# *Time-Delay Interferometry*

**John Armstrong, Frank Estabrook, and Massimo Tinto**
**Jet Propulsion Laboratory, California Institute of Technology**

- **Reminder about main noise sources and noise spectra**

- **Time-Delay-Interferometry**
  - **View unequal arm LISA as symmetric system of one-way links**
  - **Exploit transfer function of signals and noises to construct TDI- observables which cancel the leading noises while keeping GW signals**
  - **Derive basis functions for cancellation of principal noises (X, Y, Z), ($\alpha$,$\beta$,$\gamma$), $\zeta$, etc.**

- **Applications of TDI**
  - **LISA sensitivities to periodic waves: conventional and unconventional geometries**
  - **Simulated time-series**

- **Practical Problems**

# *References*

## *For LISA configuration, orbits, and noises:*

*LISA Pre-Phase A Report* http://lisa.jpl.nasa.gov/documents.html

Folkner, W. et al. 1997 *Classical Quantum Grav.* 14, 1543

Ph237b talks by William Folkner, Robert Spero, Bonny Schumaker http://elmer.tapir.caltech.edu/ph237

## *For TDI:*

Tinto, M., and Armstrong, J. W. 1999 "Cancellation of Laser Noise in an Unequal-Arm Interferometer Detector of Gravitational Radiation", *Phys. Rev. D*, 59, 102003.

Armstrong, J. W., Estabrook, F. B., and Tinto, M. 1999 "Time-Delay Interferometry for Space-Based Gravitational Wave Searches", ApJ, 527, 814.

Estabrook, F. B., Tinto, Massimo, and Armstrong, J. W. "Time-Delay Analysis of LISA Gravitational Wave Data: Elimination of Spacecraft Motion Effects", *Phys. Rev. D.* 62, 042002.

Tinto, Massimo, Armstrong, J. W. and Estabrook, F. B. "Discriminating a Gravitational Wave Background from Instrumental Noise in the LISA Detector", *Phys Rev. D.* 63, 021101(R) (2000)

Tinto, M., Estabrook, F. B. and Armstrong, J. W. " Time-Delay Interferometry for LISA", *Phys. Rev. D*, 65, 082003 (2002).

Tinto, M. "Time Delay Interferometry", CaJAGWR lecture (April 2002) http://www.cco.caltech.edu/~cajagwr/scripts/seminars.html

Dhurandhar, S., Nayak, K., and Vinet, J-Y, gr-cq/0112059, 22 December 2001

## *LISA and Its Main Noise Sources*

- **GW detector in the LF (~millihertz) band with expected sensitivity to periodic waves ~$10^{-23}$ in a one year integration**
- **3 spacecraft, six lasers, six optical benches, 3 USOs; nominally equilateral triangle (time-variable arm lengths with $\delta L/L \sim 2\%$ and $L \approx 5 \times 10^9$ meters),**
- **Laser signals exchanged between spacecraft pairs**
- **Nominal noise spectra (one-sided)**
  - **Raw laser phase noise: 30 Hz $Hz^{-1/2}$ → $S_y(f) = 10^{-26}\ Hz^{-1}$**
  - **Optical path noise: $20 \times 10^{-12}$ m $Hz^{-1/2}$ → $S_y(f) = 1.8 \times 10^{-37}\ (f/1\ Hz)^2\ Hz^{-1}$**
  - **Optical bench noise: $10 \times 10^{-9}$ m $Hz^{-1/2}$ → $S_y(f) = 4.4 \times 10^{-32}\ (f/1\ Hz)^2\ Hz^{-1}$**
  - **Proof-mass noise: $3 \times 10^{-15}$ m/sec$^2$ $Hz^{-1/2}$ → $S_y(f) = 2.5 \times 10^{-48}\ (f/1\ Hz)^{-2}\ Hz^{-1}$**

## *Why Consider Alternatives to Conventional Michelson Interferometry?*

- **In equal-arm interferometer, laser signal experiences same delay in each arm—when signals from the two arms are combined at the detector any instability in the laser light is common and cancels exactly**

- **Because of orbital dynamics, however, LISA will necessarily have unequal and time-variable arms lengths**

- **In unequal arm length cases, signals from each arm return at different times. Straightforward differencing will then not cancel laser noise exactly. If $\varepsilon$ is the fractional arm-length imbalance and C(t) is the laser frequency random process:**

$$C(t - 2\,L_1) - C(t - 2\,L_2) \approx 2\,\varepsilon\,L_1\,dC/dt$$

- **Magnitude of the effect is huge: for $\varepsilon\,L_1 = 2\%$ arm length difference and nominal stability of lasers, imprecise phase noise cancellation introduces residual noise orders of magnitude larger than the desired GW signal sensitivity**

## *Unequal Arm Michelson Interferometry*

- **Consider a one-bounce, unequal arm transponding Michelson interferometer**

- **Measure phase difference between transmitted and received signals in each arm separately, then combine data in post-processing to cancel the laser noise**

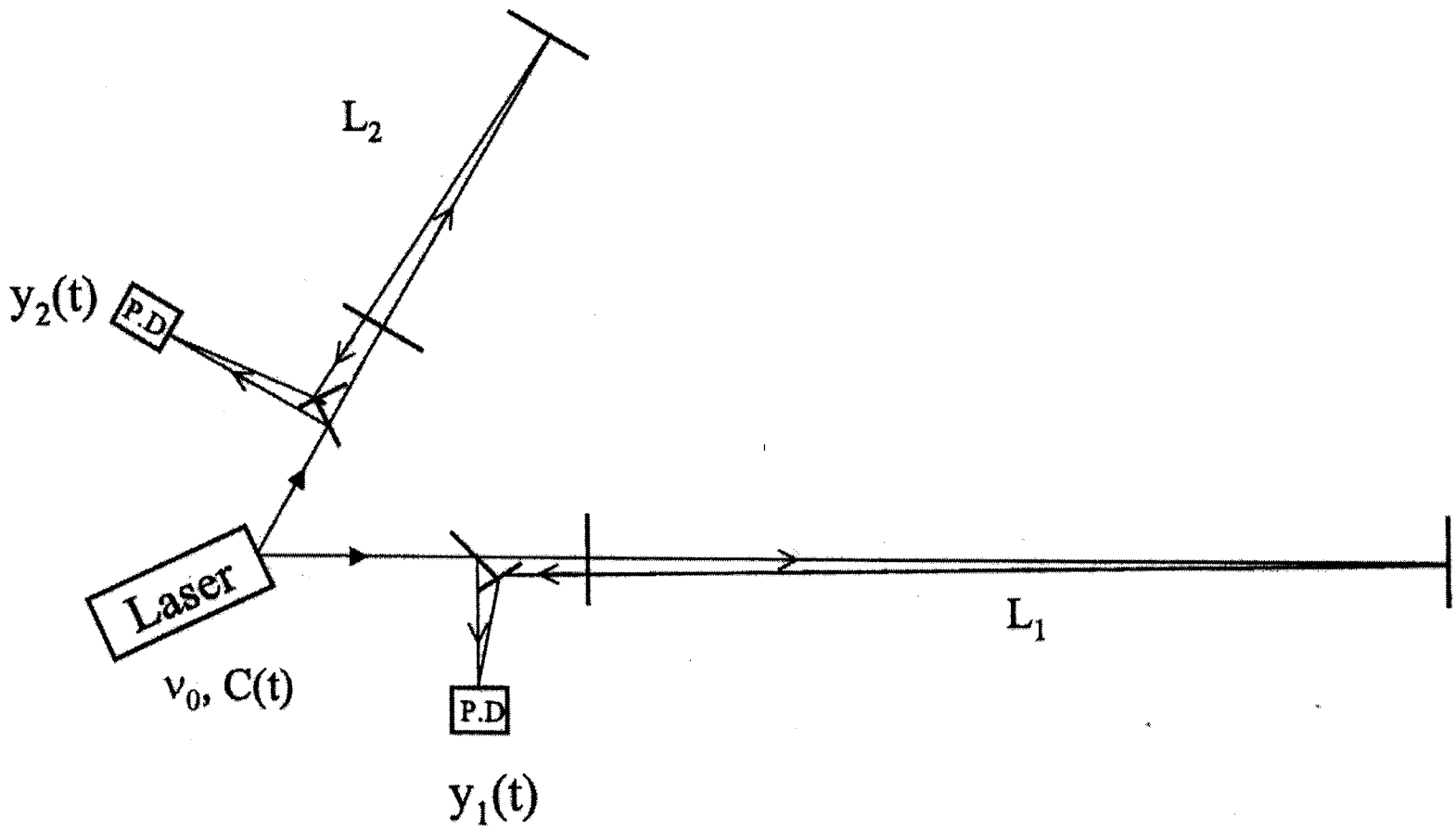
- **If $\phi_i$ is the phase difference in the arm i (i= 1,2) then the data combination**

