A fast implementation of the backpropagation and canonical transform methods for GPS occultations

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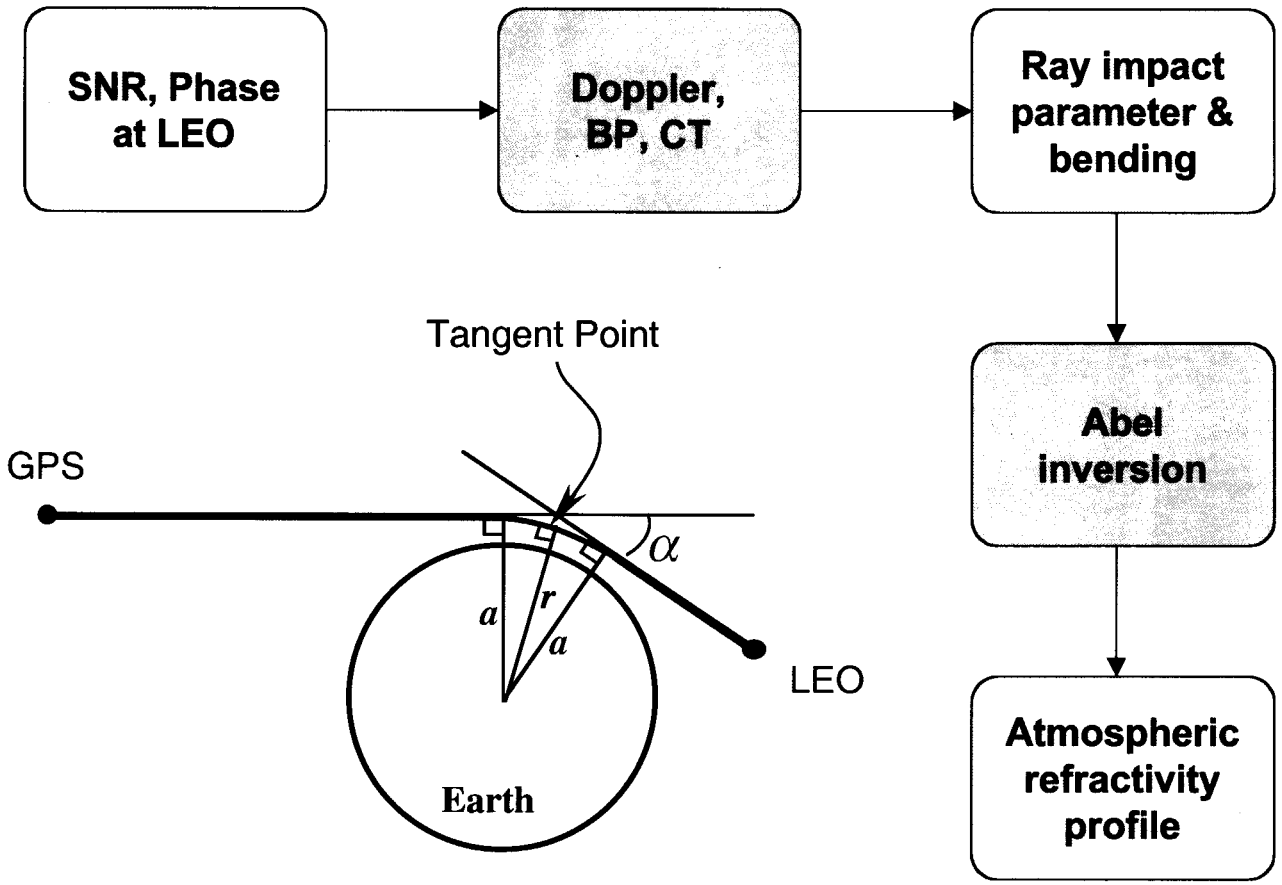
Overview

1. **Retrieval methodology**
   - Backpropagation & canonical transform
   - Need for a fast implementation
2. **Spectral formulation of backpropagation**
   - FFT implementation
   - Making it work for real data
3. **Results**
   - Validation with simulations
   - CHAMP & SAC-C
4. **Summary**

