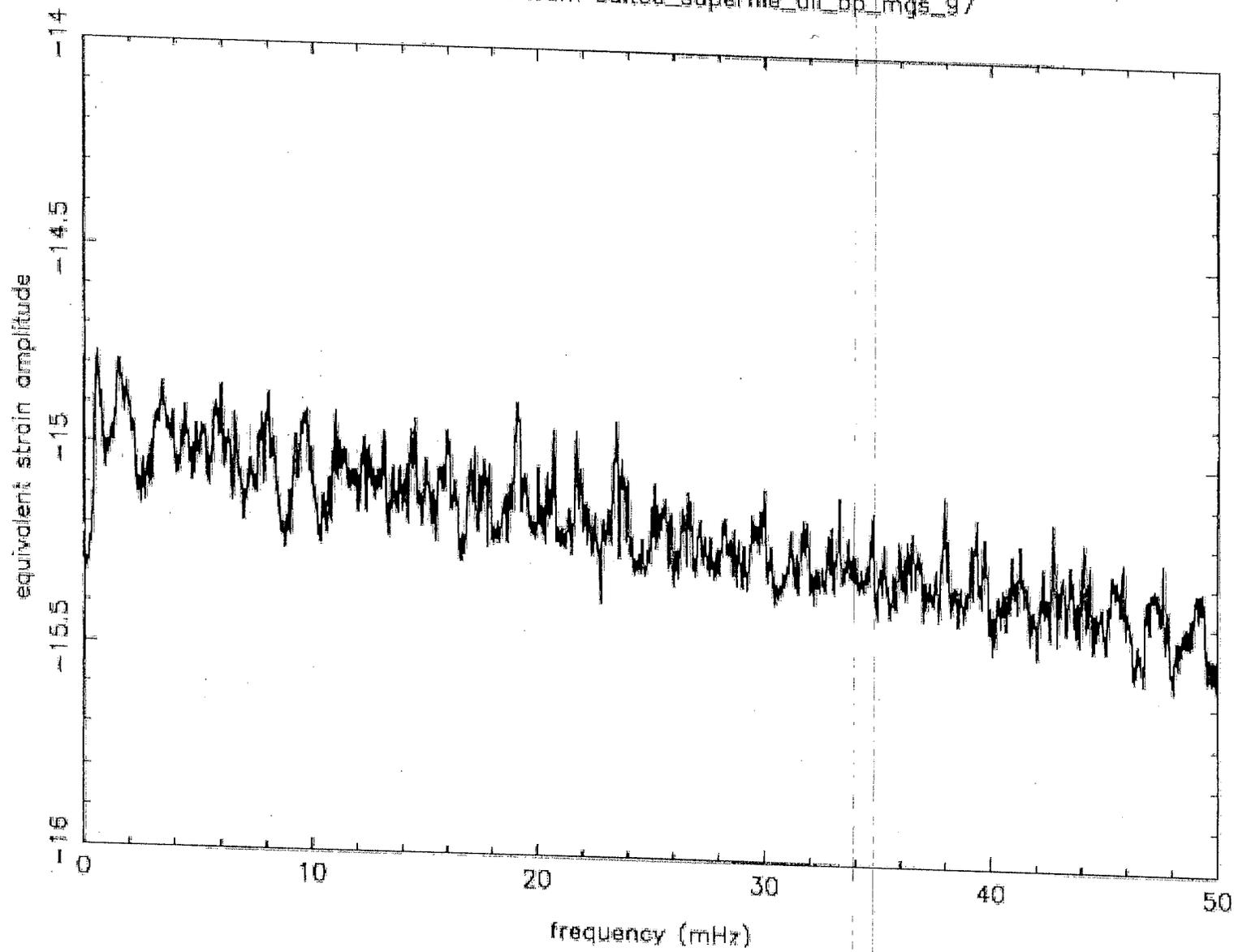
positions of dodecahedron vertices on celestial sphere

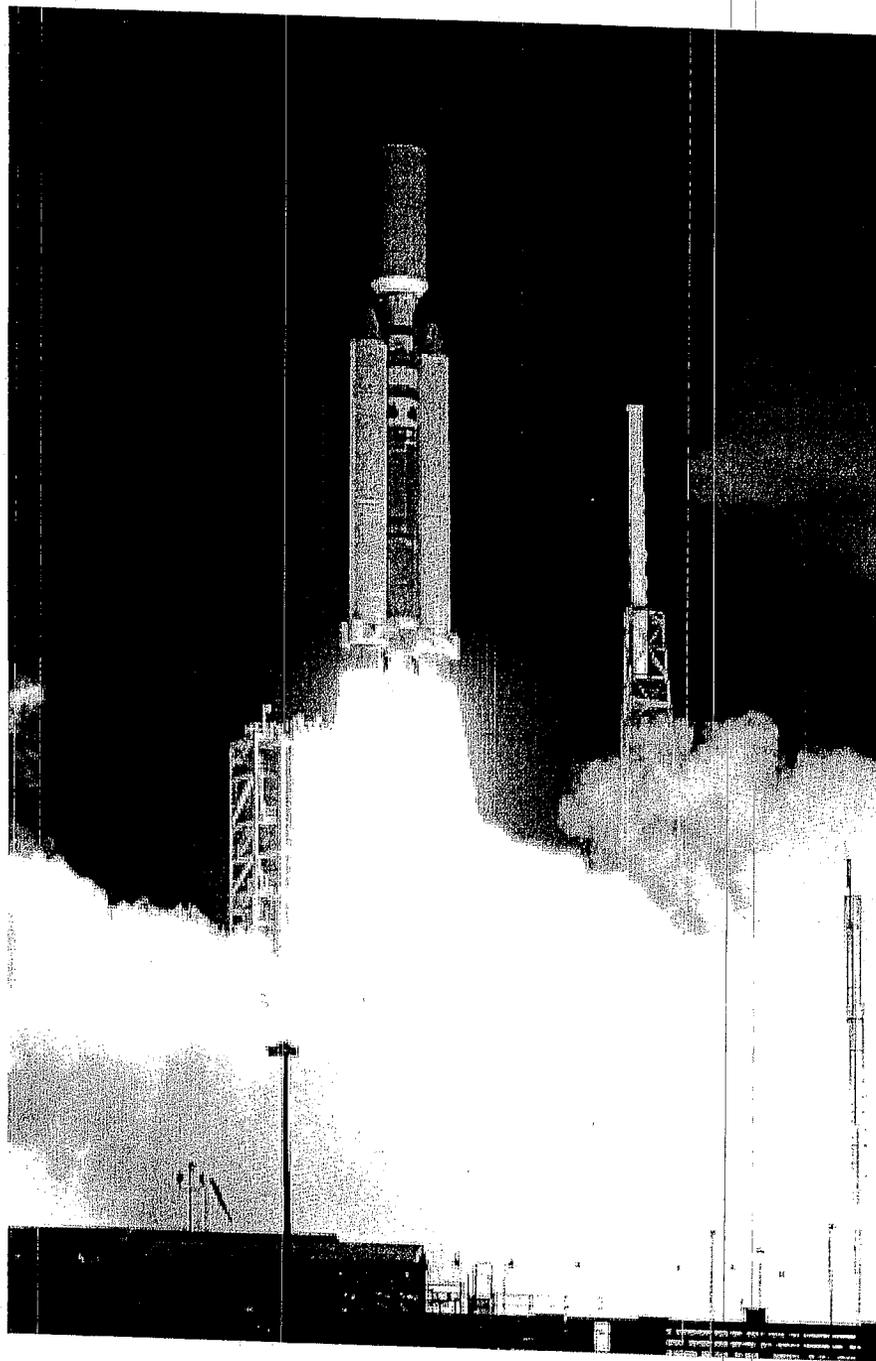






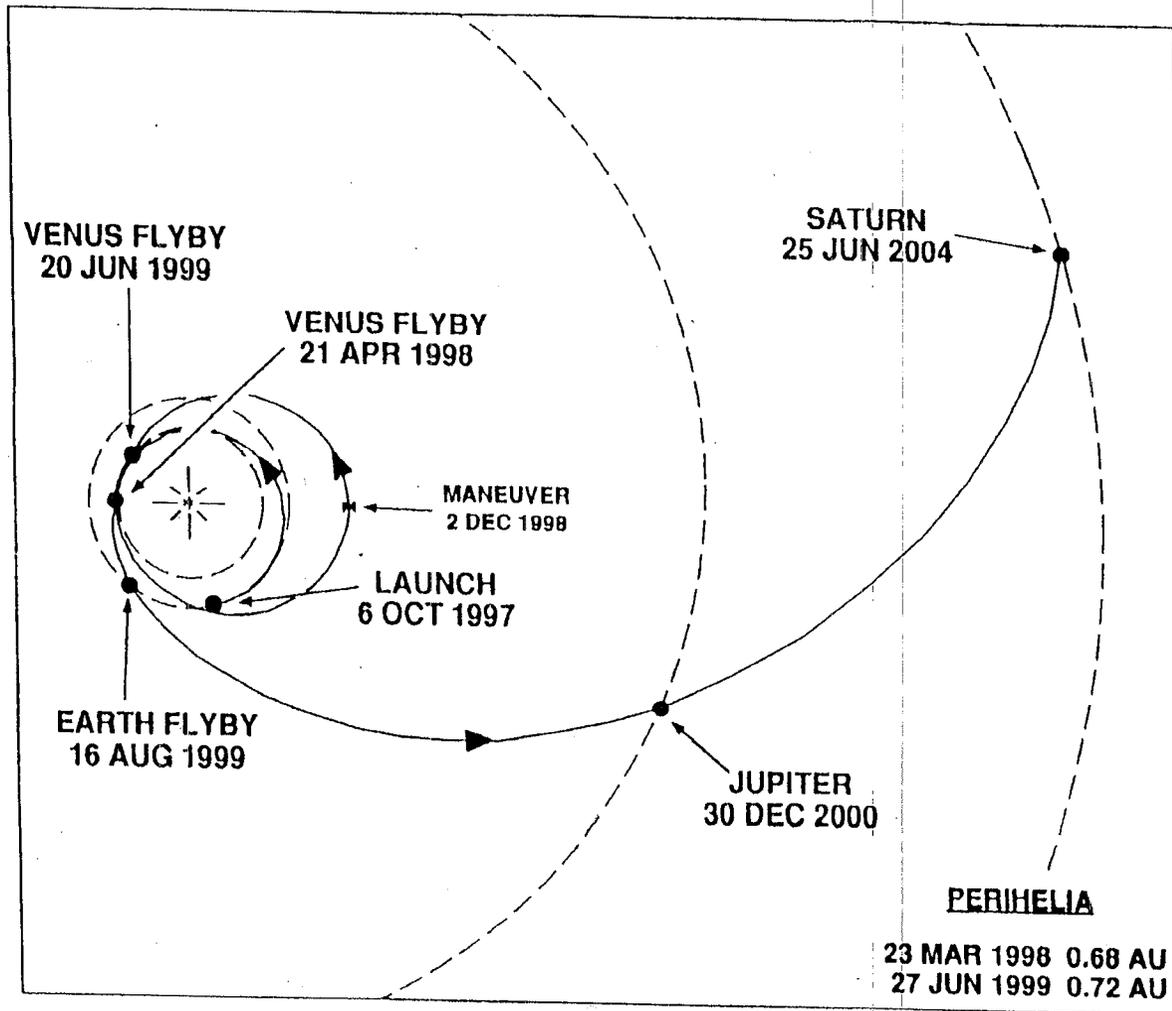
MGS 1997 data from edited\_superfile\_all\_bp\_mgs\_97