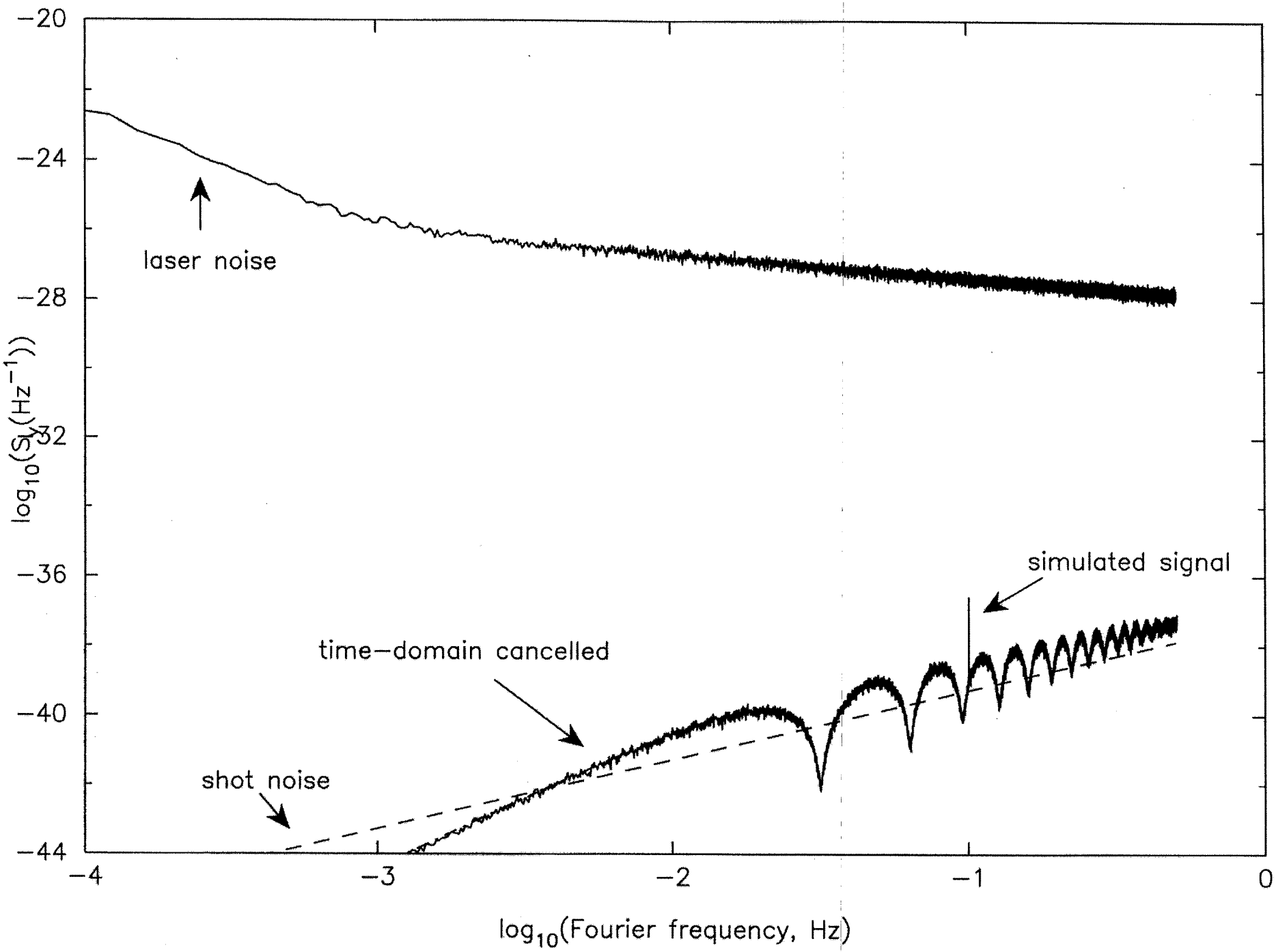
$$[\ \phi_1(t - 2L_2/c) - \phi_1(t)\ ] - [\phi_2(t - 2\ L_1/c) - \phi_2(t)\ ]$$

  **cancels the phase noise, retaining the gravity wave**

- **Tolerance on arm length knowledge can be determined by requirement that imperfection in laser noise cancellation be no worse than the uncancelled noises (e.g. shot noise). For nominal LISA parameters, this means the arms must be known to about 30 meters.**

- **Tinto &Armstrong 1999 *Phys. Rev. D*, 59, 102003 and Tinto, M. "Time Delay Interferometry", CaJAGWR lecture** http://www.cco.caltech.edu/~cajagwr/scripts/seminars.html


L2
y2(t)
P.D
Laser
ν0, C(t)
P.D
y1(t)
L1


−20
−24
−28
−32
−36
−40
−44
−4
−3
−2
−1
0
log10(Sy(Hz−1))
log10(Fourier frequency, Hz)
laser noise
shot noise
time−domain cancelled
simulated signal

## *The Main Ideas*

- LISA's can be analyzed symmetrically in terms of Doppler shifts on 6 one-way laser links connecting 6 optical benches on the three spacecraft
- Time-Delay interferometry = methods to cancel principal noises by time-shifting/adding data from single laser links, while retaining the signal
- Use linear systems ideas to build up aggregate GW signal and aggregate instrumental noise for each data combination
- Multiple noise-canceling data streams can be formed simultaneously; robustness against some classes of subsystem failure
- Applications
    - On-orbit noise calibration/isolation of confusion-limited GW background
    - Sensitivity computations
    - Simulations of what the LISA time series will look like (e.g. bursts)

## *One-Way Links: GW Response*

- **Notation and conventions:**
  - **Three LISA spacecraft, equidistant from point "O"**
  - **Unit vectors $p_i$ locate the spacecraft in the plane; $\mu_i = k \cdot p_i$**
  - **Unit vectors $n_i$ connect spacecraft pairs with the indicated orientation**
  - **$y_{21} = \Delta\nu/\nu_o$ is fractional Doppler shift on link originating at s/c 3, measured at s/c 1; $y_{31}$ is fractional Doppler shift on link originating at s/c 2, measured at s/c 1**
  - **Cyclic permutation of the indices (1 → 2 → 3 → 1) for the other $y_{ij}$**

- **GW response is "two-pulse" (Wahlquist, GRG, 19, 1101, 1987), e.g.:**

$$y_{21}(t) = \left[1 - \frac{\ell}{L_2}(\mu_3 - \mu_1)\right]\left[\Psi_2(t - L_3 - \mu_3\ell) - \Psi_2(t - \mu_1\ell)\right]$$

$$\Psi_i(t) = \frac{1}{2}\frac{n_i \cdot h(t) \cdot n_i}{1-(k \cdot n_i)^2}$$

## *One-Way Links: Noise Response*

- **Doppler link noises, in simplest case:**

  **laser phase noise:** $y_{31} = C^*_2(t - L_3) - C_1(t) = C^*_{2,3} - C_1$

  **shot noise:** $y_{31} = shot_{31}(t)$

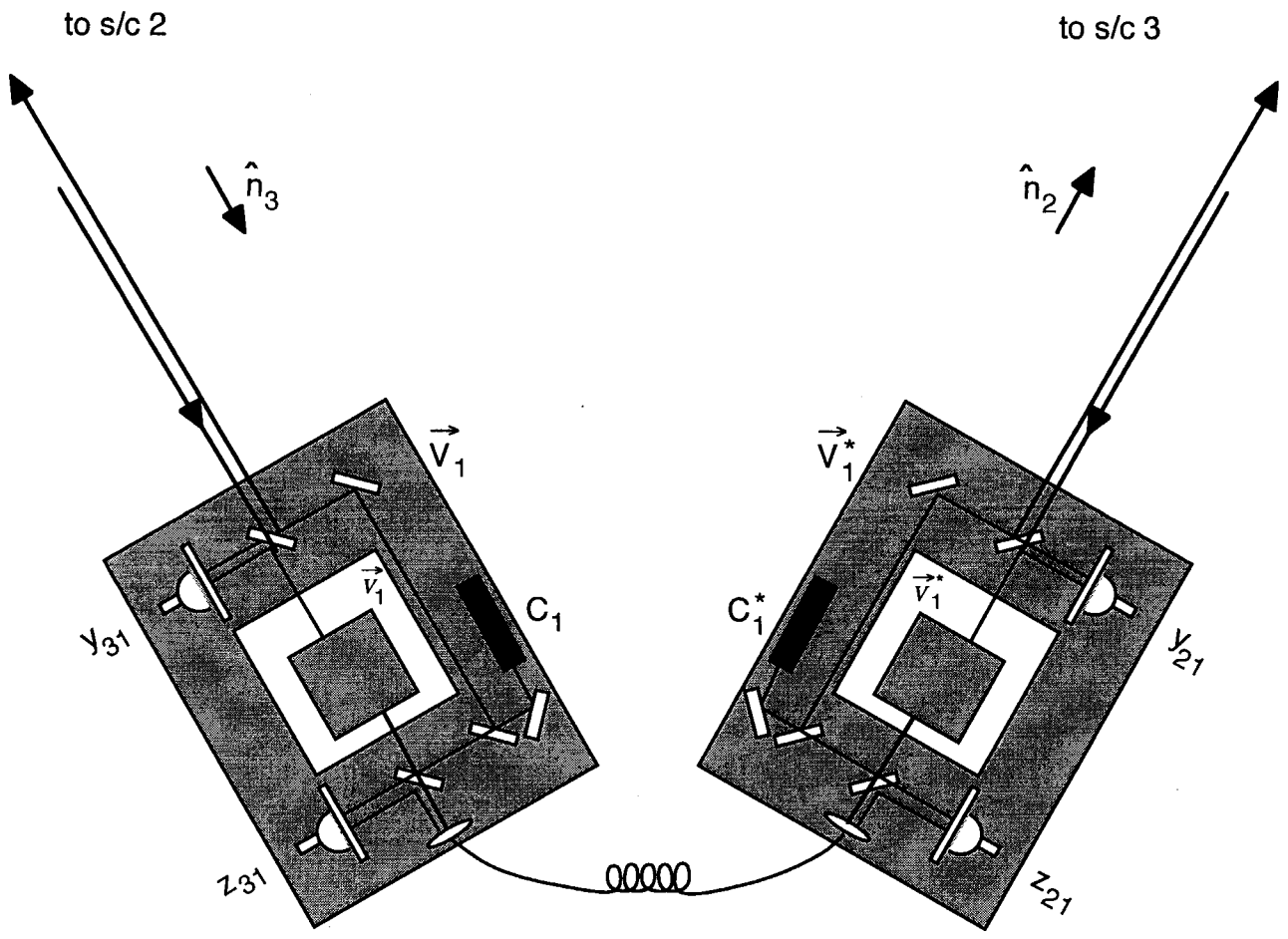
  **optical bench and proof mass noise:**