Retrieval methodology

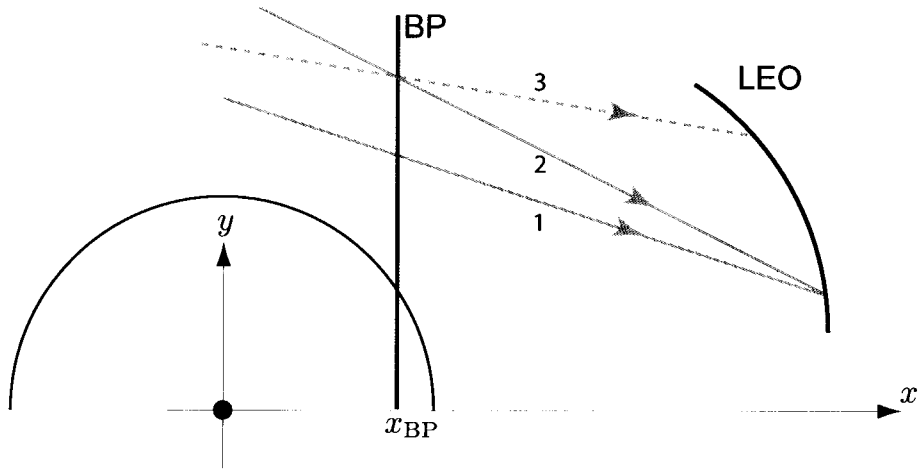


Backpropagation

- **Amplitude and phase are continued backward to a plane closer to ray tangent point using diffraction theory [*Gorbunov et al. 1996; Karayel & Hinson, 1997*].**

$$u_{\mathrm{BP}}(\mathbf{r}) = \frac{i}{2}\int_{\mathrm{LEO}} d\mathbf{r}' \; (\hat{n}' \cdot \nabla') H_0^{(1)}(-k|\mathbf{r}-\mathbf{r}'|) \; u(\mathbf{r}')$$

$$H_0^{(1)}(-k|\mathbf{r}-\mathbf{r}'|) \approx \sqrt{\frac{2i}{\pi k}} \frac{\exp(-ik|\mathbf{r}'-\mathbf{r}|)}{|\mathbf{r}'-\mathbf{r}|^{1/2}}$$

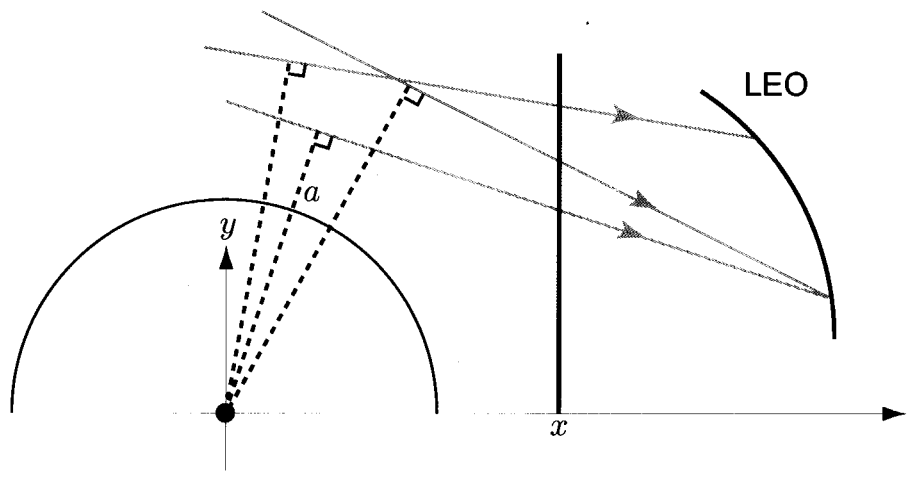


Canonical transform

- "Backpropagate" each ray to its impact parameter, ensuring that no multipath exists (spherically symmetric atmosphere) [*Gorbunov, 2001*].

$$v(a) = \int \frac{dk_y}{2\pi} \sqrt{\frac{k_x}{k}} \exp\left[ika \sin^{-1}\frac{k_y}{k} - ik_x x\right] U_x(k_y)$$

$$U_x(k_y) = \int dy\; e^{-ik_y y} u_x(y), \qquad k_x = \sqrt{k^2 - k_y^2}$$



Why is a fast method needed?

- **BP and CT processing are more computationally intensive than standard Doppler method.**
- **For one occultation (BP):**
  - $P = M \times N \approx 5000 \times 40000 = 2 \times 10^8$ operations.
  - Integration over subset: $M \approx 500$ (or less).
  - CPU time (1.4 GHz Pentium 4) = 5 min.
- **Processing 2000 occultations take 1 week.**
  - Some applications (e.g. weather prediction) require retrievals to be available within hours.
  - Further enhancements of BP and CT will likely increase computational time.