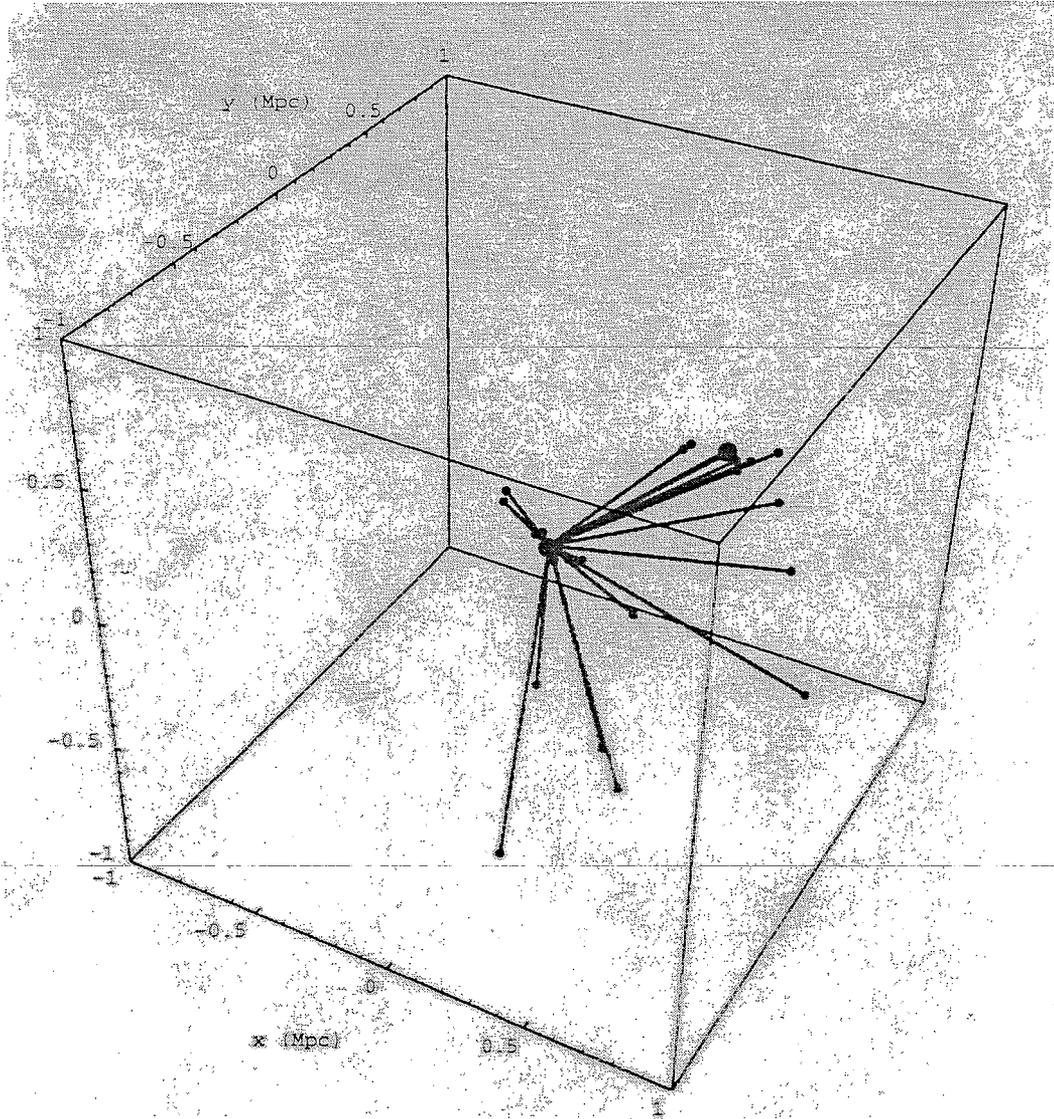
CaJAGWR-36  
11/3/00

# CASSINI OCT 1997 VVEJGA INTERPLANETARY TRAJECTORY

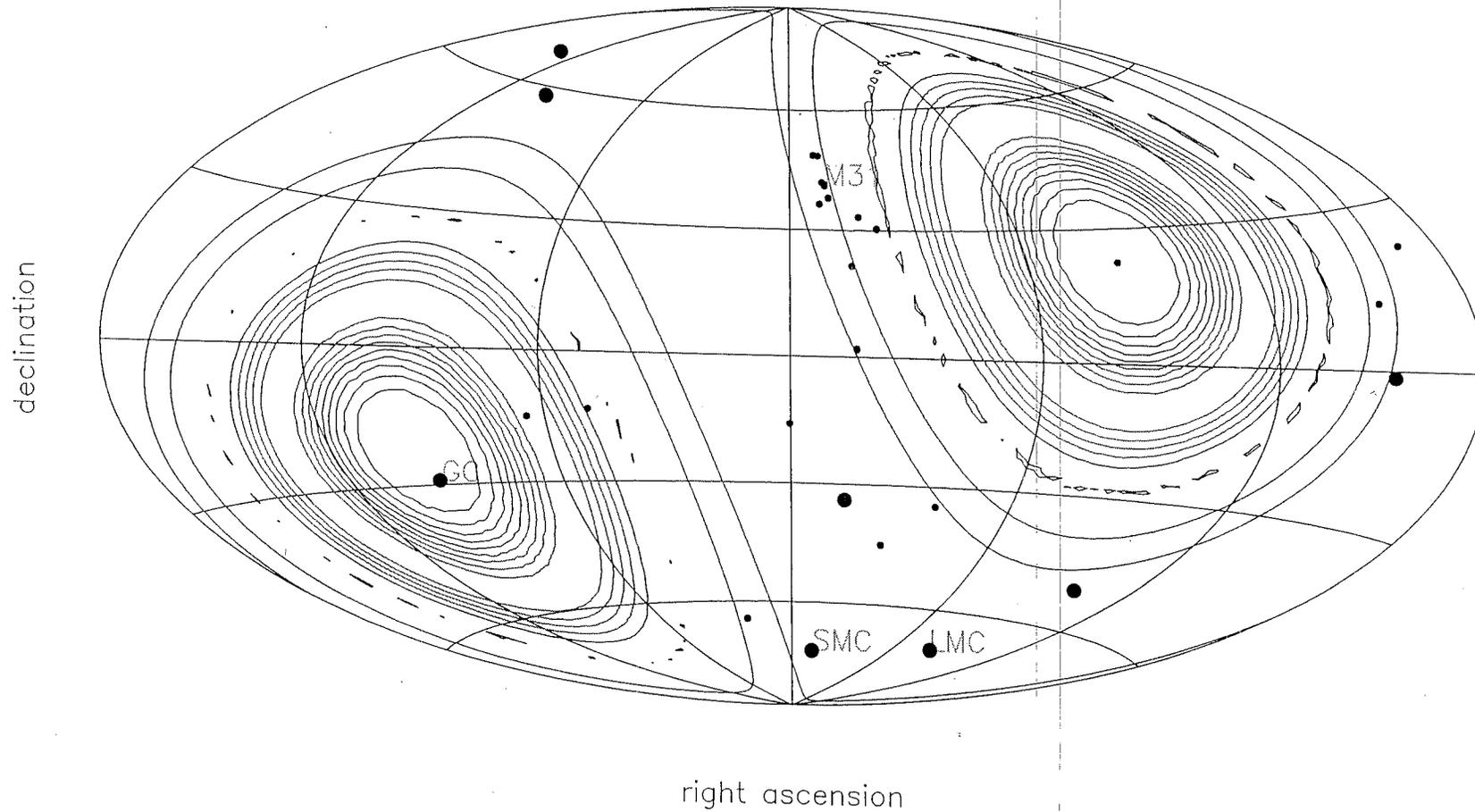


13 JUL 1992

# The Local Group

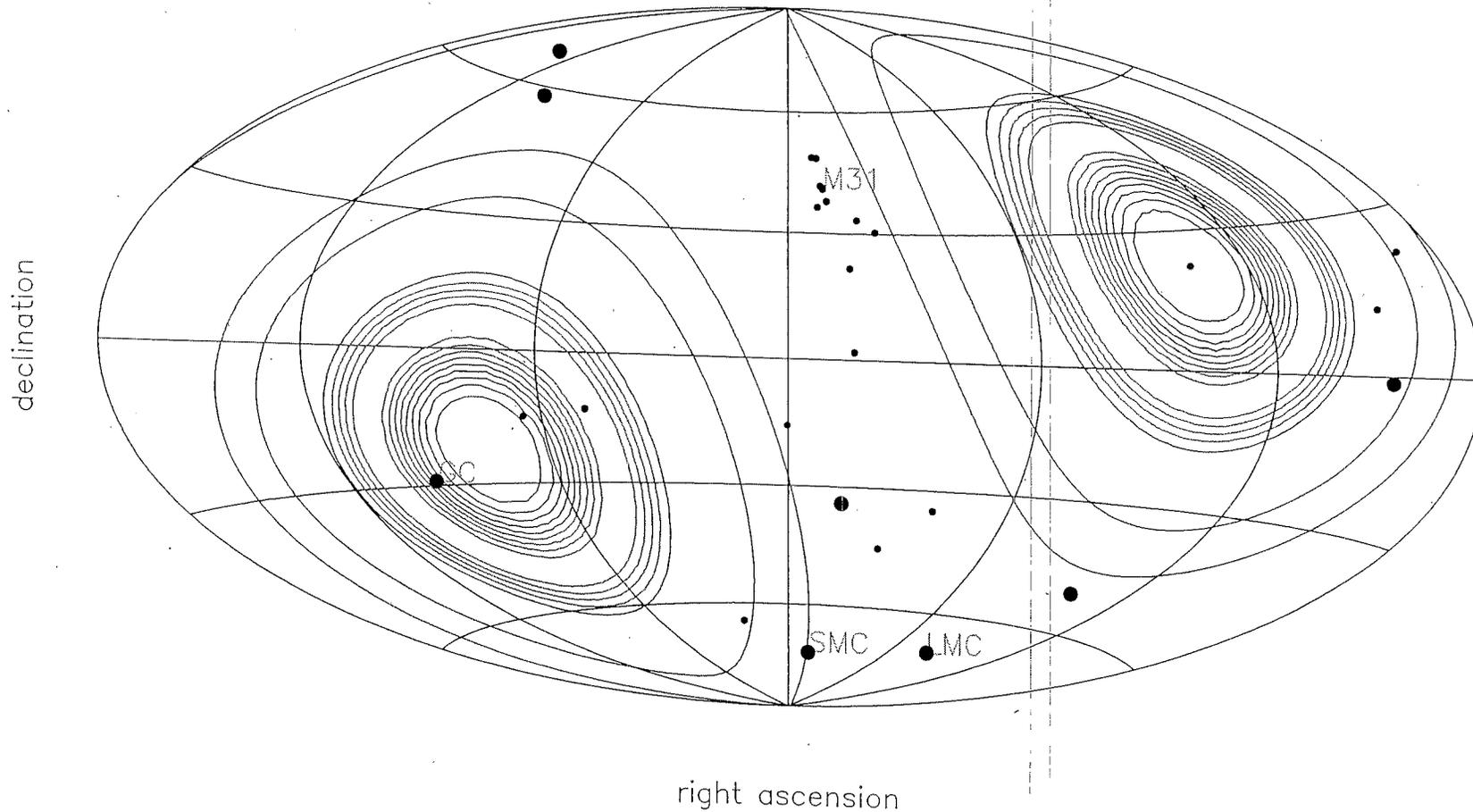


relative energy response for Cassini 2001 December 16  
circular-pol:  $\sin(2 \pi (0.001 \text{ Hz}) t) \cdot \exp(-t/1000 \text{ sec})$

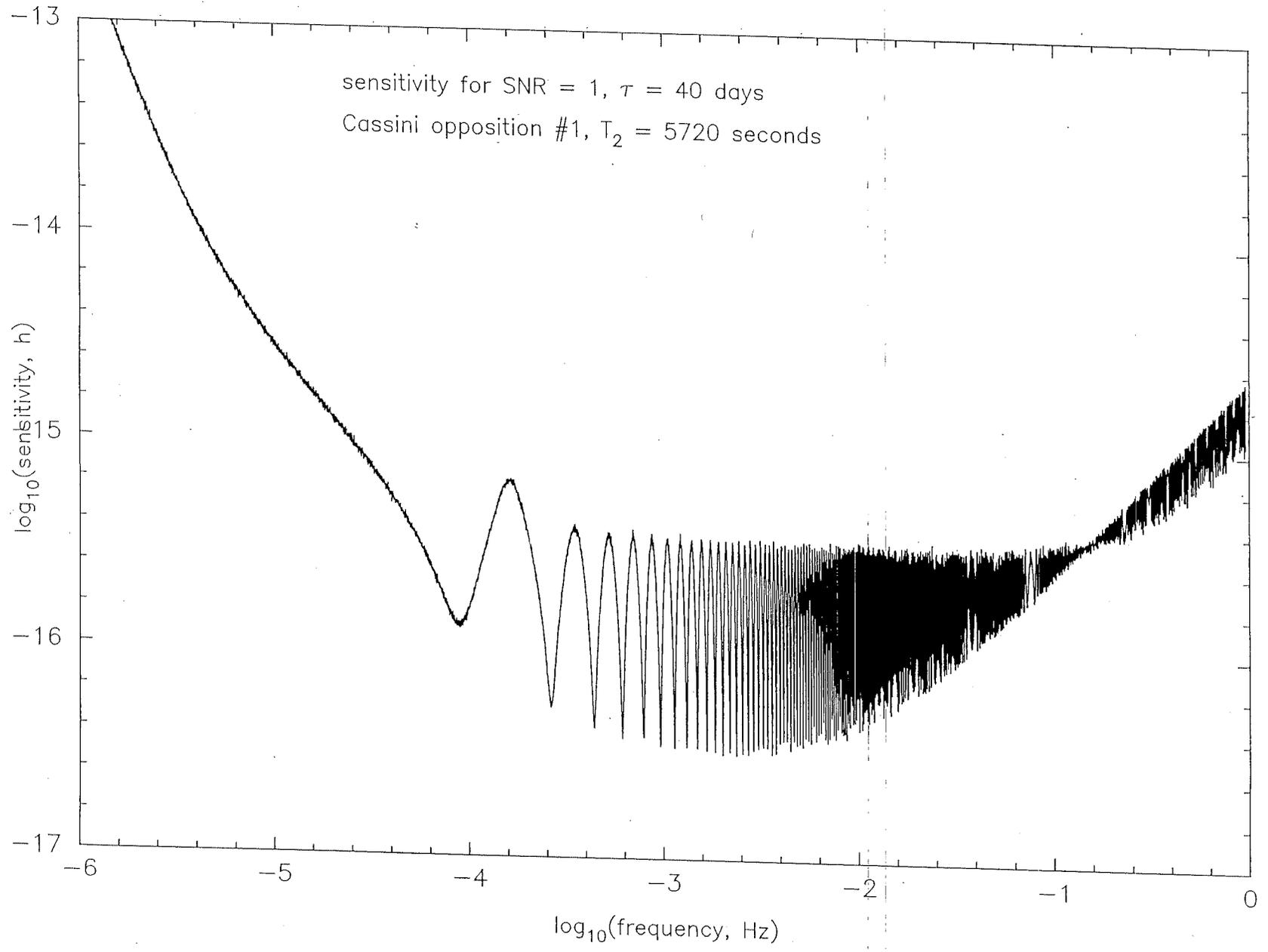


hammer-aitoff equal-area projection (center of plot is RA = 0, dec = 0)

relative energy response for Cassini 2004 January 4  
circular-pol:  $\sin(2 \pi (0.001 \text{ Hz}) t) \cdot \exp(-t/1000 \text{ sec})$



hammer-aitoff equal-area projection (center of plot is RA = 0, dec = 0)



## Backgrounds

$$\Omega = \frac{\rho}{\rho_c}$$

$$\rho_c = \frac{3H_o^2}{8\pi G} \approx 1 \times 10^{-29} \text{ grams/cm}^3, \text{ if } H_o = 80 \text{ km/sec/Mpc}$$

$$\rho = \left(\frac{243}{208}\right) \pi^3 \frac{f_c^4}{G} \int df \frac{S_y(f)}{(4\pi^2 f^2)}$$

$$= \left(\frac{243}{208}\right) \frac{f_c^3 S_y(f_c)}{(4G)}$$

Expected Cassini spectrum for  $S_y$  implies  $\Omega \lesssim 10^{-2}$  for  $f_c \approx 10^{-4}$  Hz

## **Topics**

- **Short review of the GWE (what we are doing; why we are doing it; signal and noise transfer functions; new instrumentation, etc.)**
- **Quick-look data: noise statistics**
- **Some interesting things in the quick-look data**
  - **Propagation noise/media cal data**
  - **Antenna mechanical noise (things that go bump in the night...) examples**
  - **FTS noise examples**
  - **Spacecraft noise**
  - **Lunar occultation**
- **Discussion**

## Cassini GWE

- 40 days around the clock tracking of Cassini, searching for gravitational waves in the "low-frequency" ( $\sim 0.0001$ - $0.01$  Hz) band
- New equipment at Goldstone and elsewhere
  - Ka-band up- and down-link, Advanced Media Calibration [AMC] units, new radio science receivers, new FTS, aberration correction, monopulse pointing, more)
  - Supports several Cassini RS investigations (GWE, SCE, ring occultations, atmosphere occultations, precision Doppler during Tour)
- GWE is first beneficiary of this work—main utility for GWE:
  - Ka-band ( $\sim 32$  GHz) reduces plasma scintillation noise to negligible levels
  - AMCs: measure/calibrate tropospheric scintillation

## Cassini GWE (cont.)

- **Two-way Ka-band supported on the spacecraft (by hardware supplied by Italian Space Agency--the "KaT") and on the ground (by the DSS25 Cassini radio science upgrade supplied by NASA)**

- **Discussion of error budget and sensitivity at:**

<http://www.its.caltech.edu/~cajagwr/pdf/Armstrong.pdf>

- **Discussion of Doppler tracking in context of overall GW astronomy strategy:**

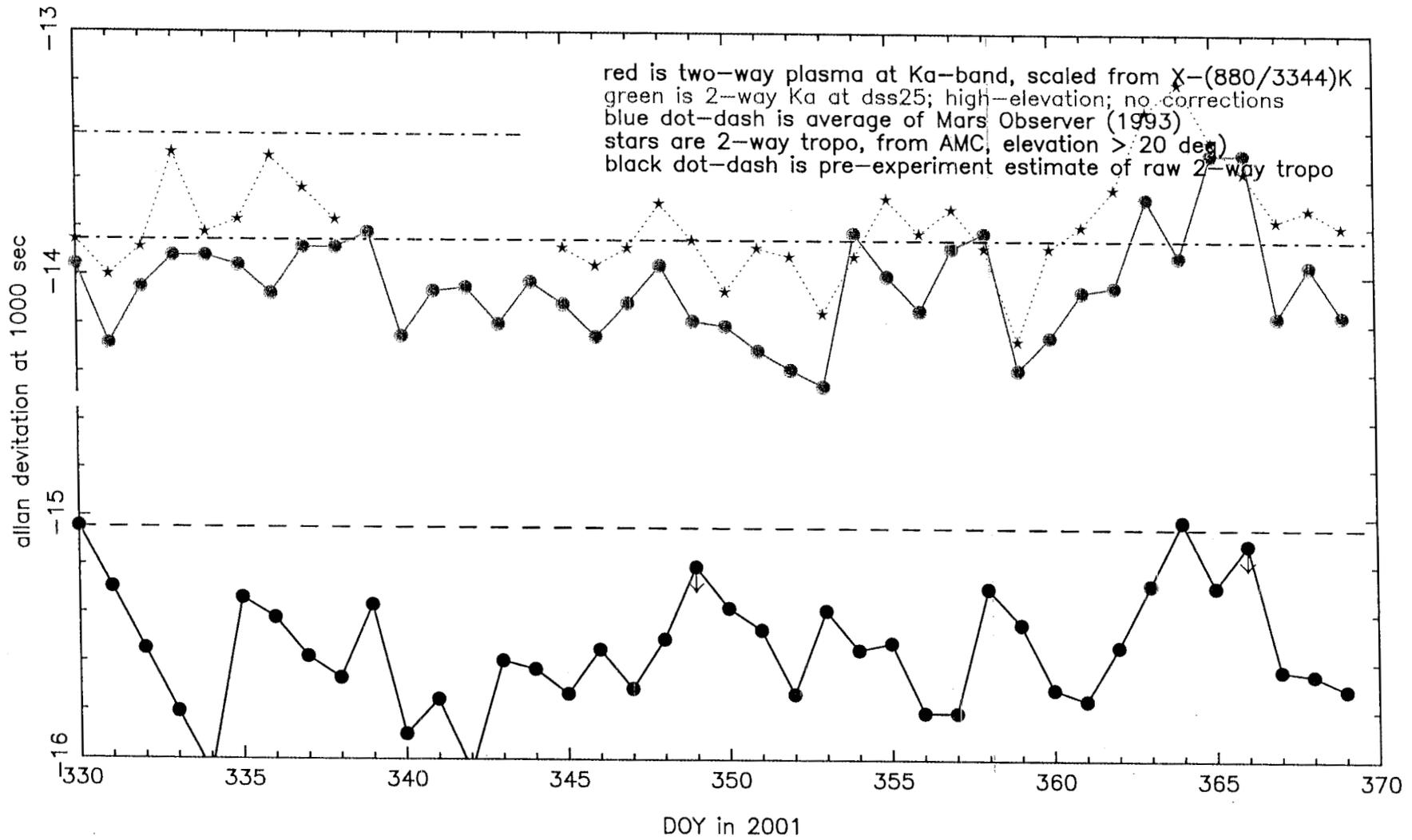
[http://www.its.caltech.edu/~cajagwr/scripts/participating\\_projects.html](http://www.its.caltech.edu/~cajagwr/scripts/participating_projects.html)