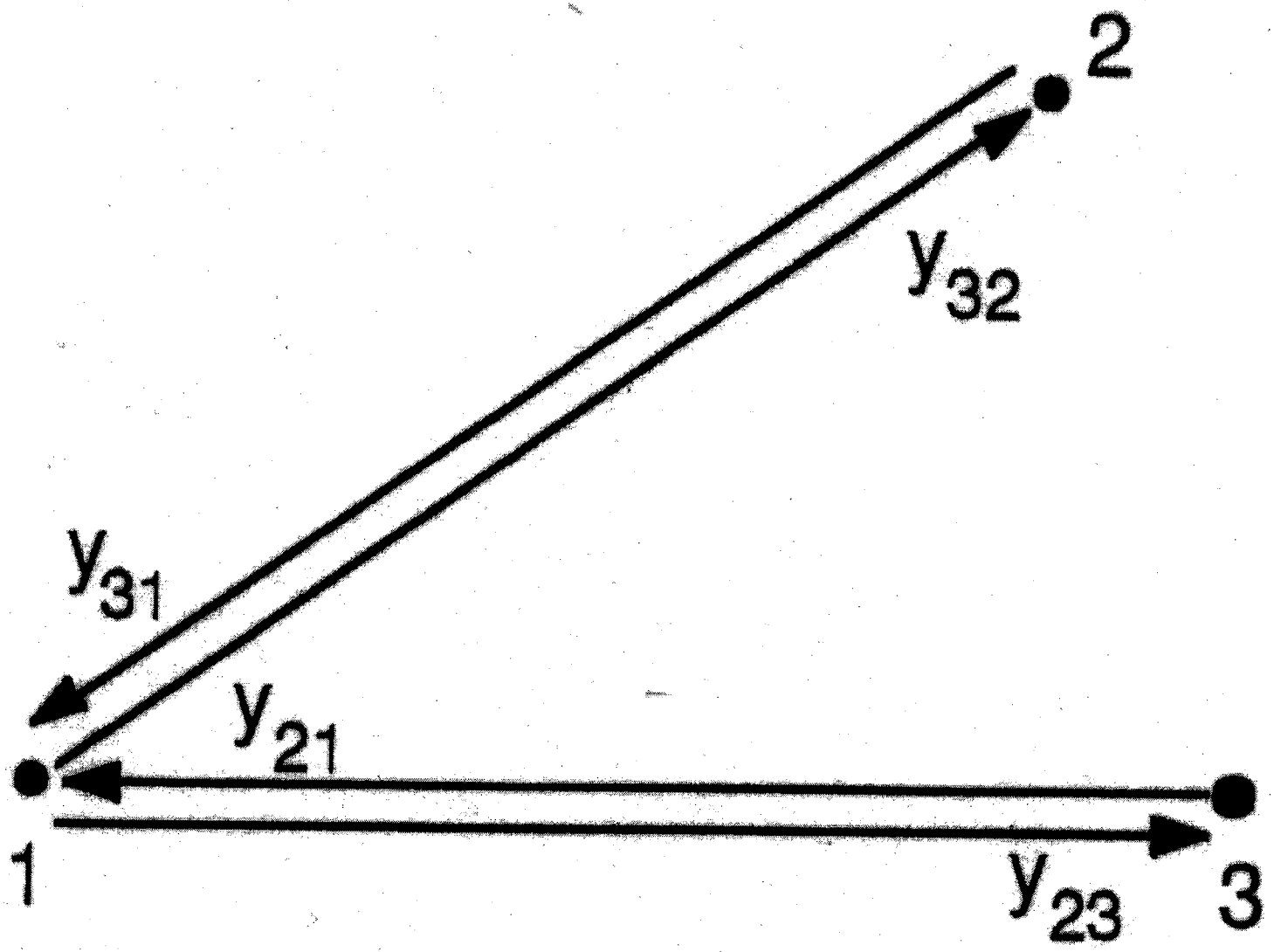
$$y_{31} = n_3 \bullet V^*_2(t - L_3) - 2\, n_3 \bullet v_1(t) + n_3 \bullet V_1(t)$$

  **metrology data:** $z_{31} = C^*_1(t) - 2\, n_2 \bullet (v^*_1 - V^*_1) + \eta - C_1(t)$

- **Introduced comma-notation, e.g.** $C_2(t - L_3) = C_{2,3}$

- **$C_i$'s, $V_i$'s, $v_i$'s, $shot_i$, etc. are random variables; ensemble average of squared Fourier transforms of time-domain transfer functions gives spectral modulation**


to s/c 2
to s/c 3
$\hat{n}_3$
$\hat{n}_2$
$\vec{v}_1$
$\vec{v}_1^*$
$\vec{v}_1$
$\vec{v}_1^*$
$C_1$
$C_1^*$
$y_{31}$
$y_{21}$
$z_{31}$
$z_{21}$


2
$y_{32}$
$y_{31}$
$y_{21}$
1
$y_{23}$
3

## UNEQUAL-ARM LENGTH INTERFEROMETRIC COMBINATIONS (X,Y,Z)

The unequal-armlength interferometric combinations are (X,Y,Z):

$$
\begin{aligned}
X \;=\; & y_{32,322} - y_{23,233} + y_{31,22} - y_{21,33} + y_{23,2} - y_{32,3} + y_{21} - y_{31} \\
& + \frac{1}{2}(-z_{21,2233} + z_{21,33} + z_{21,22} - z_{21}) \\
& + \frac{1}{2}(+z_{31,2233} - z_{31,33} - z_{31,22} + z_{31}) \; . \qquad (1)
\end{aligned}
$$

Combinations Y and Z are given by cyclic permutation of the indices in X. The gravitational wave signal maps to X as a superposition of eight realizations

$$
\begin{aligned}
X^{gw} \;=\; & \left[1 - \frac{l}{L_3}(\mu_1 - \mu_2)\right] (\Psi_3(t - \mu_1 l - 2L_3 - 2L_2) - \Psi_3(t - \mu_2 l - L_3 - 2L_2)) \\
& - \left[1 + \frac{l}{L_2}(\mu_3 - \mu_1)\right] (\Psi_2(t - \mu_1 l - 2L_2 - 2L_3) - \Psi_2(t - \mu_3 l - L_2 - 2L_3)) \\
& + \left[1 + \frac{l}{L_3}(\mu_1 - \mu_2)\right] (\Psi_3(t - \mu_2 l - L_3 - 2L_2) - \Psi_3(t - \mu_1 l - 2L_2)) \\
& - \left[1 - \frac{l}{L_2}(\mu_3 - \mu_1)\right] (\Psi_2(t - \mu_3 l - L_2 - 2L_3) - \Psi_2(t - \mu_1 l - 2L_3)) \\
& + \left[1 + \frac{l}{L_2}(\mu_3 - \mu_1)\right] (\Psi_2(t - \mu_1 l - 2L_2) - \Psi_2(t - \mu_3 l - L_2)) \\
& - \left[1 - \frac{l}{L_3}(\mu_1 - \mu_2)\right] (\Psi_3(t - \mu_1 l - 2L_3) - \Psi_3(t - \mu_2 l - L_3)) \\
& + \left[1 - \frac{l}{L_2}(\mu_3 - \mu_1)\right] (\Psi_2(t - \mu_3 l - L_2) - \Psi_2(t - \mu_1 l)) \\
& - \left[1 + \frac{l}{L_3}(\mu_1 - \mu_2)\right] (\Psi_3(t - \mu_2 l - L_3) - \Psi_3(t - \mu_1 l)) \; , \qquad (2)
\end{aligned}
$$

A $\delta$-function GW signal would produce eight pulses in X, at times depending on the arrival direction of the wave and the detector configuration: $\mu_1 l$, $\mu_2 l + L_3$, $\mu_3 l + L_2$, $\mu_1 l + 2L_3$, $\mu_1 l + 2L_2$, $\mu_3 l + L_2 + 2L_3$, $\mu_2 l + 2L_2 + L_3$, and $\mu_1 l + 2L_2 + 2L_3$.

**NOISE IN (X,Y,Z)**

The noise in X due to proof-mass motions, $\vec{v}_i$ and $\vec{v}_i^*$, is

$$\begin{aligned} X^{proof\ mass} &= \hat{n}_2 \cdot (-\vec{v}^*_{1,2233} + \vec{v}^*_{1,22} - \vec{v}^*_{1,33} + \vec{v}^*_1 + 2\vec{v}_{3,233} - 2\vec{v}_{3,2}) \\ &\quad + \hat{n}_3 \cdot (-\vec{v}_{1,2233} + \vec{v}_{1,33} - \vec{v}_{1,22} + \vec{v}_1 + 2\vec{v}^*_{2,223} - 2\vec{v}^*_{2,3}) \ , \end{aligned} \tag{3}$$

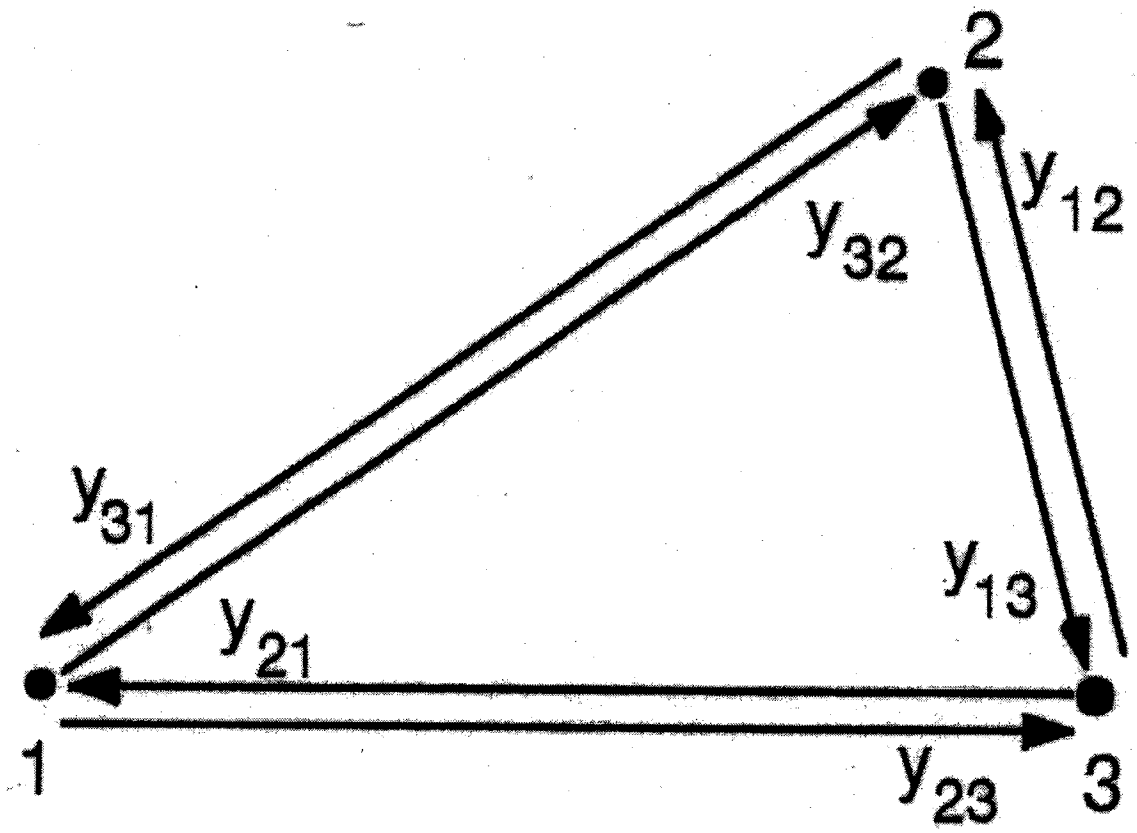
while the optical path noises (shot and beam pointing noises) affecting the measurements $y_{ij}$, enter into the interferometric combination X as follows

$$X^{optical\ path} = n_{32,322} - n_{23,233} + n_{31,22} - n_{21,33} + n_{23,2} - n_{32,3} + n_{21} - n_{31} \ . \tag{4}$$

Here $n_{ij}$ are the random processes associated with the optical path noises affecting the Doppler measurements $y_{ij}$.