Spectral formulation of BP

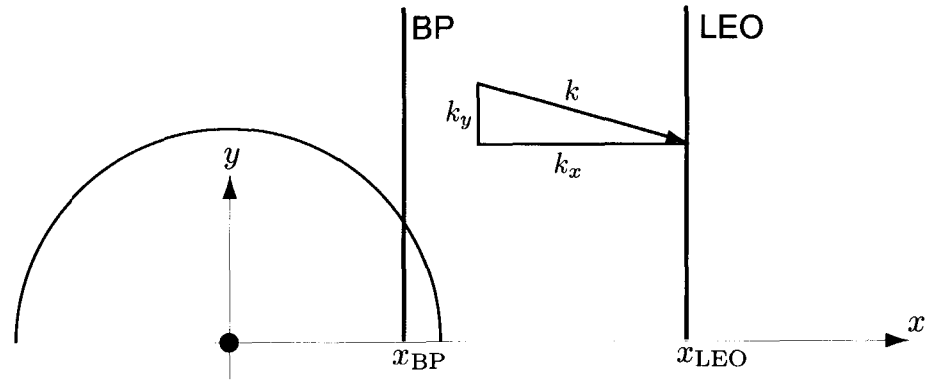
- **Consider the special case where the LEO is parallel to BP.**

$$H_0^{(1)}(-k|\mathbf{r}-\mathbf{r}'|) = \frac{1}{\pi}\int \frac{dk_y}{k_x} e^{-ik_y(y-y')} e^{-ik_x|x-x'|}$$

$$U_{\mathrm{BP}}(k_y) = U_{\mathrm{LEO}}(k_y)\exp[-ik_x(x_{\mathrm{LEO}} - x_{\mathrm{BP}})]$$

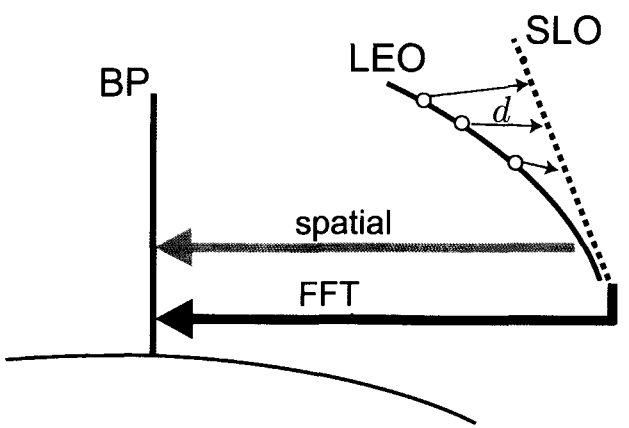
- **Implemented using FFT, achieving *N log(N)* efficiency.**

  (few CPU sec. per occultation)
- **For CT, no need to compute F.T. at BP plane.**
- **Can be generalized to non-parallel planes by rotating angular spectrum.**