- **Pre-experiment sensitivity goal, after all corrections,  
 $\sigma_y(1000 \text{ sec}) \approx 3 \times 10^{-15}$**

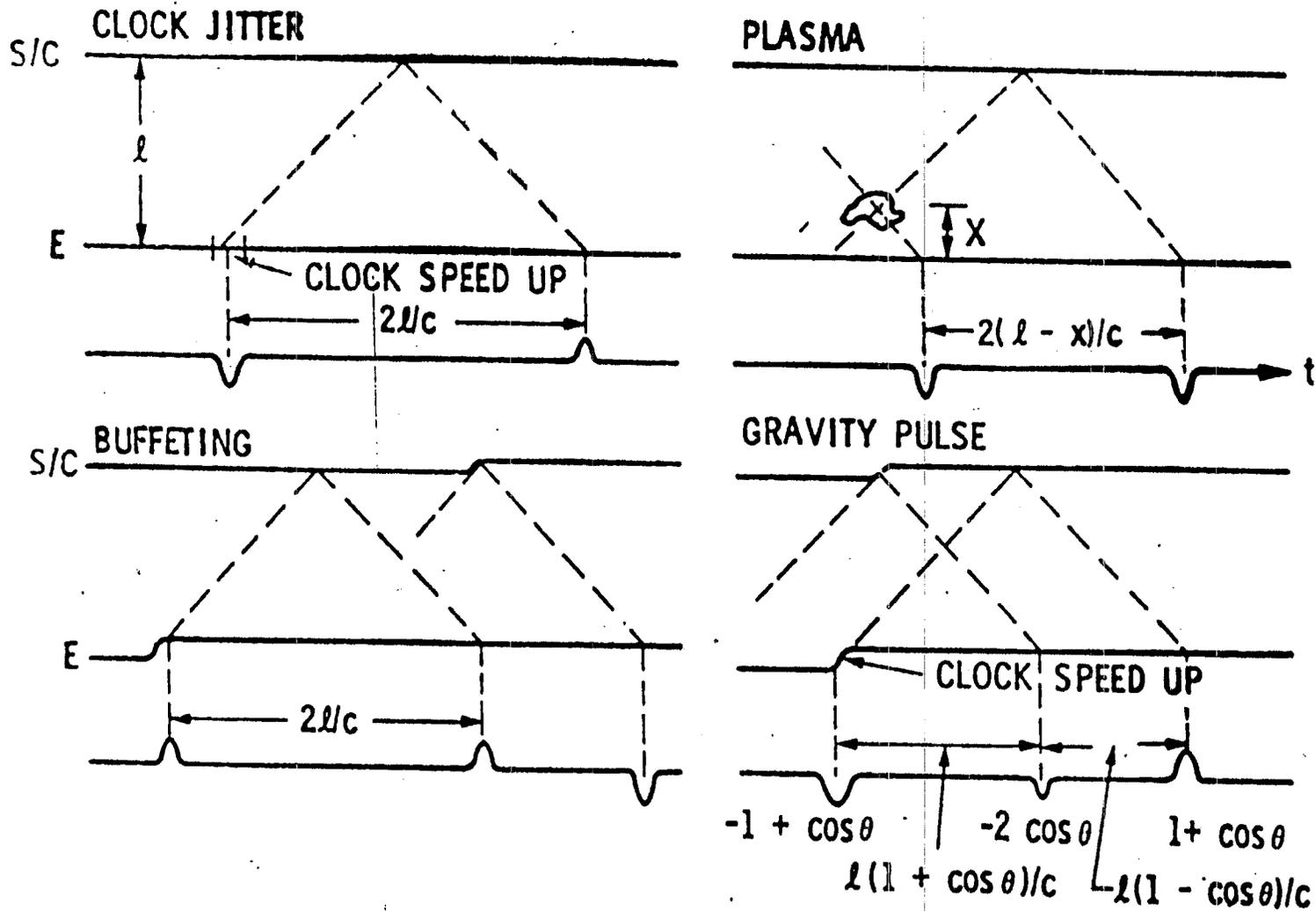
## Quick-Look Analysis

- **Quick-look data were two hours/day at DSS 25, selected to be near meridian transit**
  - **High elevation minimizes many problems: antenna mechanical systematic effects due to M1 mirror sag, minimum tropospheric noise, dry component approximately linear and thus absorbed in 4-parameter orbit fit, etc.)**
  - **PLL of open-loop data to estimate signal amplitude and frequency, solve for average velocity, average acceleration, and sine/cosine of earth rotation Doppler to remove an approximate orbit --> "residuals"**
  - **Simultaneous X and Ka1 allows simple estimate of plasma scintillation level—time series/spectra taken also**
  - **Convert AMC tropospheric delays to fractional frequency deviations—produce spectra of these to see what the troposphere is doing**

quick-look noise statistics of Cassini GWE1



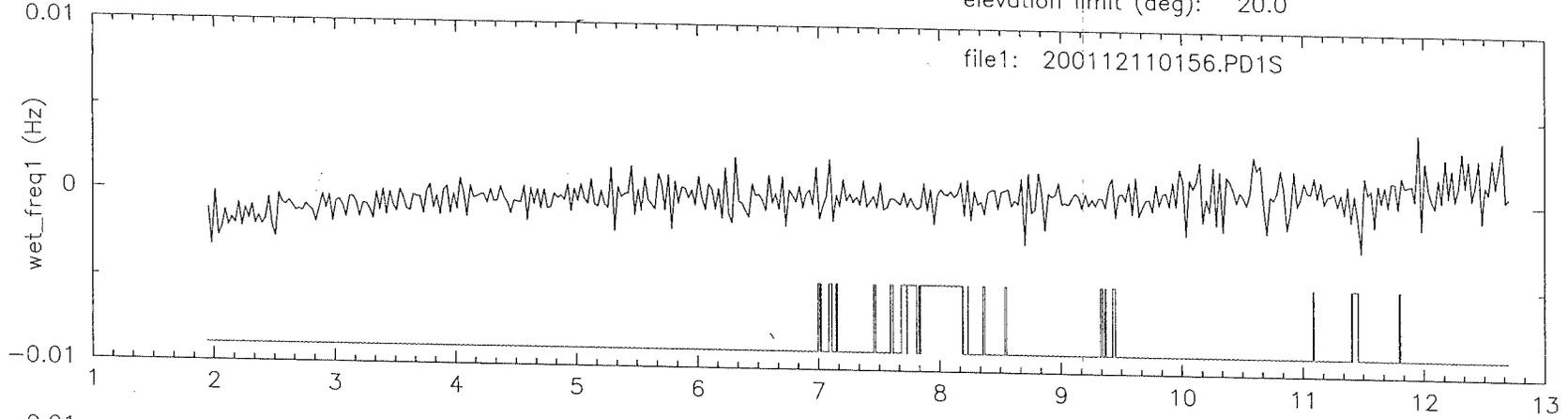
# Doppler Signals Corresponding To Different Disturbances in the Radio Link



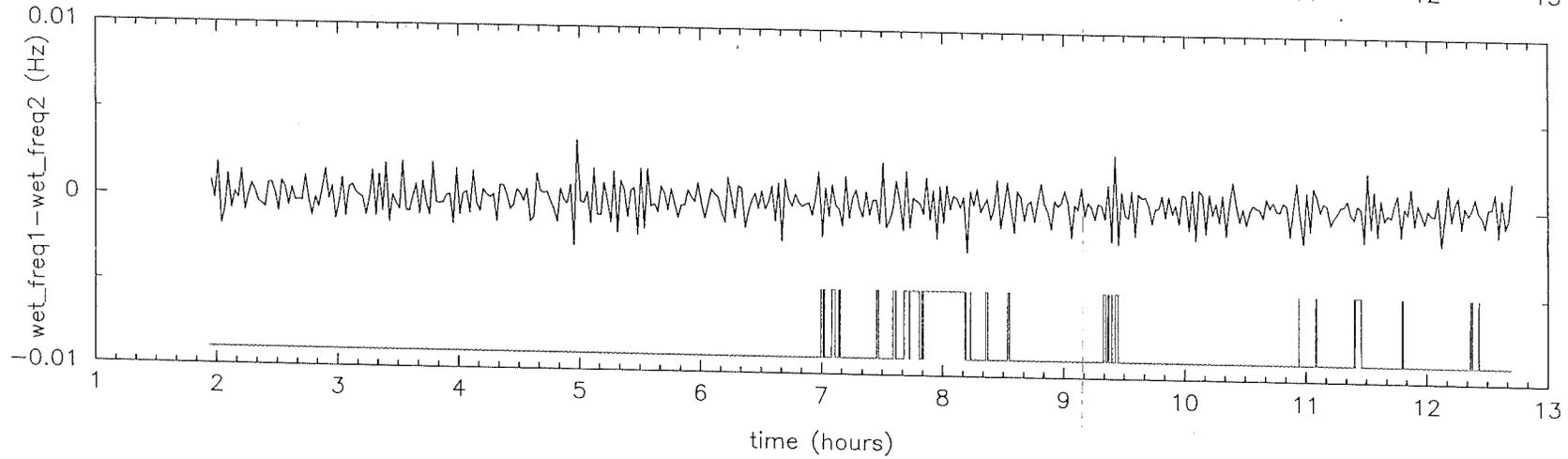
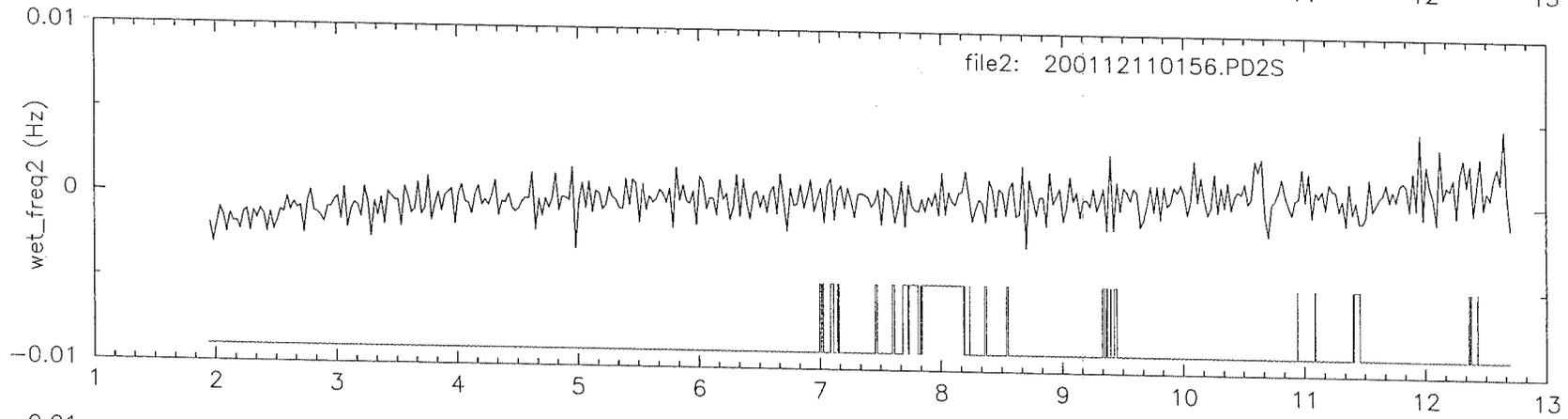
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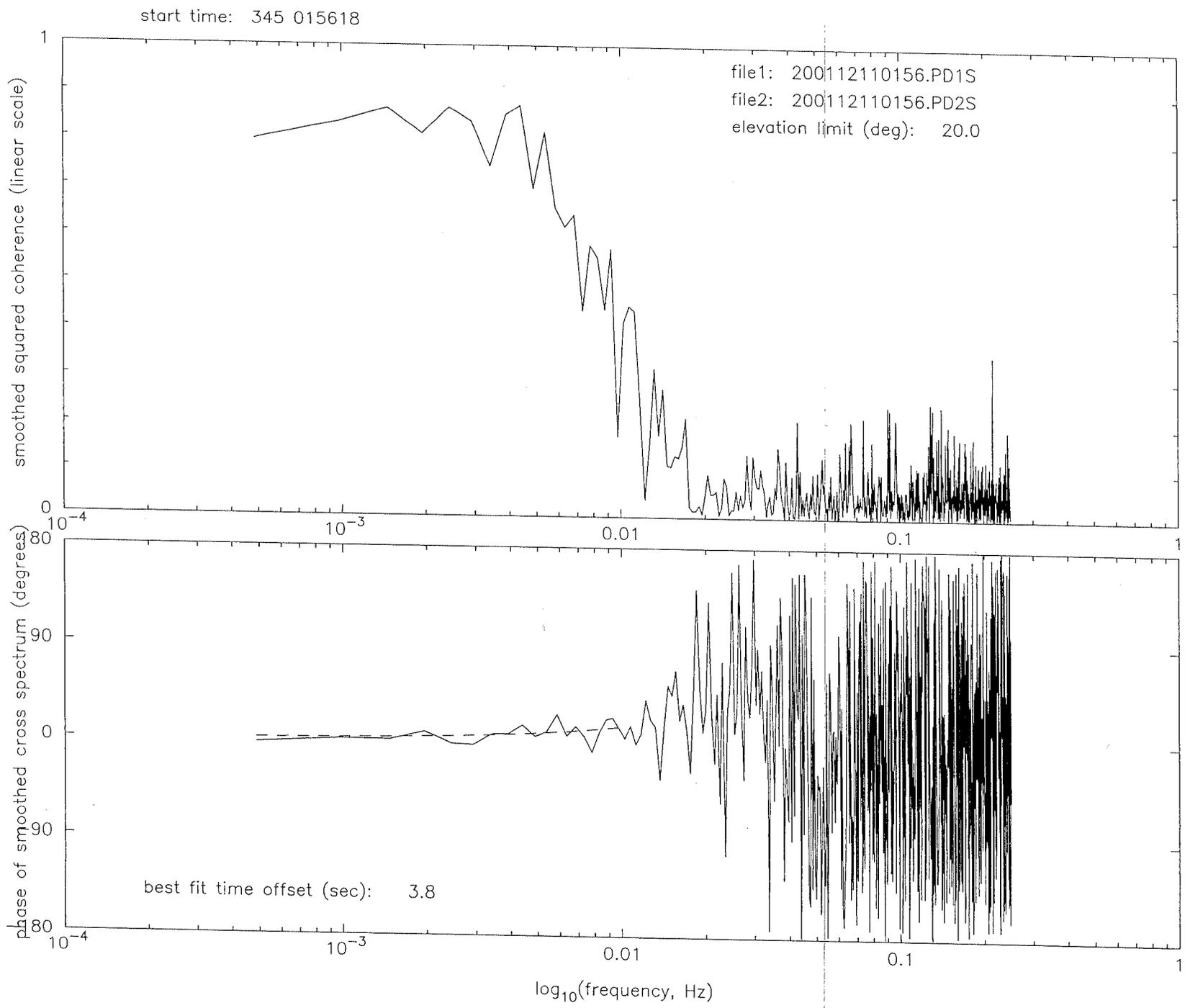
elevation limit (deg): 20.0