The power spectra of the acceleration and optical path noise components of X, assuming independent individual proof mass acceleration noises, (with equal raw spectra) and independent optical path noises (with equal raw spectra) and for the equilateral triangle ($L_1 = L_2 = L_3 = L$) case

$$S_X = [8sin^2(4\pi fL) + 32sin^2(2\pi fL)]S_y^{proof\ mass} + 16sin^2(2\pi fL)\ S_y^{optical\ path} \ . \tag{5}$$


2
y12
y32
y31
y13
y21
1
y23
3

## *Noise-Canceling Data Combinations*

**Assume unequal, but at this point not time-varying, armlengths and that the lasers all have the same center frequency. Forming, for example,**

$$\alpha_{gen} = y_{21} - y_{31} + y_{13,2} - y_{12,3} + y_{32,12} - y_{23,13} +$$
$$1/2\,[z_{13,2} + z_{13,13} + z_{21} + z_{21,123} + z_{32,3} + z_{32,12}] +$$
$$1/2\,[z_{23,2} + z_{23,13} + z_{31} + z_{31,123} + z_{12,3} + z_{12,12}]$$

**exactly cancels all laser phase ($C_i$ and $C^*_i$) and optical bench ($V_i$ and $V^*_i$) noise. Of course non-inertial motion of the proof masses do <u>not</u> cancel:**

$$\text{proof mass noise for } \alpha_{gen} = \quad n_1 \bullet (v_{2,3} - v_{2,12} + v^*_{3,2} - v^*_{3,13}) +$$
$$n_2 \bullet (v_{3,13} - v_{3,2} + v^*_1 - v^*_{1,123}) +$$
$$n_3 \bullet (v_1 - v_{1,123} + v^*_{2,12} - v^*_{2,3})$$

## *Noise-Canceling Data Combinations: Spectral Modulation of Noise*

- **If all proof-masses are independent and have same spectrum, and for the LISA equilateral case, ensemble average of FT-squared gives spectral modulation for $\alpha$'s proof mass noise:**

$$[\ 8 \sin^2(3\ \pi\ f\ L) + 16 \sin^2(\pi\ f\ L)\ ]\ S^{\text{proof mass}}$$

- **Similar argument gives proof mass noise for other combinations:**
  - **for Michelson interferometer (X):**

$$[\ 8 \sin^2(4\ \pi\ f\ L) + 32 \sin^2(\ 2\ \pi\ f\ L)\ ]\ S^{\text{proof mass}}$$

  - **for Sagnac combination ($\zeta$):**

$$[\ 24 \sin^2(\pi\ f\ L)\ ]\ S^{\text{proof mass}}$$

  - **and so forth for other combinations and for optical path noise...**

## FULLY SYMMETRIC (SAGNAC) COMBINATION, $\zeta$

Another combination of all six data streams which exactly cancels all laser and optical bench motion noises and has the property that each of the $y_{ij}$ enters exactly once and is lagged by exactly one of the one-way light times is $\zeta$

$$\begin{aligned}\zeta &= y_{32,2} - y_{23,3} + y_{13,3} - y_{31,1} + y_{21,1} - y_{12,2} \\ &\quad + \frac{1}{2}(-z_{13,21} + z_{23,12} - z_{21,23} + z_{31,23} - z_{32,13} + z_{12,13}) \\ &\quad + \frac{1}{2}(-z_{32,2} + z_{12,2} - z_{13,3} + z_{23,3} - z_{21,1} + z_{31,1}) \ . \end{aligned} \tag{11}$$

Like $\alpha$, $\beta$, $\gamma$, this combination also has a six-pulse response to gravitational radiation

$$\begin{aligned}\zeta^{gw} = &\left[1 - \frac{l}{L_3}(\mu_1 - \mu_2)\right] (\Psi_3(t - \mu_1 l - L_3 - L_2) - \Psi_3(t - \mu_2 l - L_2)) \\ &- \left[1 + \frac{l}{L_2}(\mu_3 - \mu_1)\right] (\Psi_2(t - \mu_1 l - L_2 - L_3) - \Psi_2(t - \mu_3 l - L_3)) \\ &+ \left[1 - \frac{l}{L_1}(\mu_2 - \mu_3)\right] (\Psi_1(t - \mu_2 l - L_1 - L_3) - \Psi_1(t - \mu_3 l - L_3)) \\ &- \left[1 + \frac{l}{L_3}(\mu_1 - \mu_2)\right] (\Psi_3(t - \mu_2 l - L_3 - L_1) - \Psi_3(t - \mu_1 l - L_1)) \\ &+ \left[1 - \frac{l}{L_2}(\mu_3 - \mu_1)\right] (\Psi_2(t - \mu_3 l - L_2 - L_1) - \Psi_2(t - \mu_1 l - L_1)) \\ &- \left[1 + \frac{l}{L_1}(\mu_2 - \mu_3)\right] (\Psi_1(t - \mu_3 l - L_1 - L_2) - \Psi_1(t - \mu_2 l - L_2)) \ . \end{aligned} \tag{12}$$

A $\delta$-function GW signal would produce six pulses in $\zeta$, located at: $\mu_1 l + L_3 + L_2$, $\mu_2 l + L_1 + L_3$, $\mu_3 l + L_1 + L_2$, $\mu_3 l + L_3$, $\mu_2 l + L_2$, and $\mu_1 l + L_1$.

**NOISE IN $\zeta$**

The proof-mass noise for $\zeta$ is

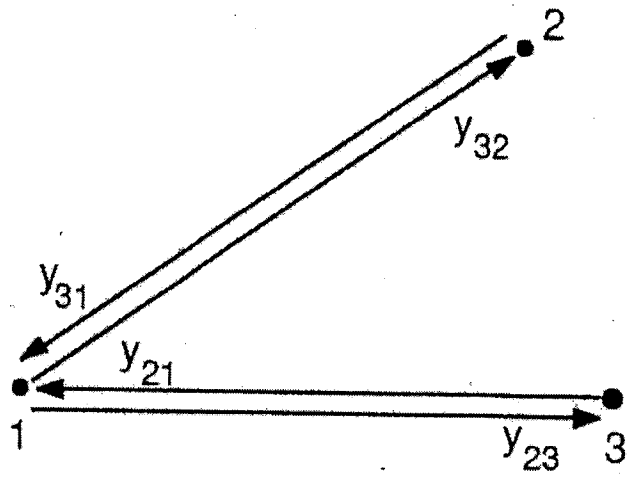
$$\begin{aligned}\zeta^{proof\ mass} &= \hat{n}_1 \cdot (\vec{v}_{2,2} - \vec{v}_{2,13} + \vec{v}^*_{3,3} - \vec{v}^*_{3,21}) \\ &+\hat{n}_2 \cdot (\vec{v}_{3,3} - \vec{v}_{3,21} + \vec{v}^*_{1,1} - \vec{v}^*_{1,23}) \\ &+\hat{n}_3 \cdot (\vec{v}_{1,1} - \vec{v}_{1,23} + \vec{v}^*_{2,2} - \vec{v}^*_{2,13}) \ , \end{aligned} \tag{13}$$

while the contribution from the optical path noise is

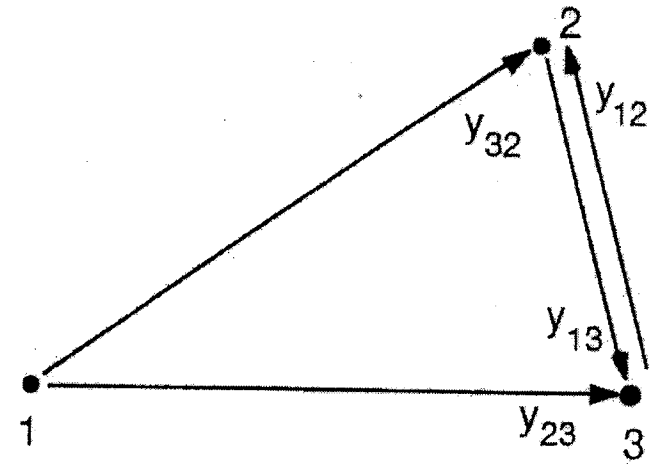
$$\zeta^{optical\ path} = n_{32,2} - n_{23,3} + n_{13,3} - n_{31,1} + n_{21,1} - n_{12,2} \ . \tag{14}$$

The power spectra of the acceleration and optical path noise components of $\zeta$, assuming equal and independent individual proof mass acceleration noises, equal and independent optical path noises, and the equilateral triangle configuration are

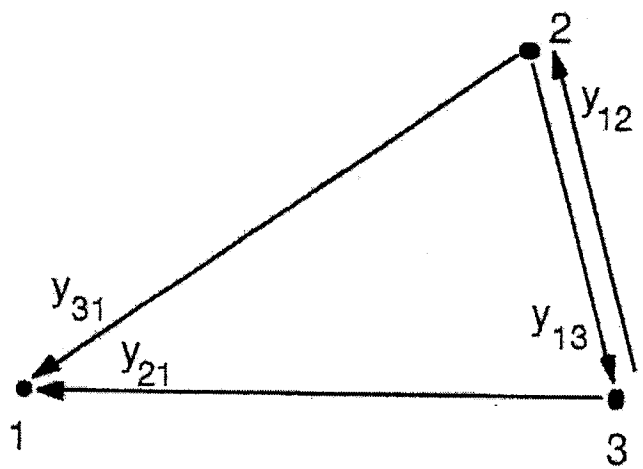
$$S_\zeta = 24 sin^2(\pi f L)\ S_y^{proof\ mass} + 6\ S_y^{optical\ path} \ . \tag{15}$$