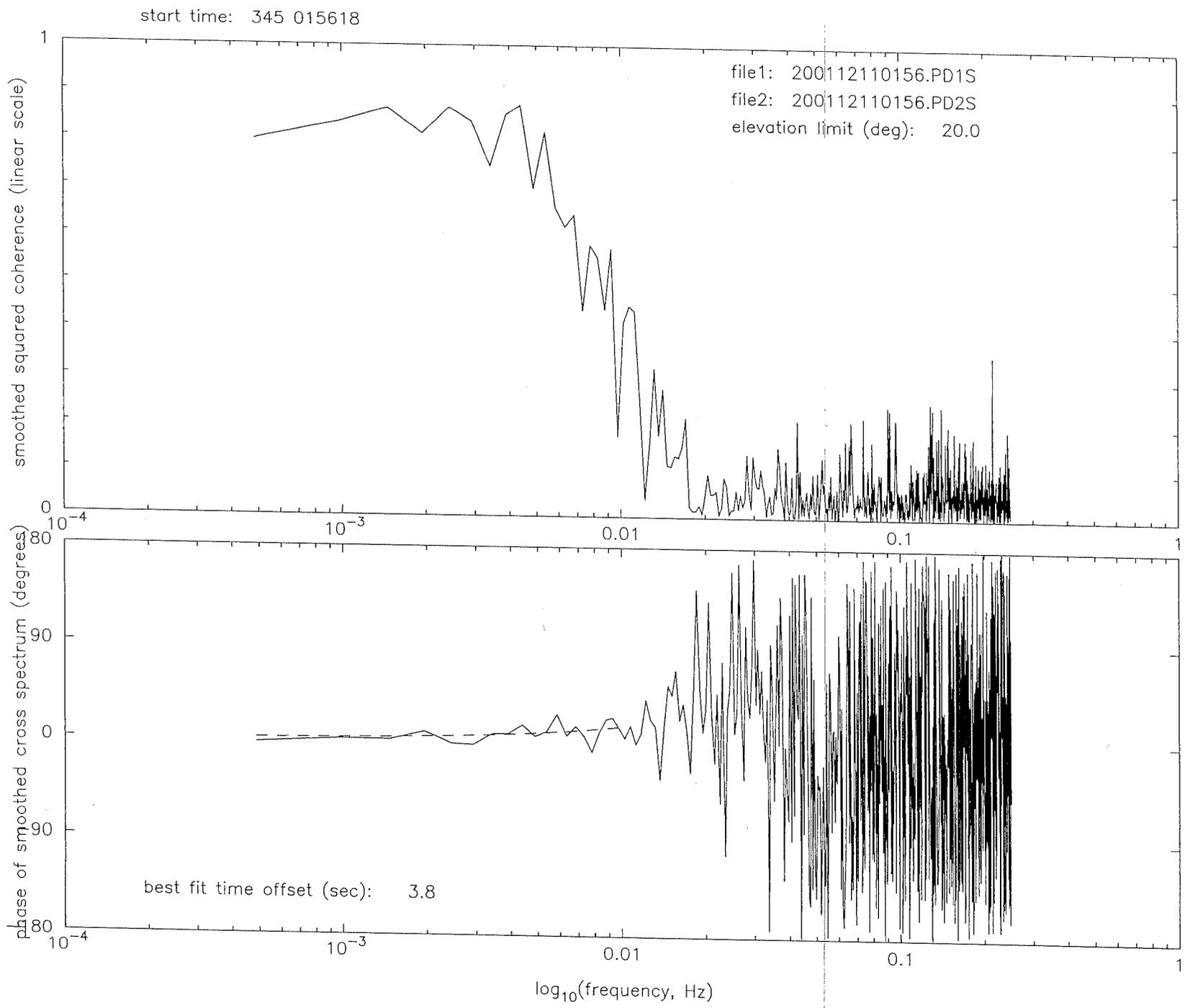
file1: 200112110156.PD1S



file2: 200112110156.PD2S

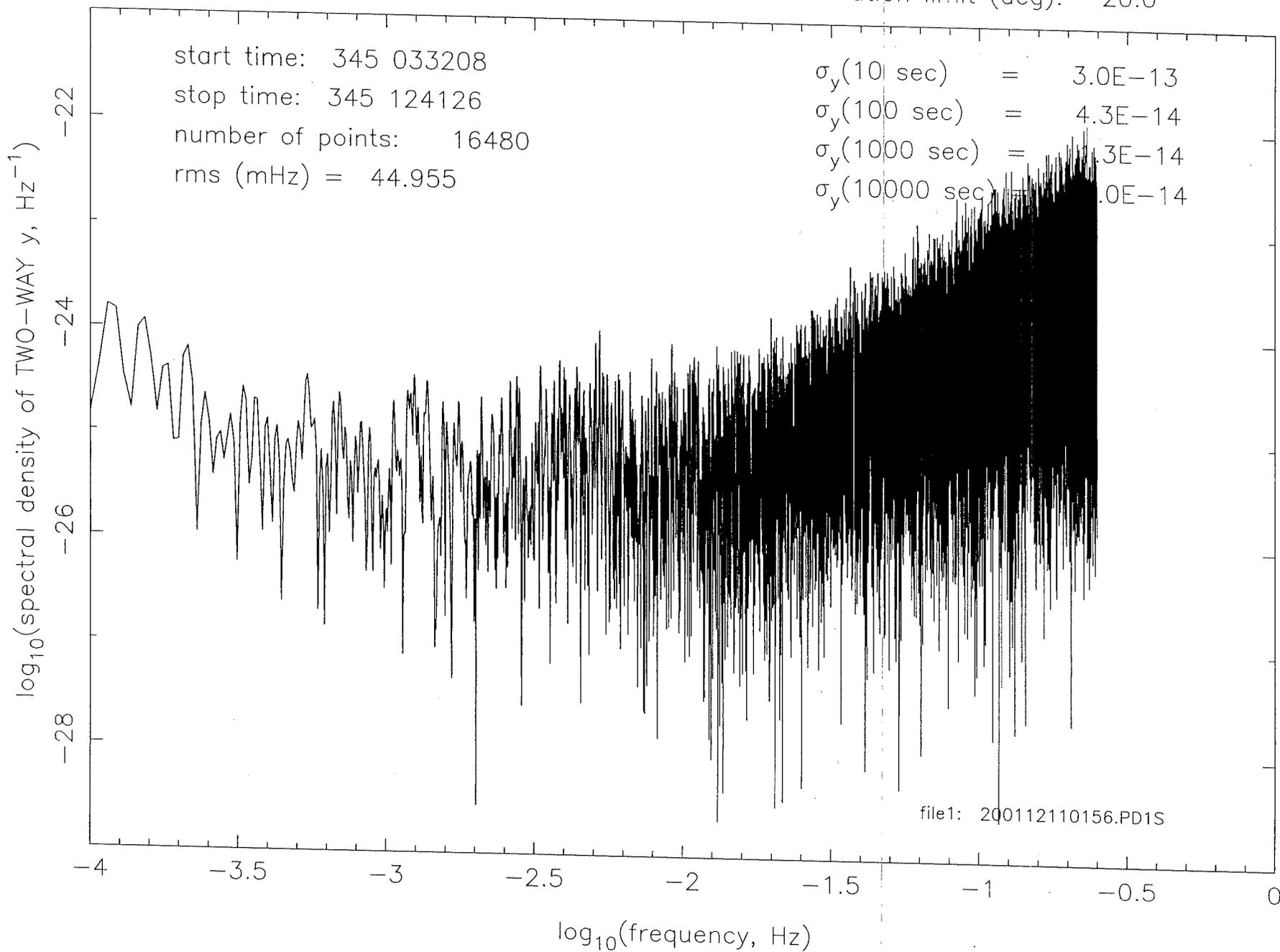


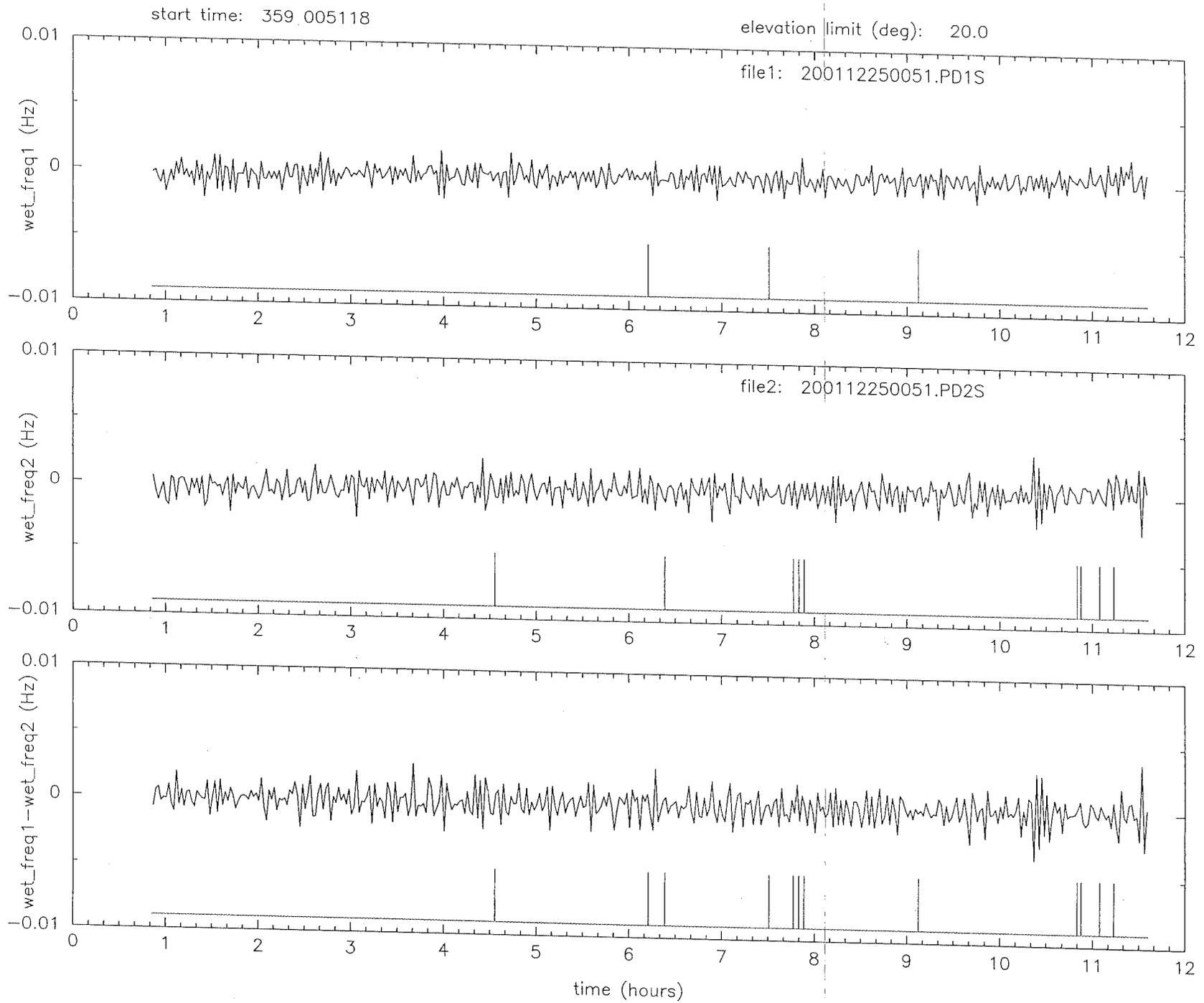




(Ka-band normalization for y)

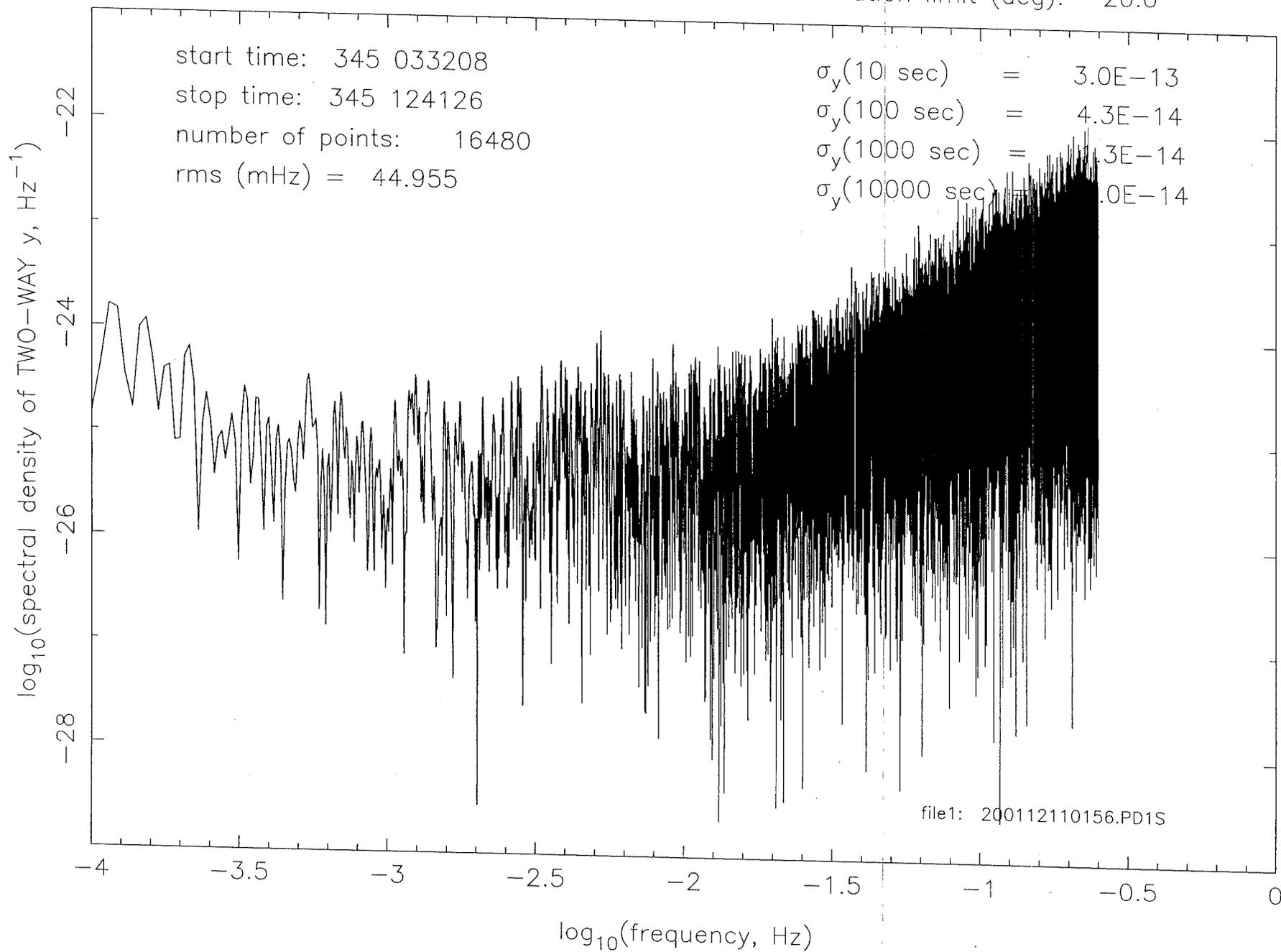
elevation limit (deg): 20.0

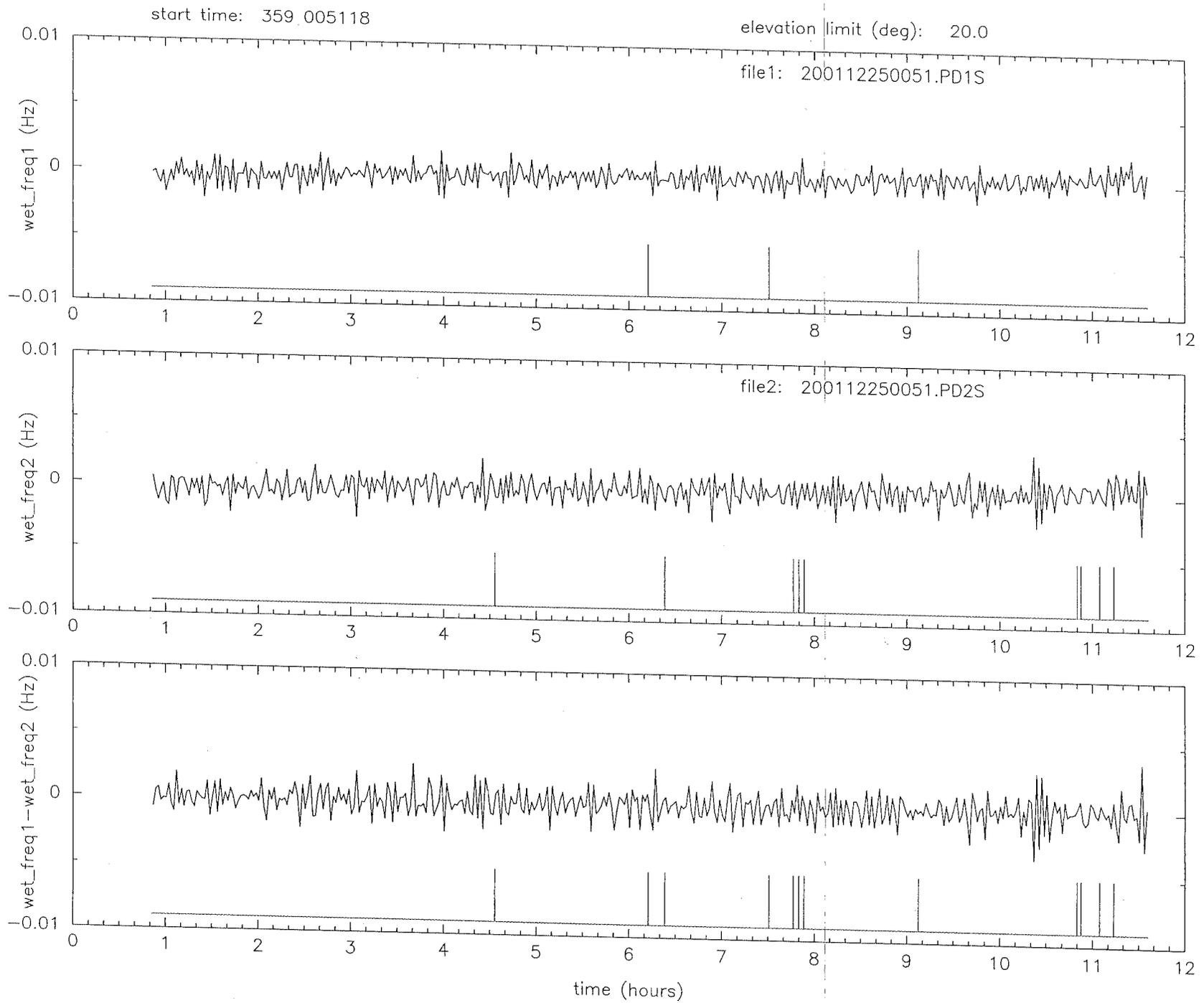




(Ka-band normalization for y)

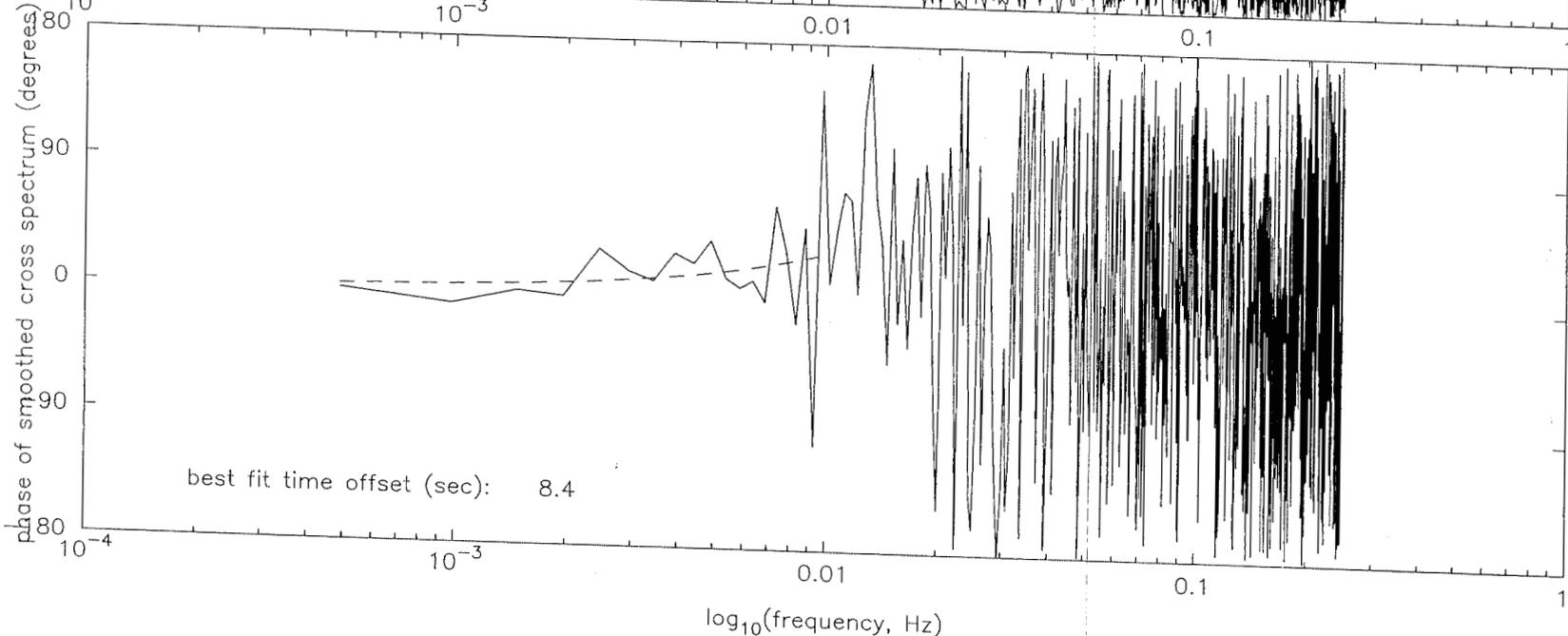
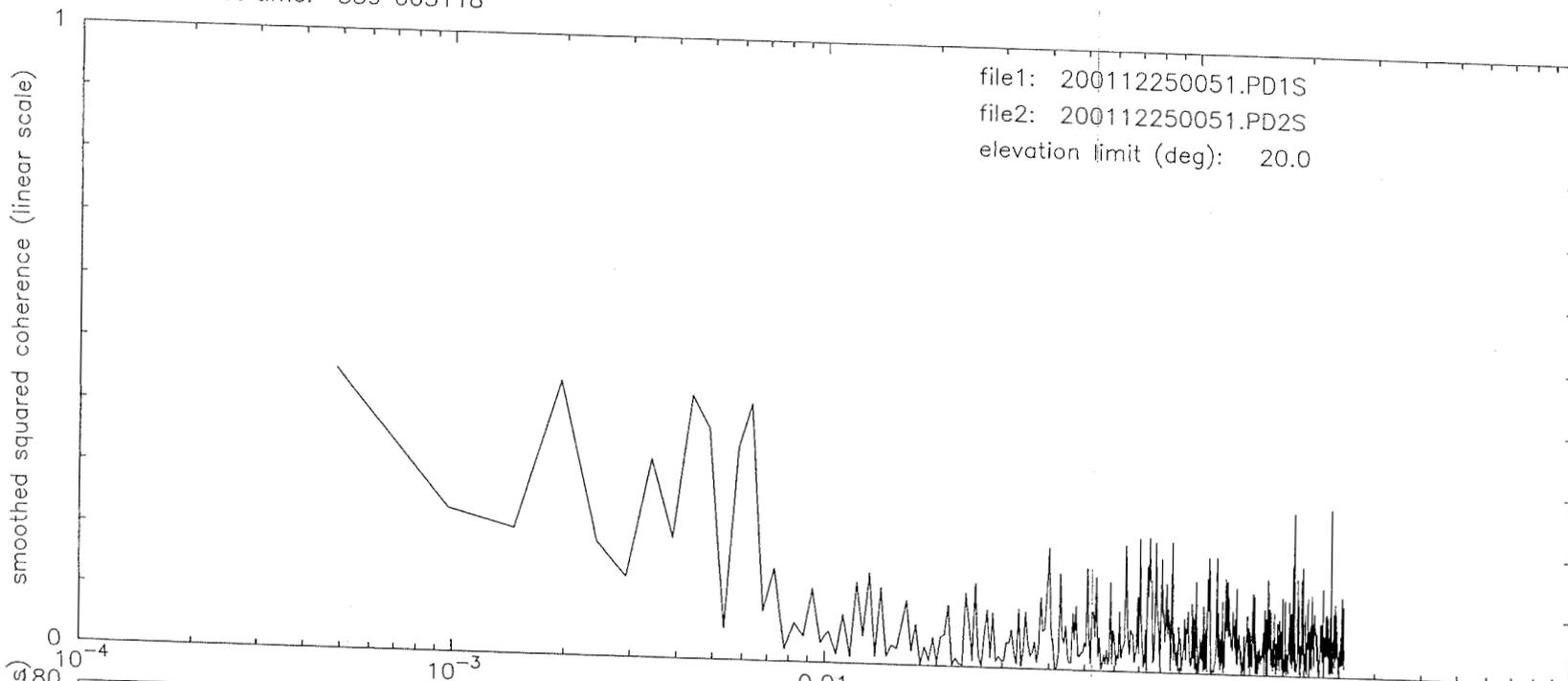
elevation limit (deg): 20.0





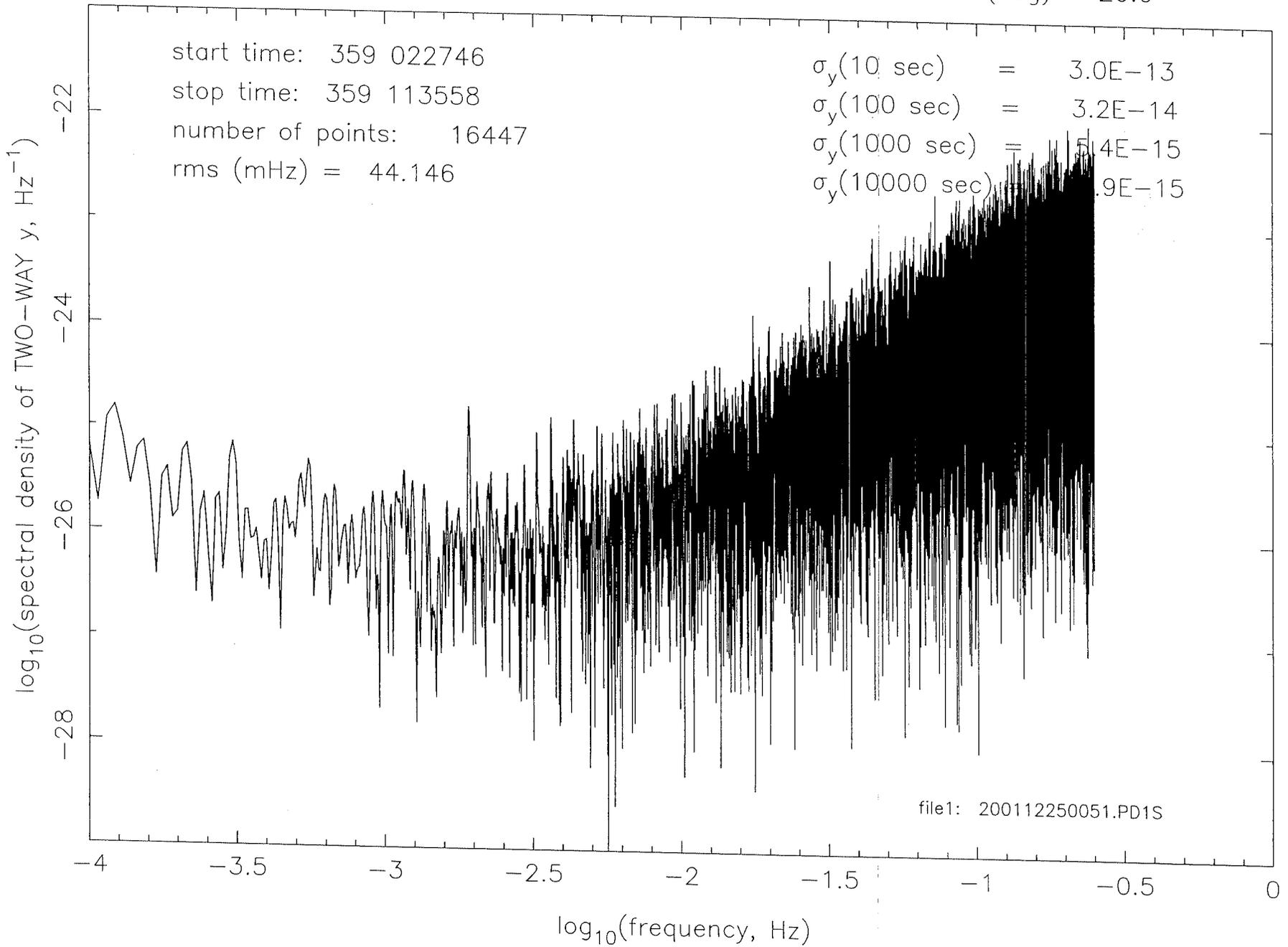
start time: 359 005118

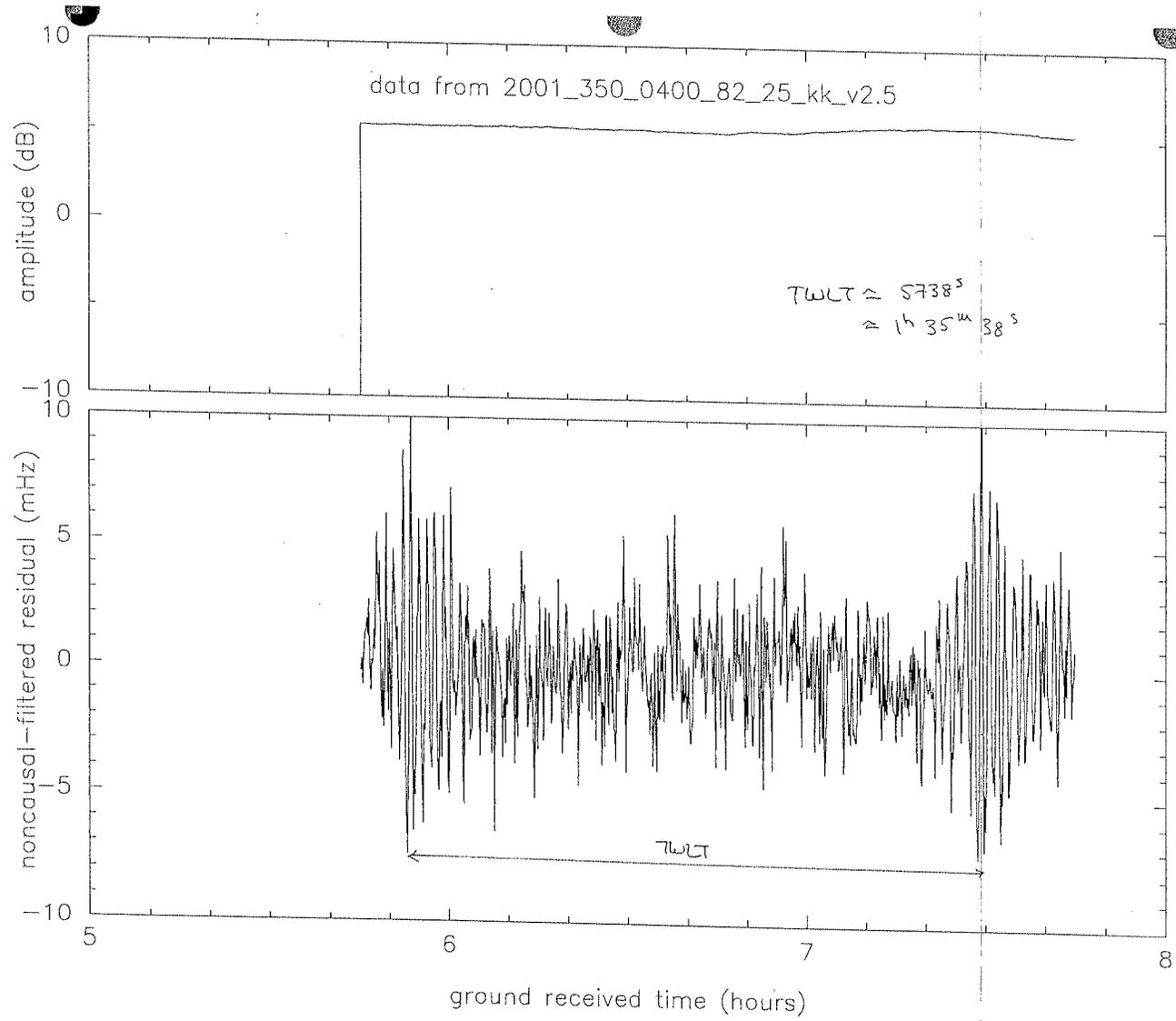
file1: 200112250051.PD1S  
file2: 200112250051.PD2S  
elevation limit (deg): 20.0