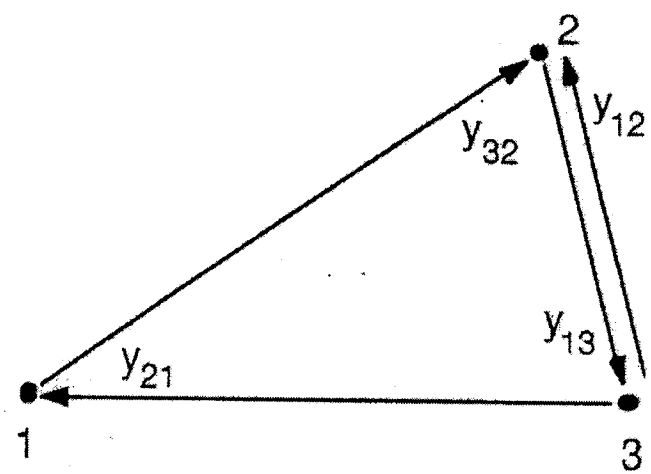

INTERFEROMETER (X,Y,Z)




BEACON (P,Q,R)




MONITOR (E,F,G)



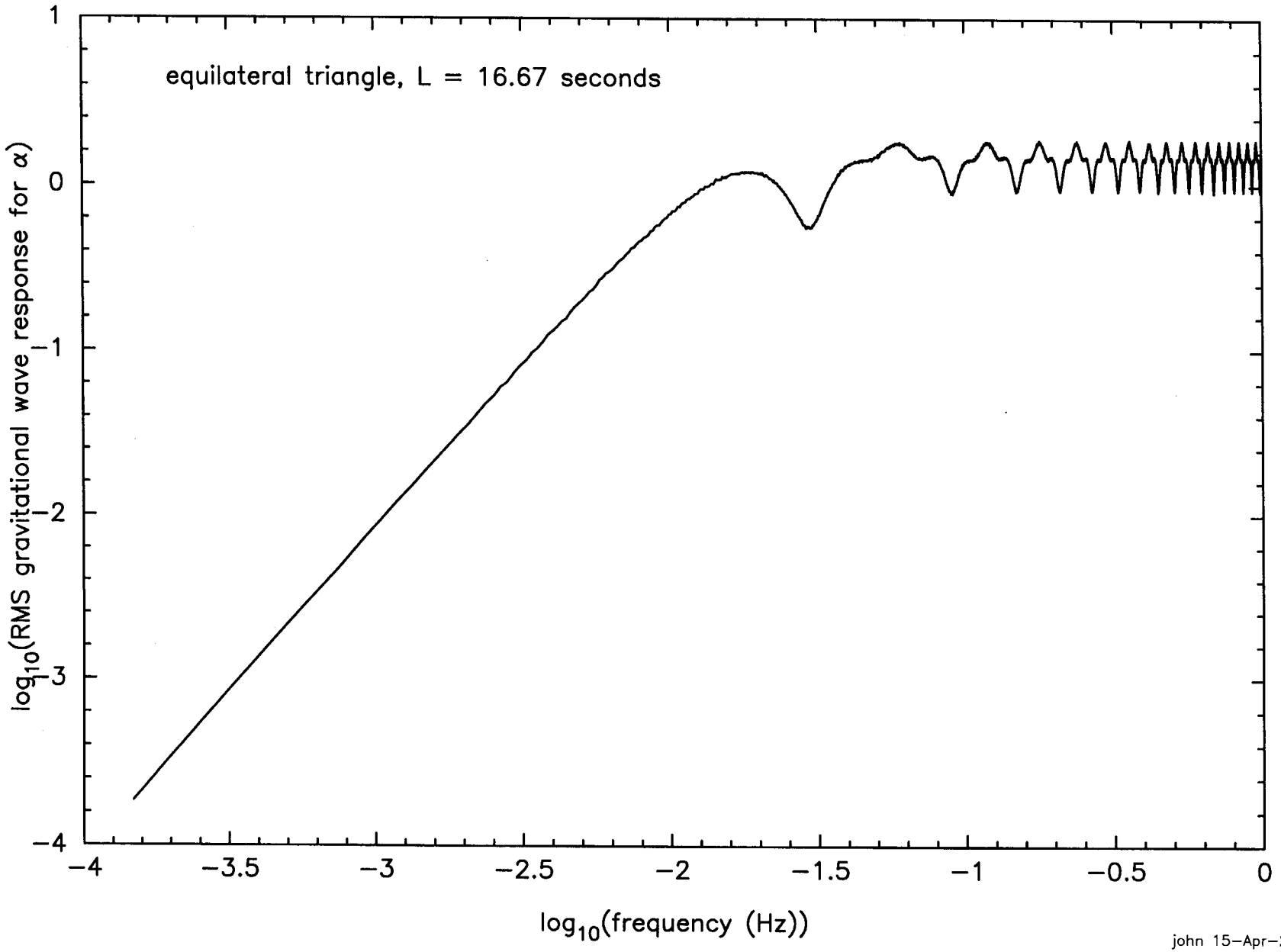

RELAY (U,V,W)

## *Application of TDI: Sensitivity Calculations*

- **Conventional figure-of-merit is RMS sensitivity required for a sinusoidal wave as a function of Fourier frequency**
  - **SNR = 5 in bandwidth B = 1/(one year)**
  - **Averaged over source position and over wave polarization state**

$$h = \frac{noise(f)}{signal(f)} = \frac{5\sqrt{S_{noise}(f)\ B}}{rms\ GW\ response(f)}$$

- **Thus need GW signal response for a given noise-canceling data combination and the noise spectra (including transfer functions for both signal and noise) for that data combination**

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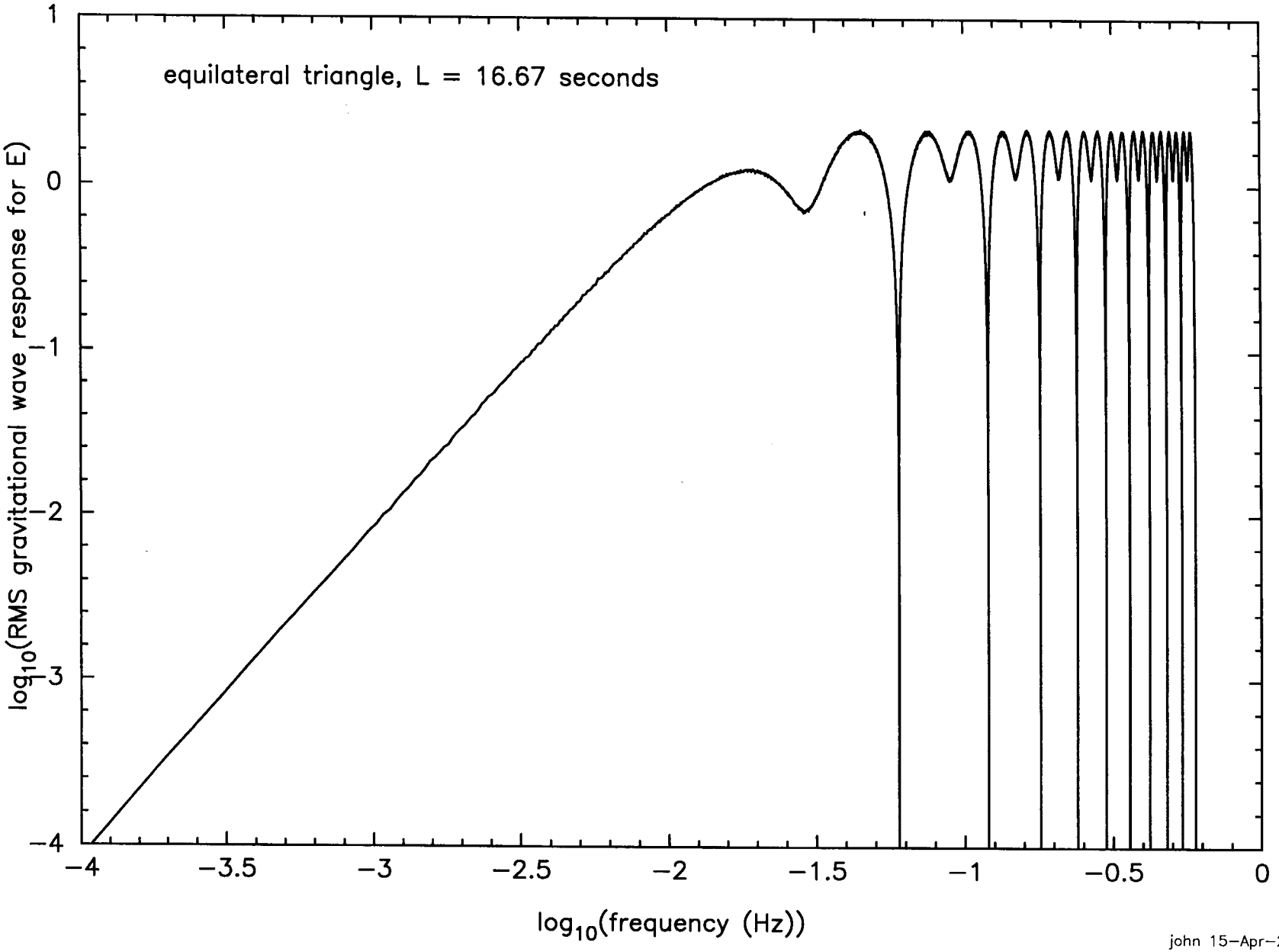