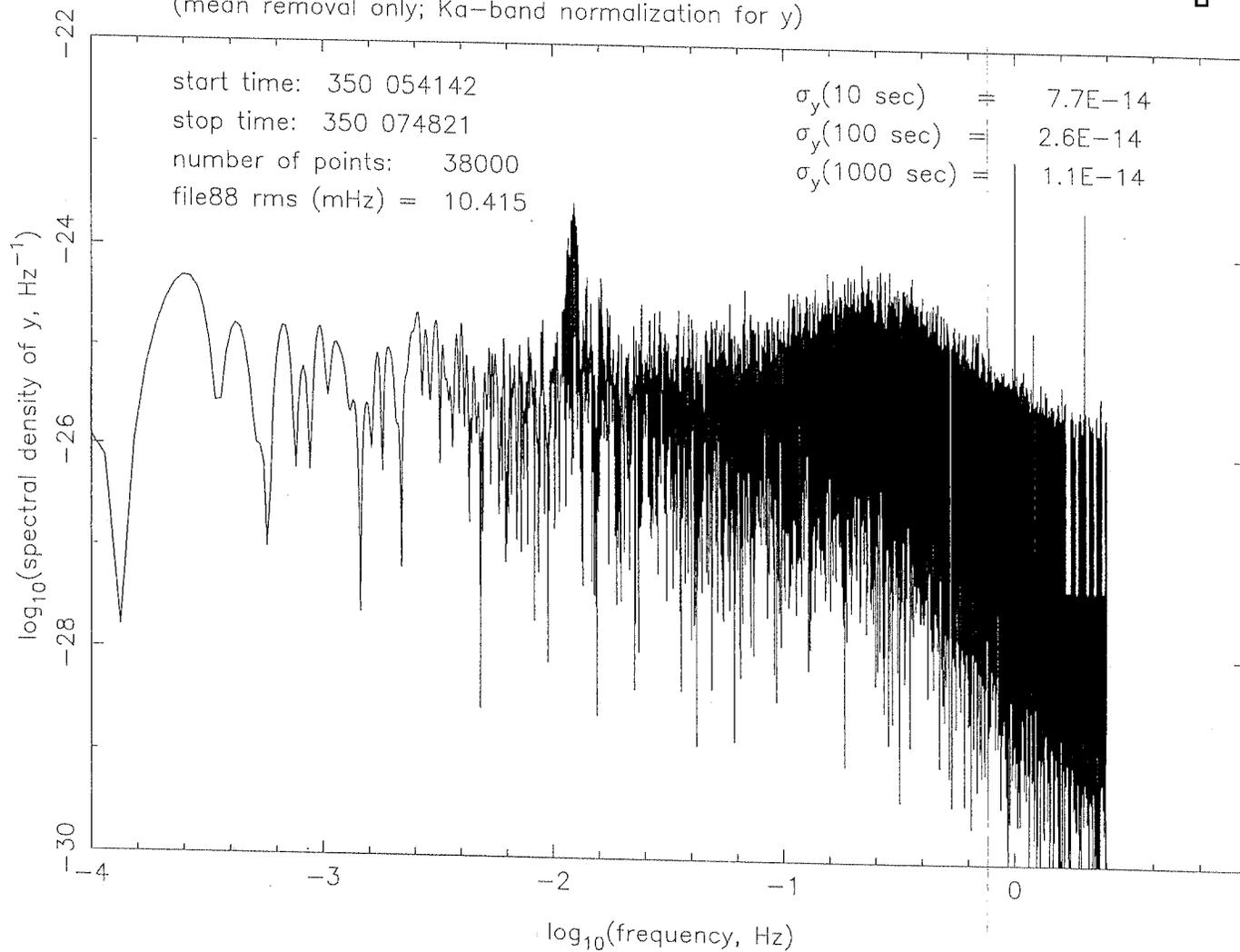
(Ka-band normalization for y)

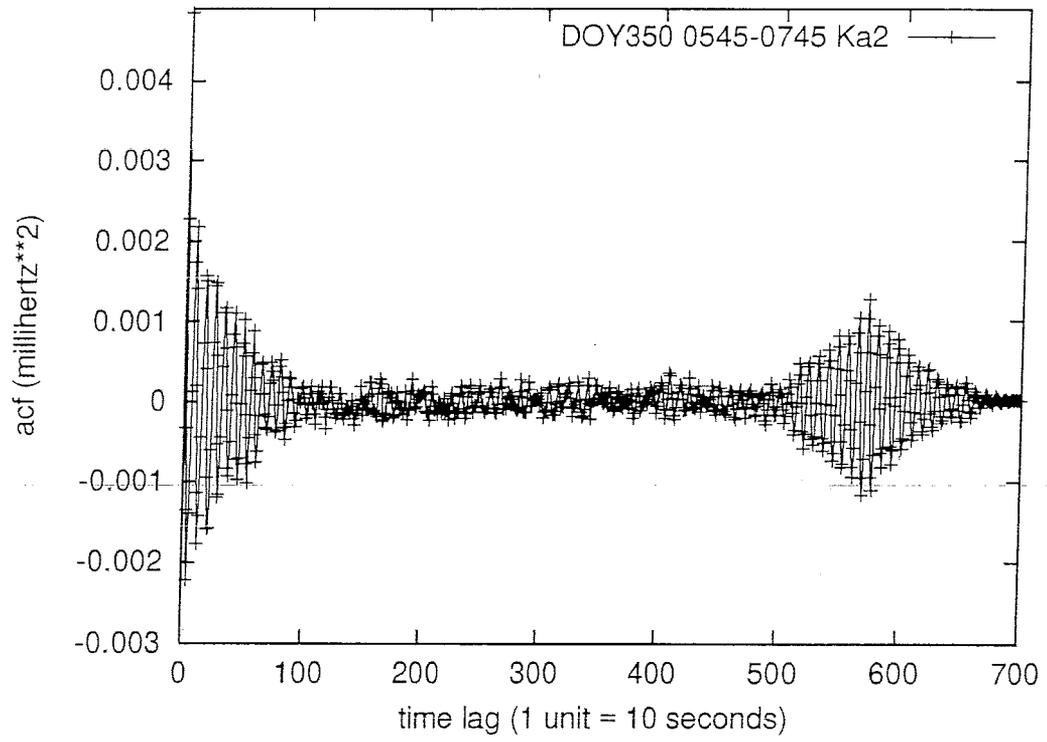
elevation limit (deg): 20.0



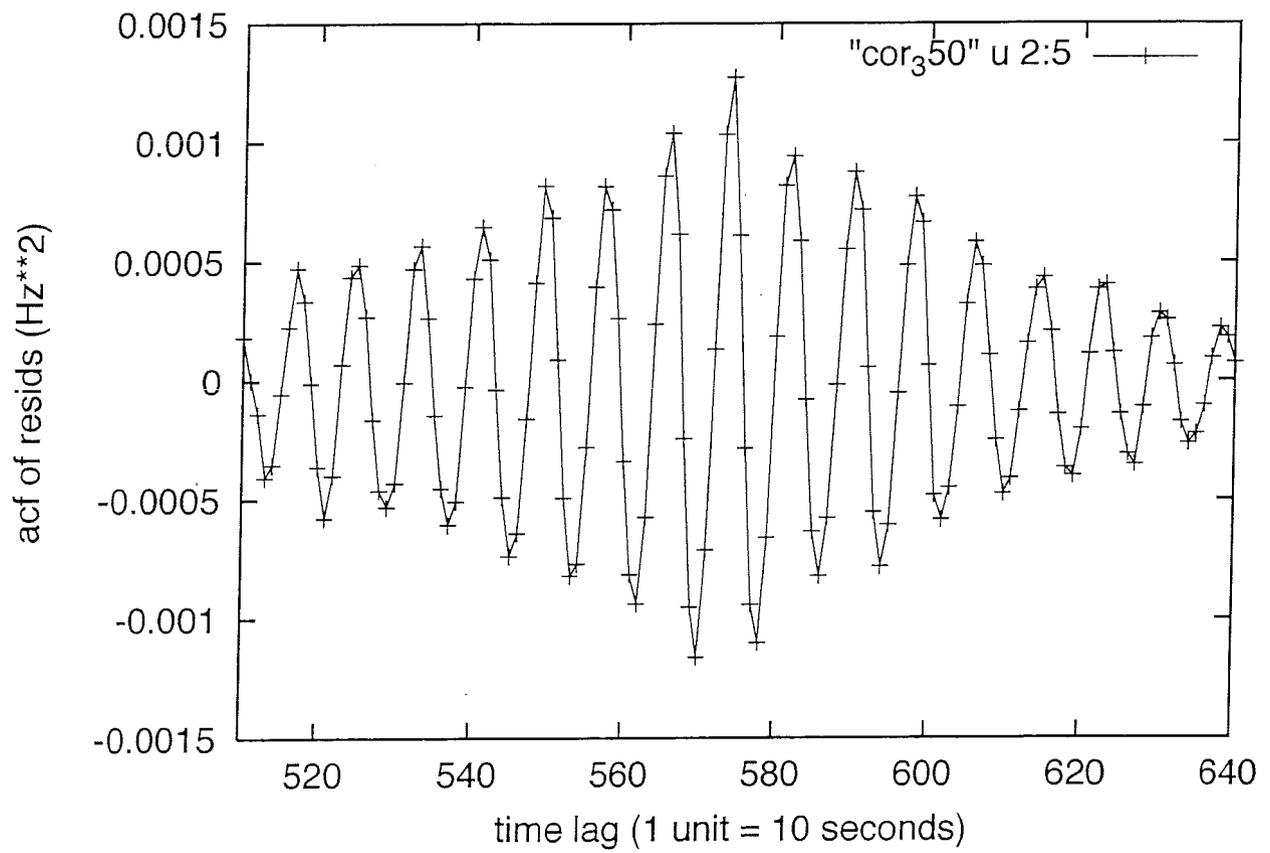


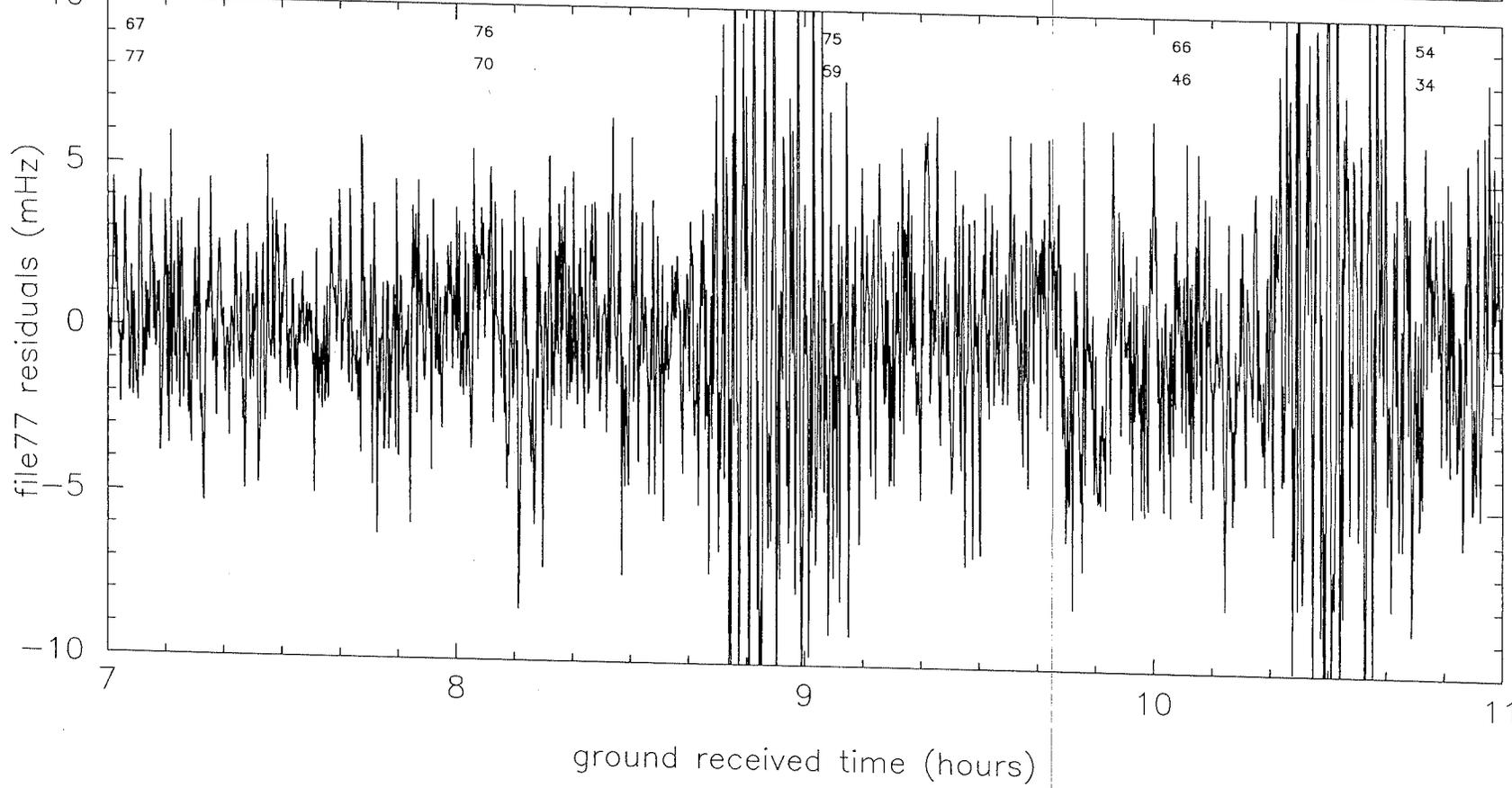
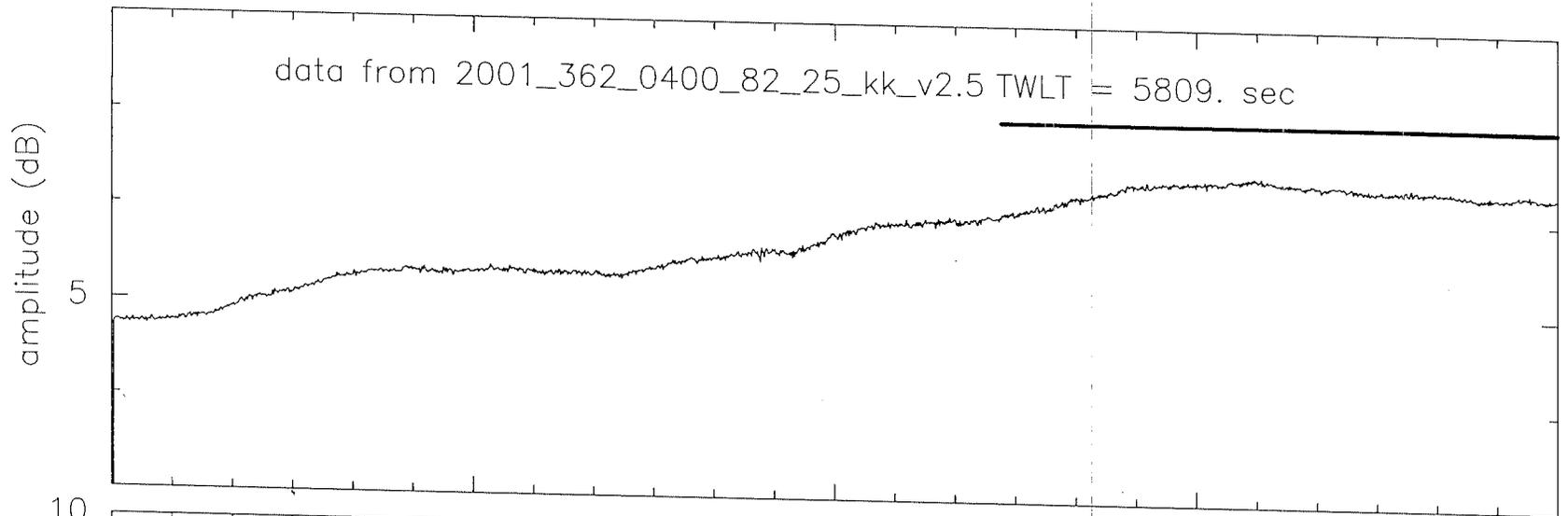
data from 2001\_350\_0400\_82\_25\_kk\_v2.5  
(mean removal only; Ka-band normalization for y)