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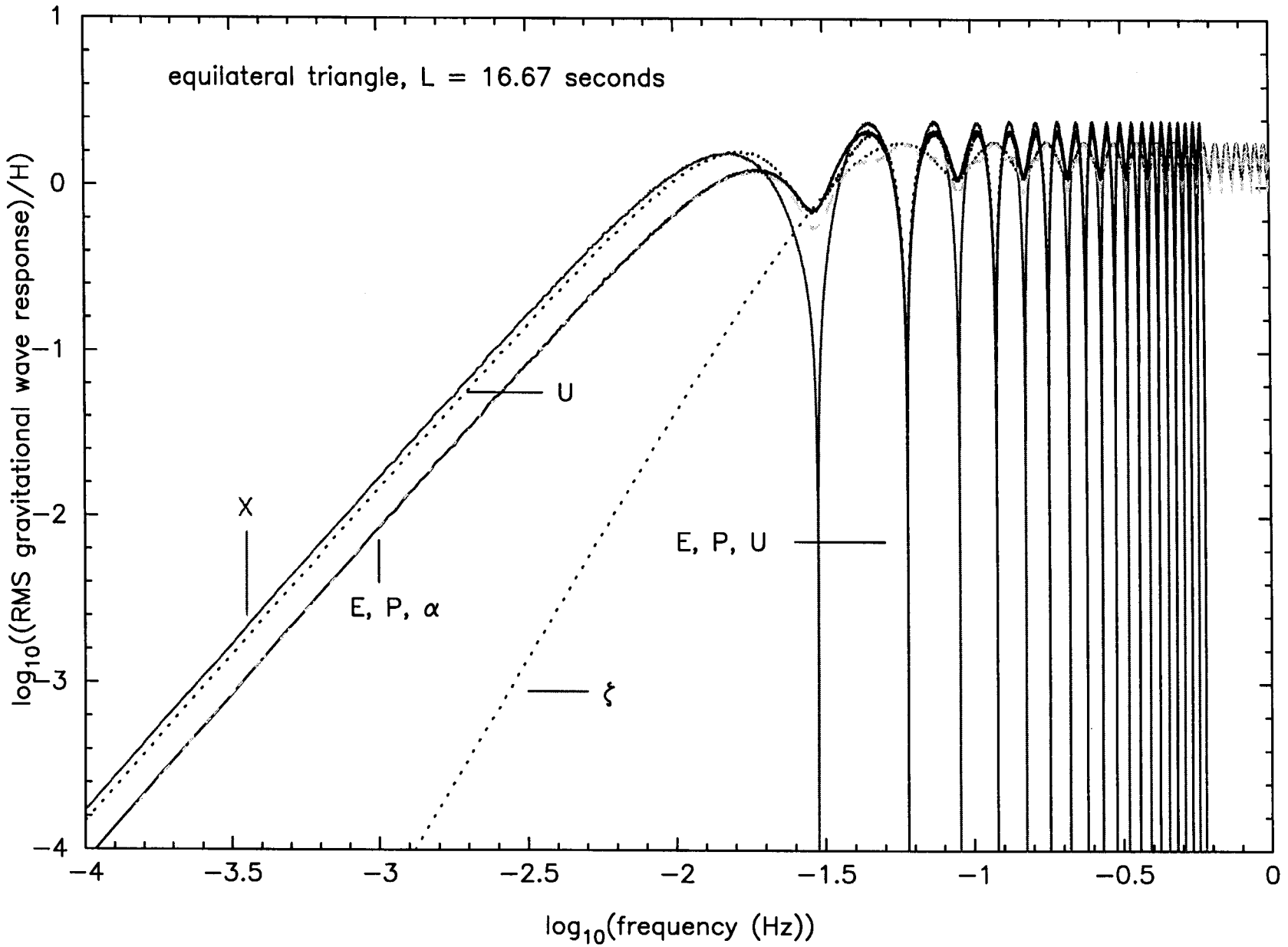

equilateral triangle, L = 16.67 seconds
U
X
E, P, α
E, P, U
ζ
log10((RMS gravitational wave response)/H)
log10(frequency (Hz))
1
0
−1
−2
−3
−4
−4
−3.5
−3
−2.5
−2
−1.5
−1
−0.5
0

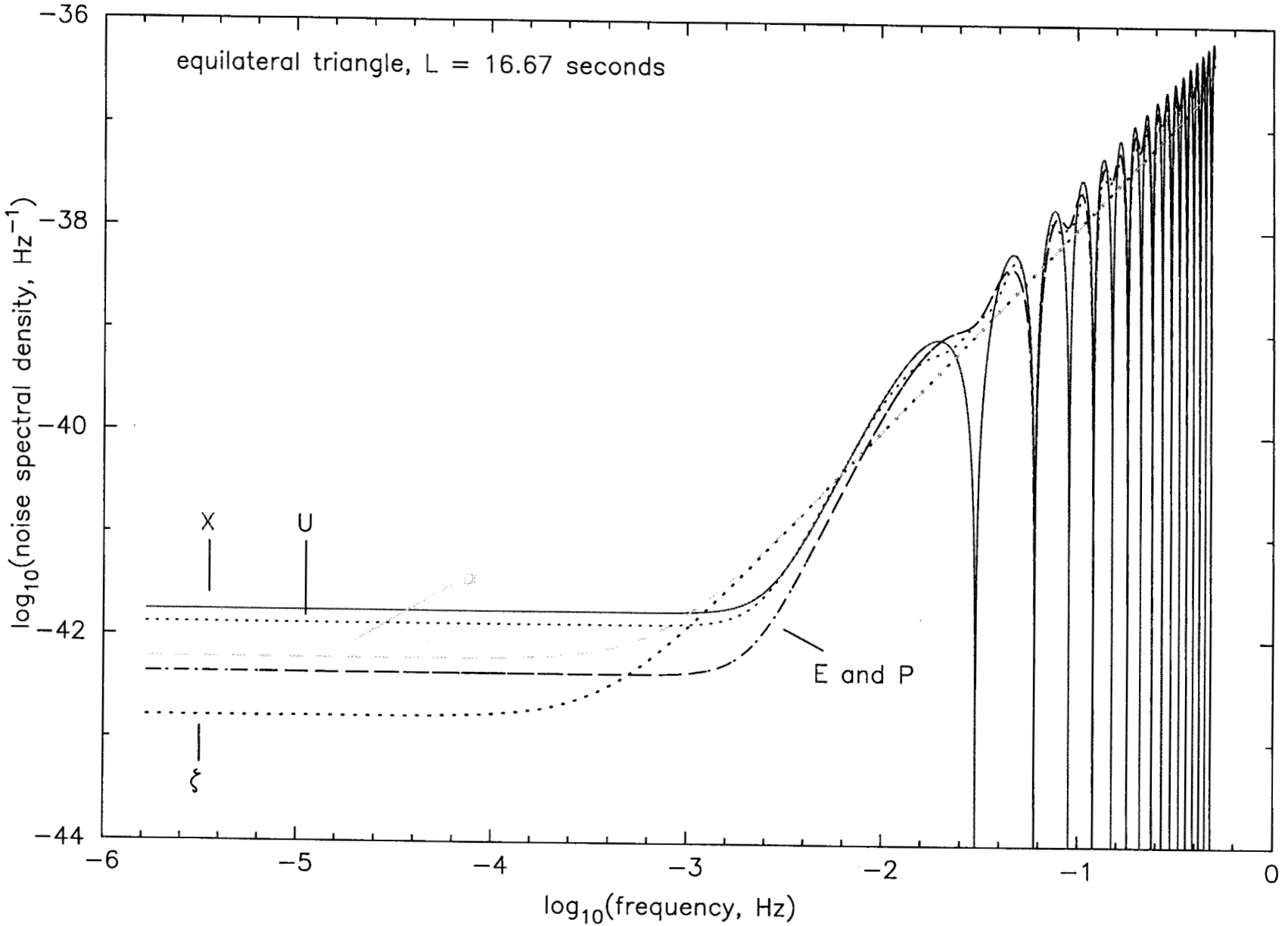
## *Procedure: Noises*

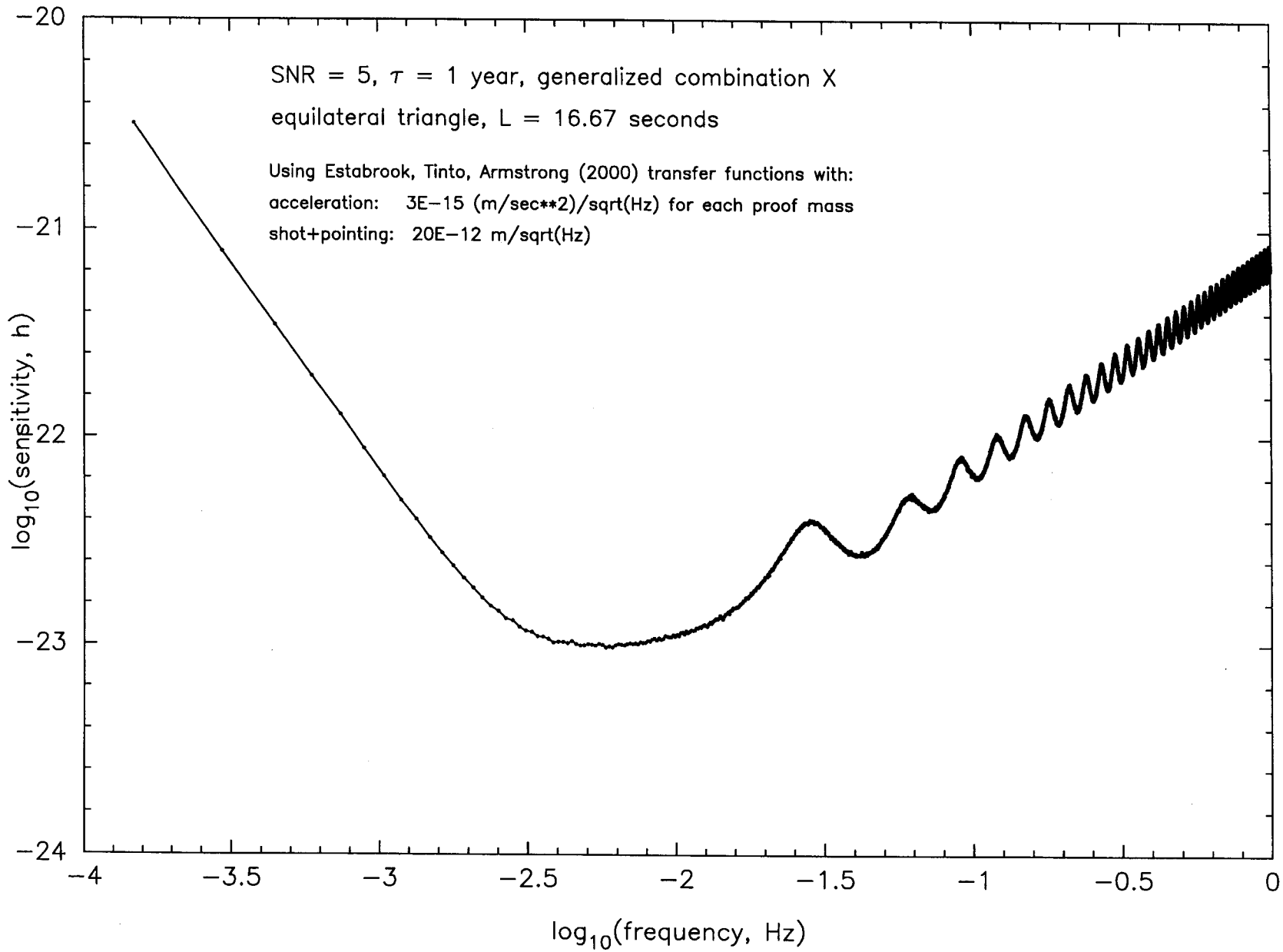
- **Spectra of individual <u>proof-mass</u> noise and single <u>optical path</u> noise (mostly shot + beam pointing) from the LISA Pre-Phase A report—these are expressed in units of length per square-root-Hz**

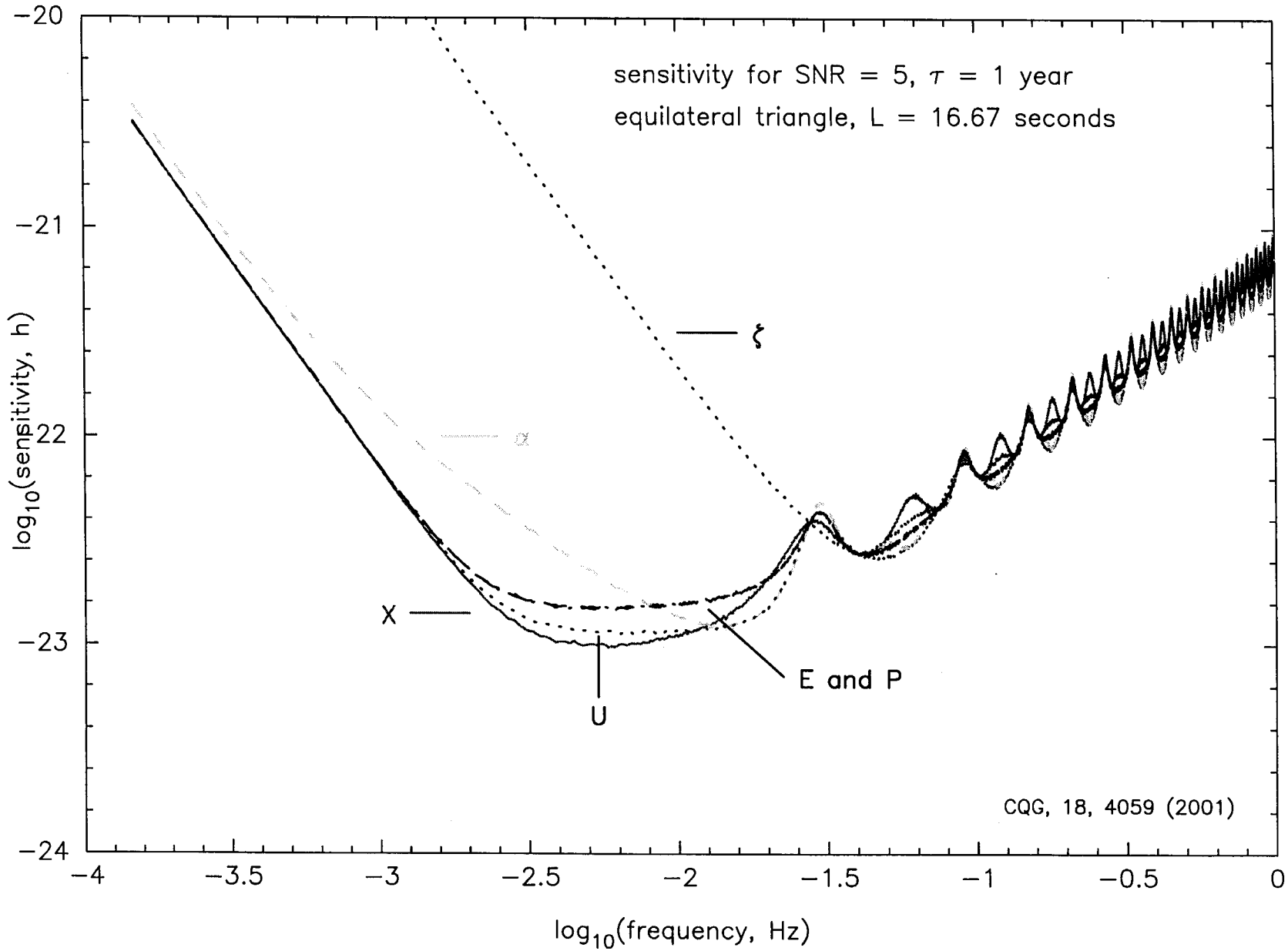
- **Square to get power spectra, convert from units of length$^2$/Hz to fractional Doppler (derivative theorem for Fourier transforms) to get equivalent velocity spectrum and divide by speed of light squared to convert to spectrum of y: $S^{\text{proof mass}}$ and $S^{\text{optial path}}$**

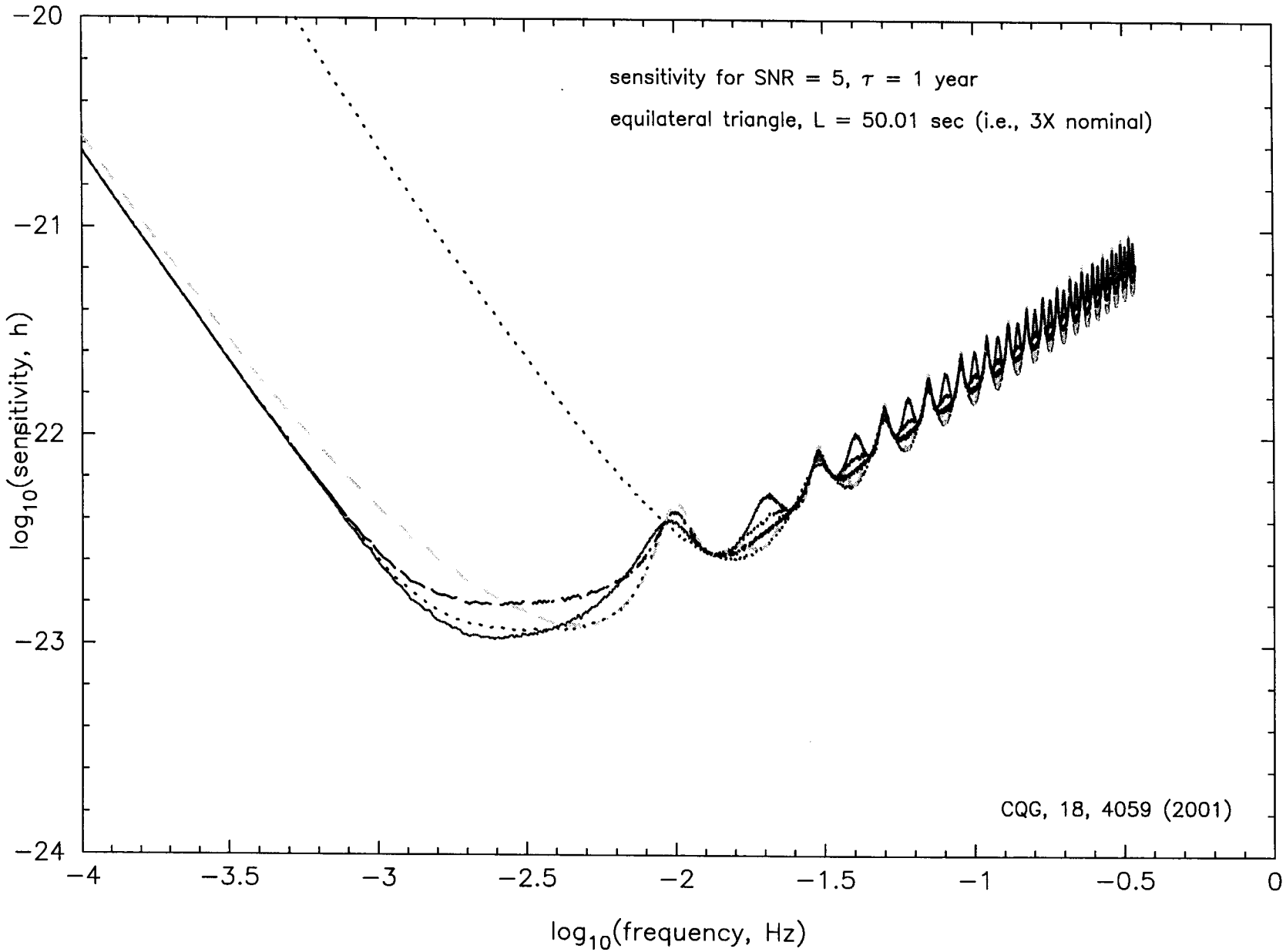
- **Write the noise-canceling data combination under consideration in terms of the defining $y_{ij}$'s. Square of Fourier-transform gives noise spectrum transfer functions multiplying $S^{\text{proof mass}}$ and $S^{\text{optial path}}$.**