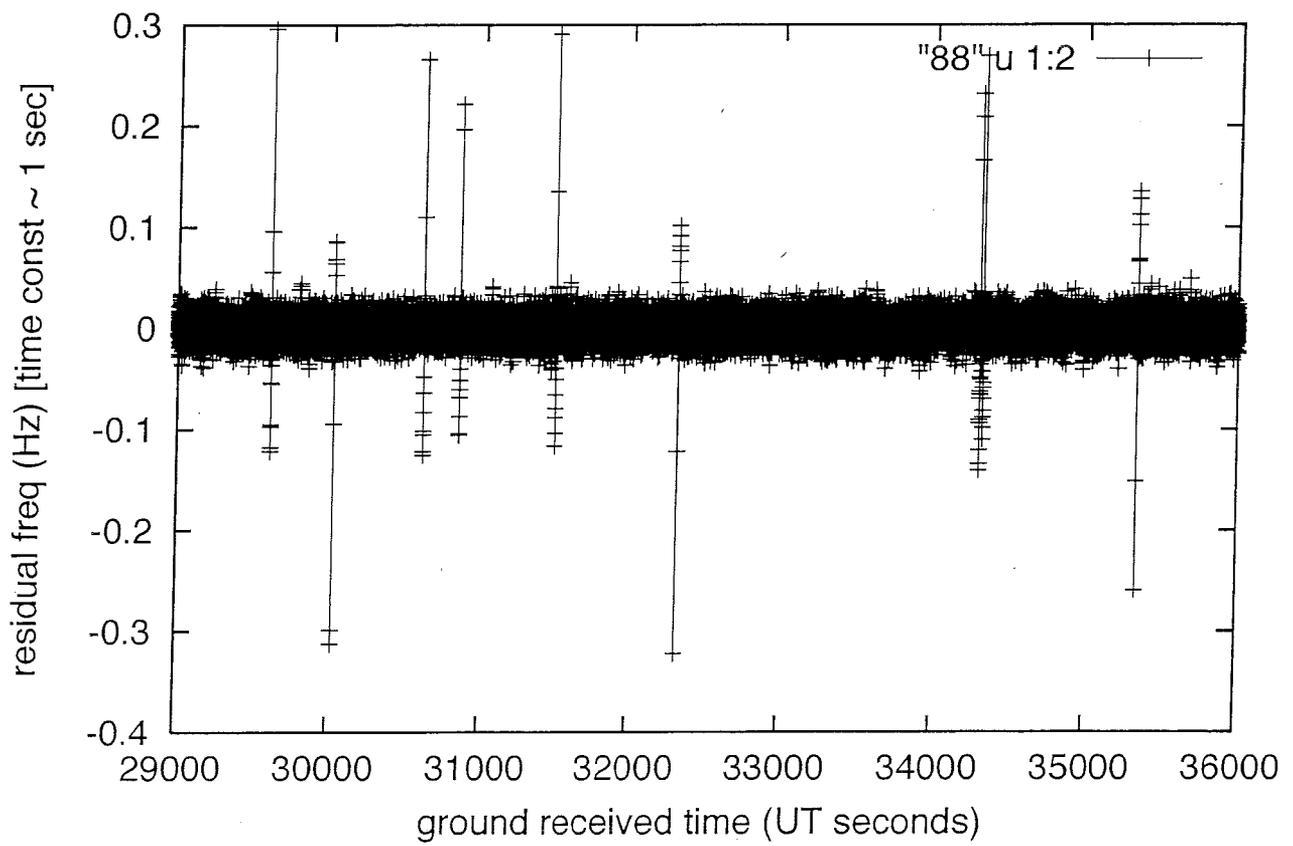


DOY 350 acf blowup near TWLT ~ 5738 sec

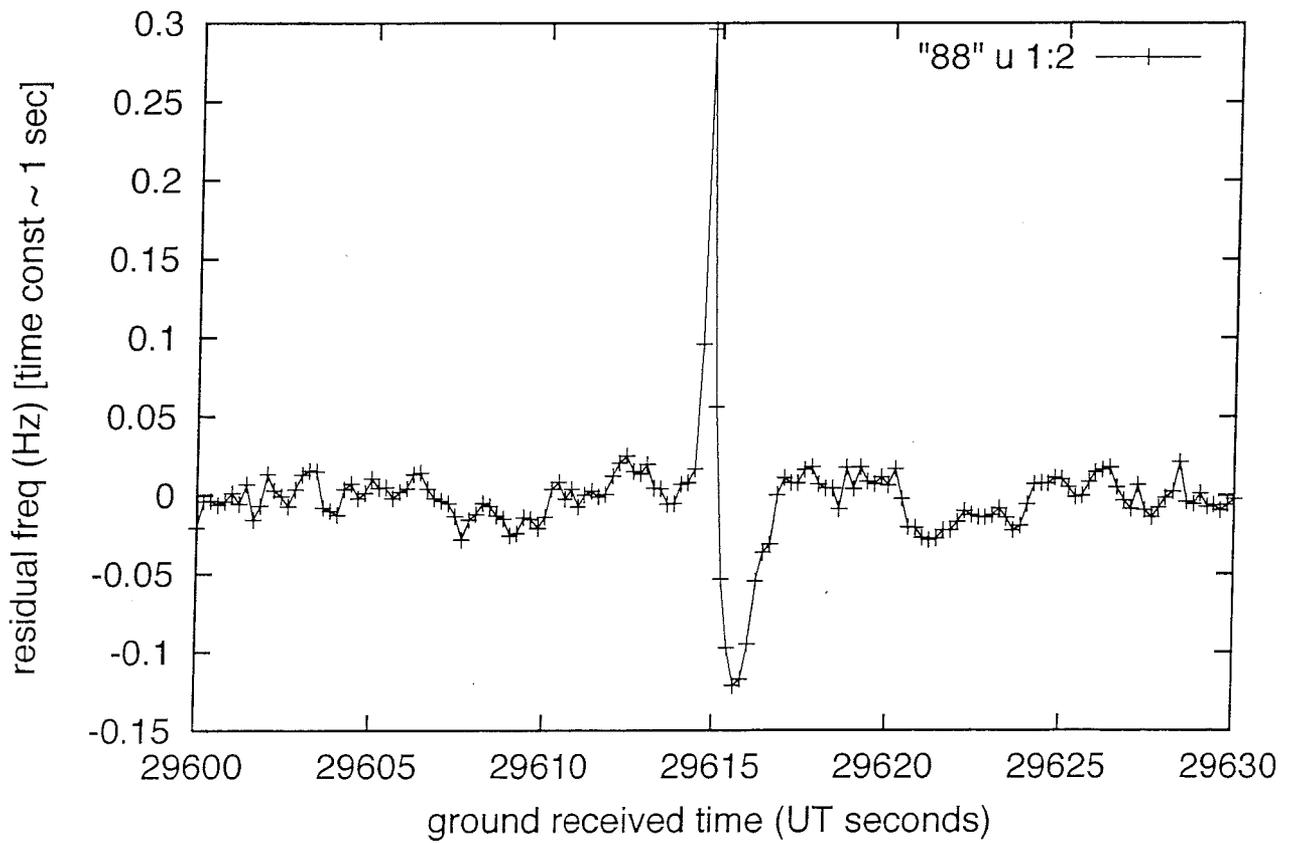




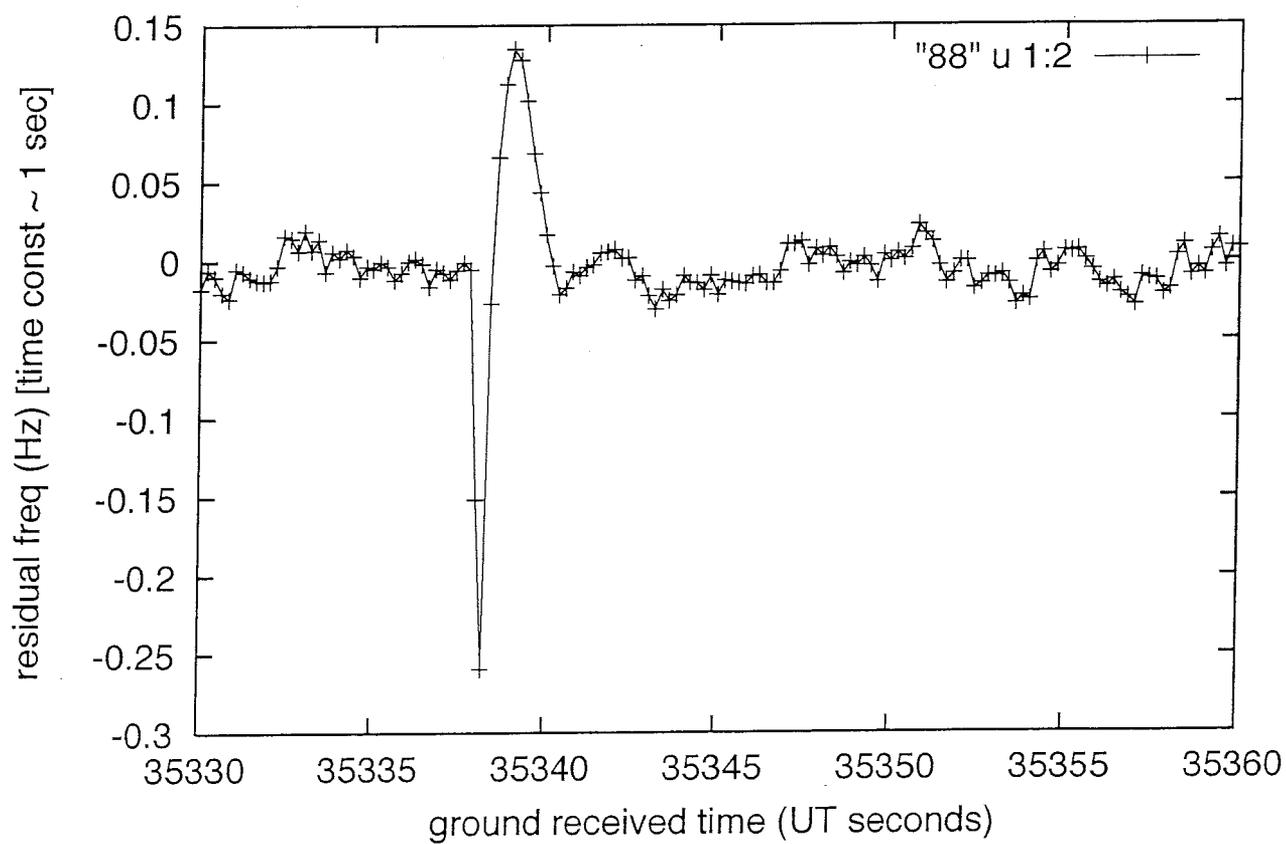
DOY 346 2-way Ka-band, two-way light time = 5723.2 sec



DOY 346 2-way Ka-band, two-way light time = 5723.2 sec



DOY 346 2-way Ka-band, two-way light time = 5723.2 sec



## Estimated Value of S/C's $\sigma_Z^{GWE}$

- Results:

- 40 hours of data beginning 2001-DOY-152/T02:00:03.558 (sampling time is 4 s)
- CAPS articulation motion was active over this time span (articulation frequency  $\approx 0.0025$  Hz)
- Estimated value of  $\sigma_Z^{GWE}$  is  $\approx 0.071 \mu\text{m}/\text{sec}$ . It is significantly better than the requirement ( $0.3 \mu\text{m}/\text{sec}$ )

- Corresponding Allan deviation

$$\approx 2.3 \times 10^{-16} < 10^{-15} \text{ (Requirement)}$$

from: Won, Hanover, Beletky & Lee 10/18/01

## DOY 363 Lunar Occultation

DOY 363 was notable for (a) very bad weather, (b) a precursor to the most noisy plasma day seen to date (possibly associated with a coronal mass ejection—see below) and (c) a lunar occultation of the Cassini signal.

**W.r.t. the weather:** this is the only day I have seen so far where the AMC data quality warning flags are set for the **whole** pass.

**W.r.t. the plasma:** this was the day before the worst plasma noise day to date (see first plot—the last red dot is the DOY 364; the ka-band plasma is still very low compared with other noises!). It is probable that this enhanced plasma is just a statistical attribute of the corotating solar wind (since the plasma noise was smoothly rising anyway, perhaps due to rotation of a noisy solar wind sector onto the line of sight, for the last several days). But Elias alertly found this information on the web (Dec 28 is DOY 362):

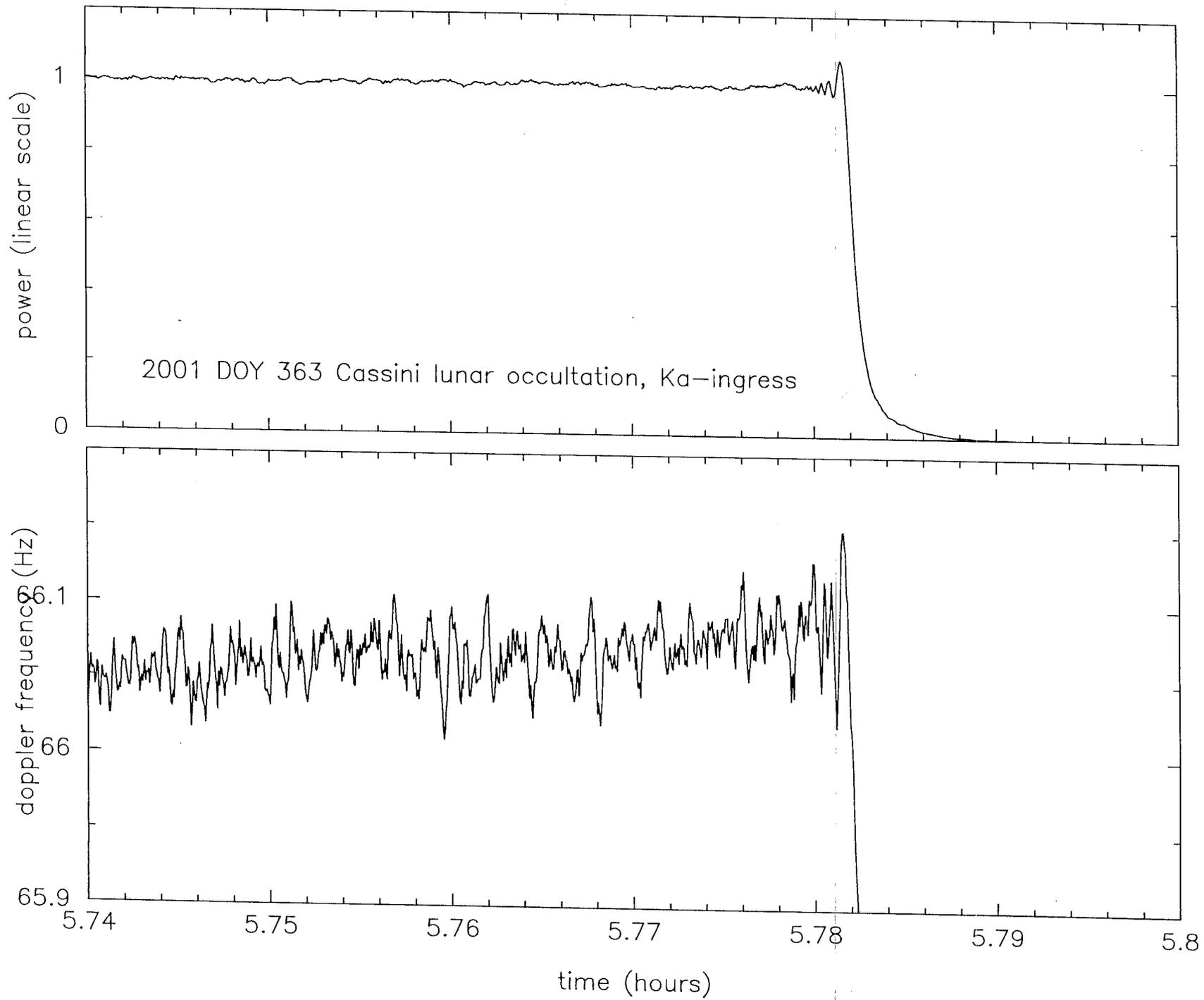
“Space Weather News for Dec. 26, 2001  
<http://www.spaceweather.com>