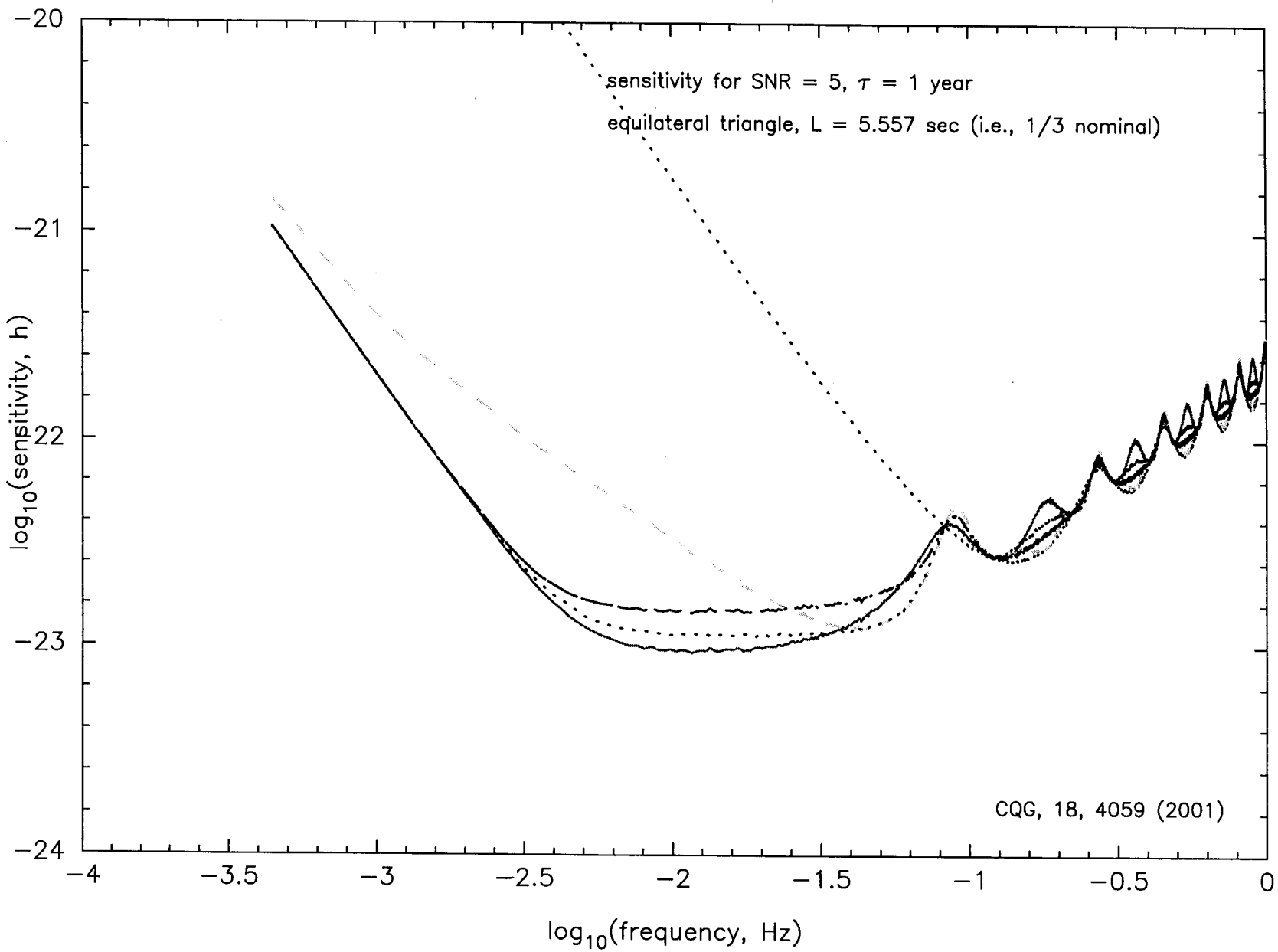
- **Examples: Aggregate noise spectrum for data combinations X, P, etc. in equilateral and non-equilateral triangle cases**

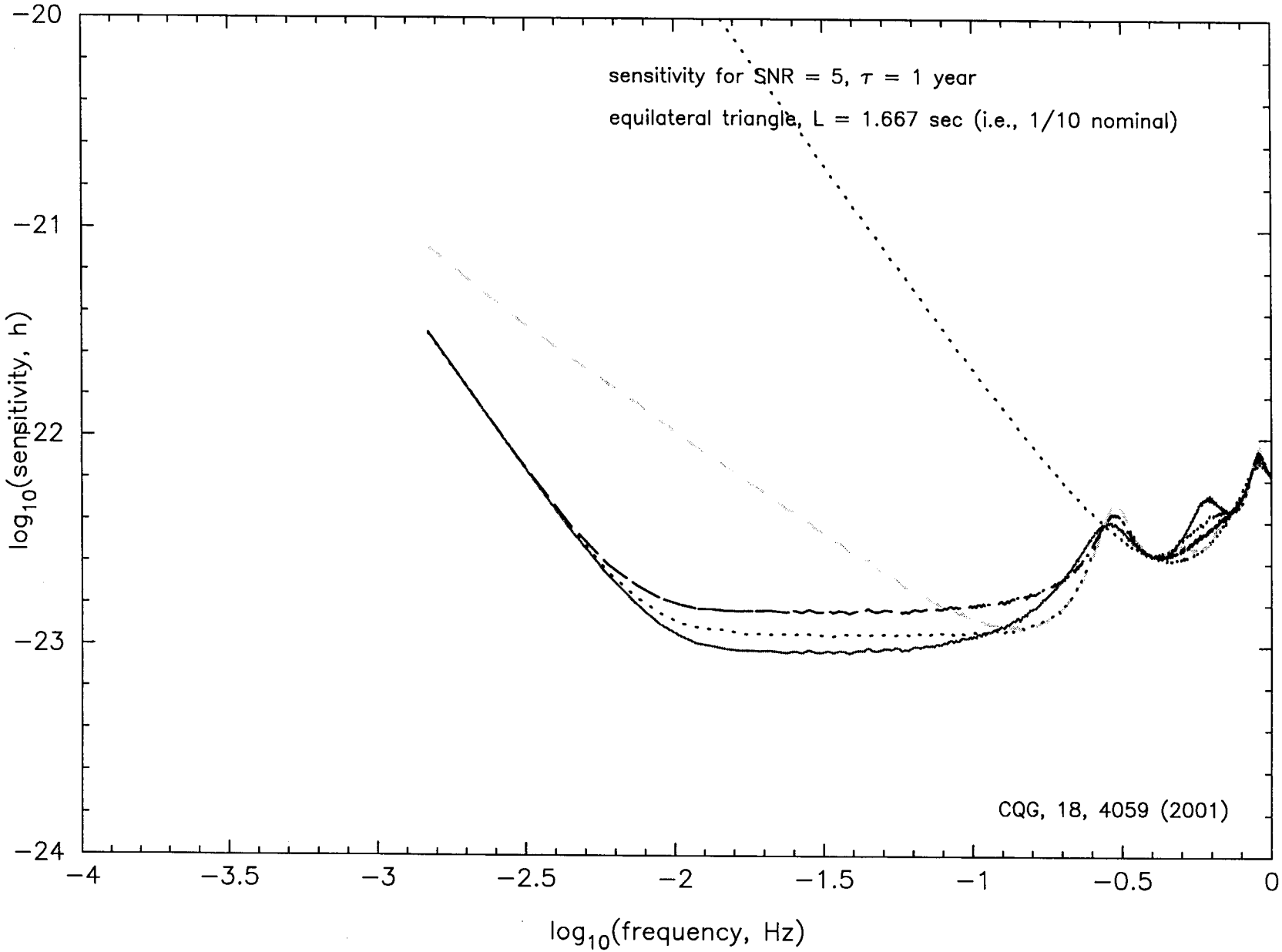

equilateral triangle, L = 16.67 seconds
X
U
E and P
ζ
−36
−38
−40
−42
−44
−6
−5
−4
−3
−2
−1
0
log10(noise spectral density, Hz−1)
log10(frequency, Hz)


SNR = 5, τ = 1 year, generalized combination X
equilateral triangle, L = 16.67 seconds
Using Estabrook, Tinto, Armstrong (2000) transfer functions with:
acceleration: 3E−15 (m/sec**2)/sqrt(Hz) for each proof mass
shot+pointing: 20E−12 m/sqrt(Hz)
−20
−21
−22
−23
−24
−4
−3.5
−3
−2.5
−2
−1.5
−1
−0.5
0
log10(sensitivity, h)
log10(frequency, Hz)


sensitivity for SNR = 5, τ = 1 year
equilateral triangle, L = 16.67 seconds
ζ
α
X
U
E and P
CQG, 18, 4059 (2001)
−20
−21
−22
−23
−24
−4
−3.5
−3
−2.5
−2
−1.5
−1
−0.5
0
log10(sensitivity, h)
log10(frequency, Hz)


sensitivity for SNR = 5, $\tau$ = 1 year
equilateral triangle, L = 50.01 sec (i.e., 3X nominal)
CQG, 18, 4059 (2001)
$\log_{10}$(sensitivity, h)
$\log_{10}$(frequency, Hz)
−20
−21
−22
−23
−24
−4
−3.5
−3
−2.5
−2
−1.5
−1
−0.5
0


sensitivity for SNR = 5, τ = 1 year
equilateral triangle, L = 5.557 sec (i.e., 1/3 nominal)
CQG, 18, 4059 (2001)
log10(sensitivity, h)
log10(frequency, Hz)
−20
−21
−22
−23
−24
−4
−3.5
−3
−2.5
−2
−1.5
−1
−0.5
0


sensitivity for SNR = 5, τ = 1 year
equilateral triangle, L = 1.667 sec (i.e., 1/10 nominal)
CQG, 18, 4059 (2001)
log10(sensitivity, h)
log10(frequency, Hz)
−20
−21
−22
−23
−24
−4
−3.5
−3
−2.5
−2
−1.5
−1
−0.5
0

## *Time-Delay Interferometry*

- **LISA's can be analyzed symmetrically in terms of Doppler shifts on 6 one-way laser links connecting 6 optical benches on the three spacecraft + metrology time series comparing the lasers and relative optical bench motions within the 3 spacecraft**

- **Time-Delay interferometry = methods to cancel principal noises by time-shifting/adding data from single laser links, while retaining the signal**

- **The functional space of noise-canceling TDI-combinations is 3-dimensional**

$$\zeta - \zeta_{,123} = \alpha_{,1} - \alpha_{,23} + \beta_{,2} - \beta_{,31} + \gamma_{,3} - \gamma_{,12}$$

$$X_{,1} = \alpha_{,32} - \beta_{,2} - \gamma_{,3} + \zeta$$

- **One TDI-combination, $\zeta$, is sensitive to proof-mass and optical path noises, but relatively insensitive to GW at low Fourier frequencies**

- **Many TDI-combinations can be formed simultaneously**

## *Summary*

- LISA can be analyzed symmetrically in terms of Doppler shifts on 6 one-way laser links connecting 6 optical benches on the three spacecraft.
- Time-delay interferometry provides many ways—not just Michelson—to combine these links with appropriate time delays, canceling principal noises even with unequal and time-varying arms.
- Provides a framework for analysis of signals, noises, sensitivity, and some design tradeoffs (e.g. Doppler shifts due to orbits, USO stability, offsets in laser center frequencies)
- Results valid across the whole LISA band, not just in the LWL
- Multiple noise-canceling data streams can be formed simultaneously
    - Differing sensitivity to GWs can be exploited—e.g. TDI-combination $\zeta$ for assessing on-orbit instrumental noise and isolating it from GW background
    - Robustness against some classes of subsystem failure—e.g. the 4-link combinations (X, Y, Z), (E, F, G), (P, Q, R), (U, V, W)
    - Design for specific waveforms
    - Discriminate signals and noises based on differing transfer functions