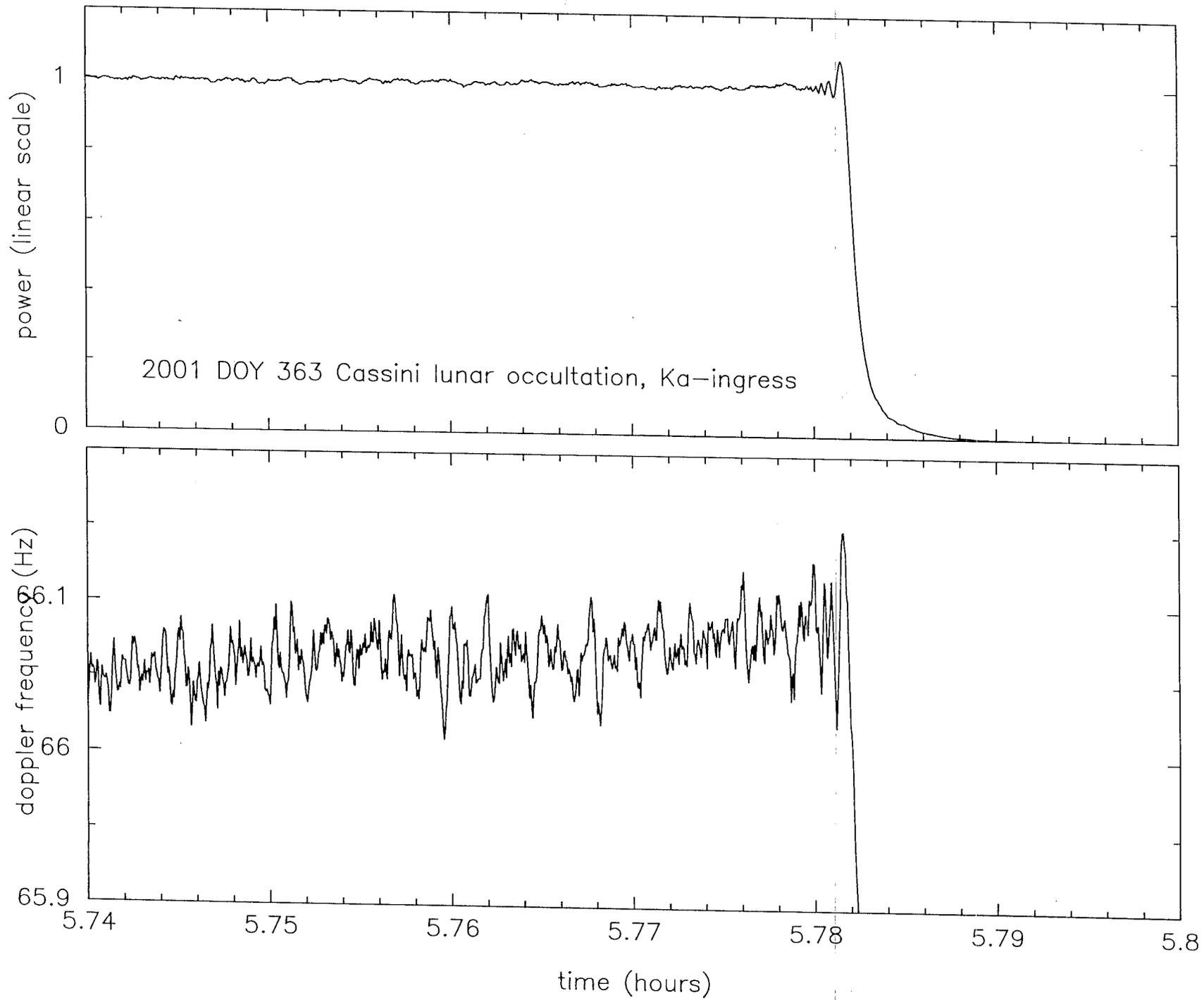
GEOMAGNETIC STORM WARNING: An explosion on the Sun today triggered a solar proton storm around our planet and hurled a bright coronal mass ejection (CME) into space. Although the fast-moving (~1100 km/s) CME was not squarely Earth-directed, it could deliver a glancing blow to Earth's magnetosphere as early as Dec. 28th Universal Time -- that is, Thursday night for North Americans, Friday morning for Europeans. NOAA forecasters estimate a 10% chance of severe geomagnetic storms at middle latitudes when the expanding cloud sweeps past Earth. Sky watchers along the northern tier of US states (and similar latitudes) should be alert for auroras during the nights ahead.”

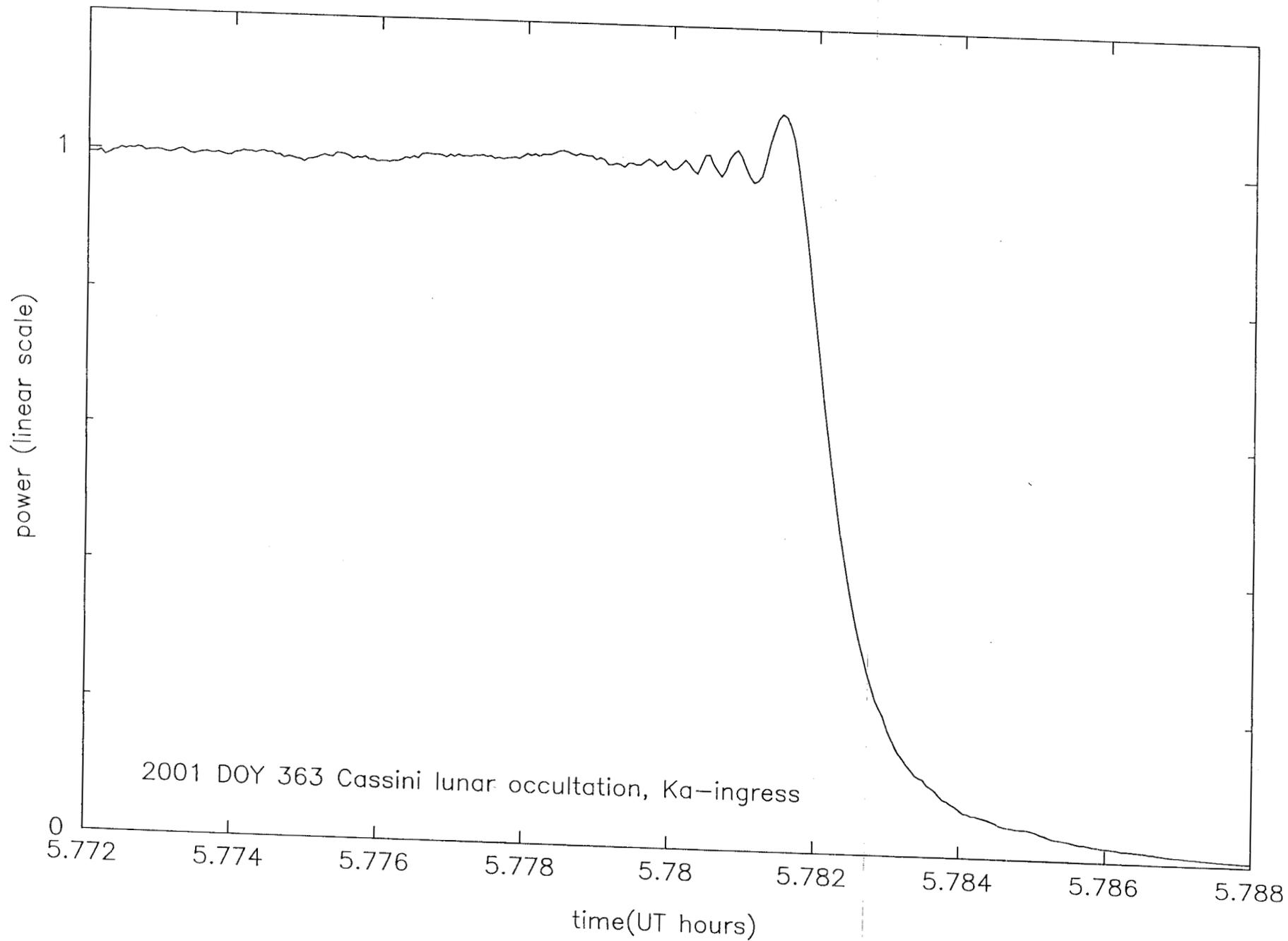
[If the CME was directed to the trailing side of the Earth in its orbit, I would expect it to influence the Cassini line-of-sight 1-3 days **after** it hit the earth, i.e., roughly consistent with larger plasma noise on DOY 364. But I don't know if the CME was actually directed to the trailing side and in any case maybe this is a coincidence.]

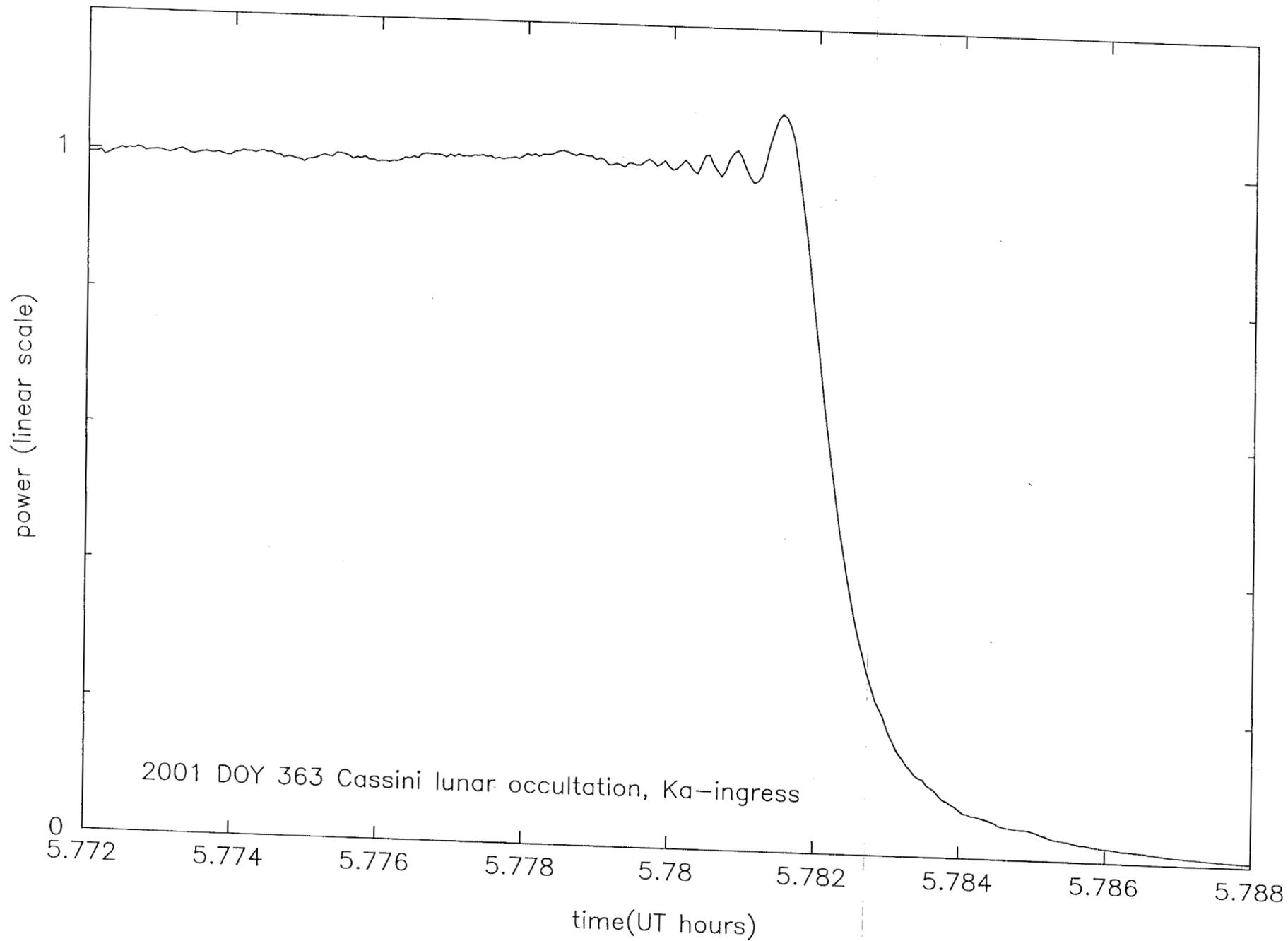
**W.r.t. lunar occultation:** For fun (and because it comes out almost for free in the quick-look analysis) I plotted the power versus time for the entry occultation. I show here the Ka-band ingress occultation. Time constants for amplitude estimation are about 1 second in these plots. The Fresnel structure is clearly evident (also evident in the X-band power and frequency/phase, not shown here because of the poorer SNR on that link.)

Two plots follow, an “overview” plot and a zoom in on the time around the actual occultation.









## Discussion

- **Ka-band up/down, as it had to, knocked the plasma scintillation noise out of the error budget**
  - **X – (880/3344)K independently estimates the downlink plasma**
  - **Two-way Ka-band plasma consistent with pre-experiment expectation**
- **2-way Ka-band (uncorrected, selected for high elevation angles) limited by nondispersive process (e.g. some combination of troposphere, antenna mechanical, FTS, KaT instability, s/c motion noise,...)**
  - **Level is consistent with independent estimate of the troposphere, therefore should be able to correct for this to about the target sensitivity level**
  - **Potential problem: significant fraction of AMC data flagged—liquid water in AMC beam may degrade AMC correction of the data**

## Discussion (continued)

- **KaT**
  - After very difficult experience through most of 2001 the KaT worked for GWE1 (see Randy's presentation for operational issues)
  - When in the locked state, the stability seems fine (at least as good as  $5E-15$  at 1000 seconds and probably  $\ll 5E-15$ )
- **AMCs discussed earlier**
  - Still learning best way to use these data
  - Weather (e.g. liquid water in the beam) degrades calibration some of the time
- **RSRs worked well (useful real-time diagnostics, in addition to primary function—see Randy's presentation)**

## Discussion (concluded)

- **Antenna mechanical**
  - Have clear examples of "events" — Bad news: affect ~1% of the data; Good news: "wrong" correlation structure and thus won't affect GW search [will look for lower-level *random* process after systematics taken out]
- **Systematics (largely elevation angle dependent)**
  - General antenna sag (dominated by M1 mirror sag?) will have to be modeled to correct low-elevation data to sub- $10^{-14}$  level
  - Dry troposphere more subtle than is generally acknowledged (at better than  $10^{-14}$  level)
- **Removal of the average orbit and other systematics w/o introducing artifacts in the Fourier band of interest may be tricky**

## **The Story So Far...**

- **Quick-look noise statistics are consistent with pre-experiment noise budget**
- **Data quality as we expected**

## Can We Do Better than Cassini?

### Problems Are

- tropospheric scintillation
- plasma scintillation
- antenna mechanical noise
- frequency standard noise
- spacecraft position noise

### Possible Fixes

- better calibrations and/or Estabrook/Hellings idea
- higher radio frequency and/or Cassini-style multi-frequency links
- look in nulls of transfer function (?)
- 30X better clocks are "straightforward"
- very careful design (?)

Conclusion: *Maybe* 10-fold improvement—to  $\sim 3 \times 10^{-18}$  for periodic sources at selected Fourier frequencies—is possible using spacecraft Doppler tracking from an Earth-based station. However the cost to achieve this would be very high.

## Concluding Ideas

- Doppler tracking of Cassini can be used as a broadband gravity wave detector
  - Apparatus is large compared with GW wavelength; thus detector properly described in terms of three pulses GW response
  - Low-frequency band edge is  $\approx 1/(\text{two-way-light-time})$  set by pulse-overlap
  - High-frequency band edge is  $\sim 10^{-1}$  to  $10^{-2}$  Hz, set by combination of downlink SNR, FTS, ability to calibrate troposphere
  - Not an interferometer: coherence maintained by excellent frequency standard on the ground
- Main noise sources
  - FTS stability
  - Plasma scintillation (dominates S-band; secondary at Ka-band)
  - Tropospheric scintillation (nondispersive)
  - Antenna mechanical stability

## Concluding Ideas (continued)

- Signals and noises enter with different transfer functions—a *very* useful discriminator
- Cassini experiment will be  $\approx 10$ -fold more sensitive than previous observations
  - Ka-band lowers plasma noise at opposition to below troposphere noise
  - Sophisticated tropospheric scintillation calibration
  - Sensitivity:
    - $\approx 3 \times 10^{-15}$  for bursts (i.e.,  $\sigma_y(\tau \approx 1000 \text{ sec})$ )
    - $\Omega \lesssim 10^{-2}$  for backgrounds ( $f_c \approx 10^{-4} \text{ Hz}$ )
    - $\approx 3 \times 10^{-17}$  for periodic waves (at selected Fourier frequencies);
    - $\approx 1.5 \times 10^{-16}$  averaged over the band