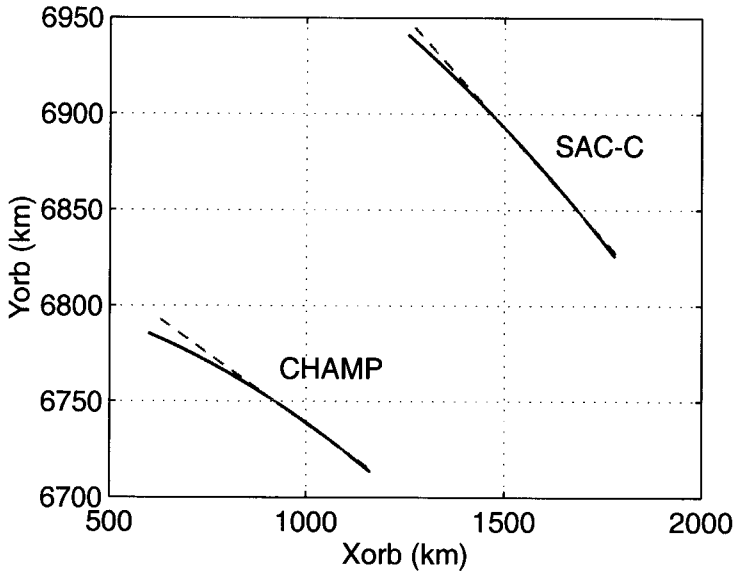


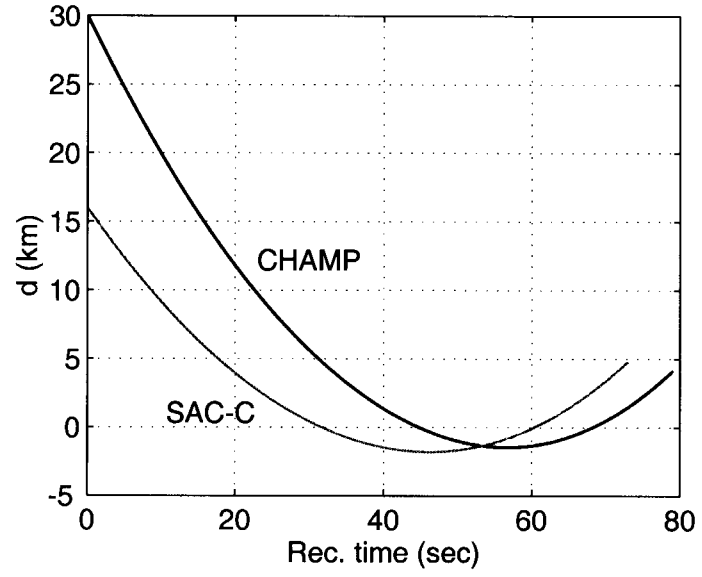
Real data preprocessing

- **How to apply the FFT method to real data when LEO is curved?**
- **Phase extrapolation method**
  1) Find "best-fit" straight line orbit (SLO);
  2) Associate each data point with a propagation direction;
  3) "Propagate" data from LEO to SLO locations;
  4) Add/subtract corresponding optical path to the phase. (Amplitude is unchanged.)
- **Apply BP from SLO to BP plane (or apply CT directly from SLO).**



SLO examples



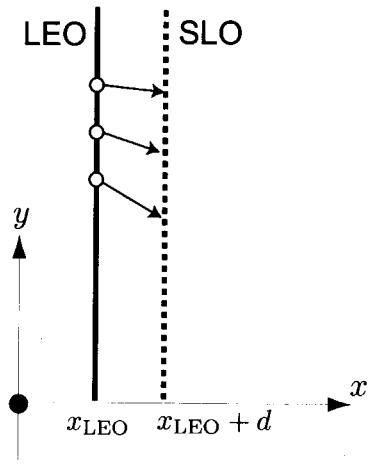


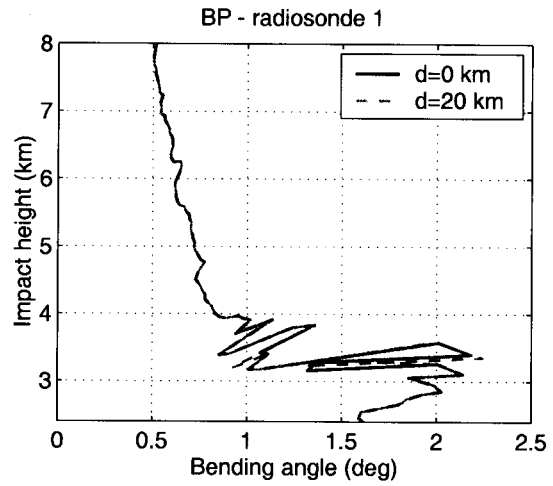
- **Propagation direction is determined by moving window average of the phase.**

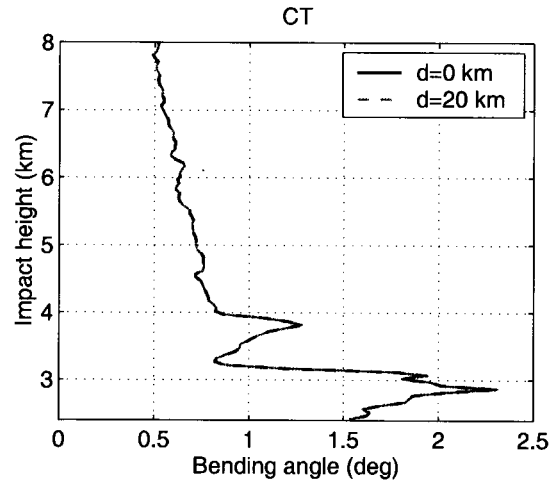
- **Error in bending:**

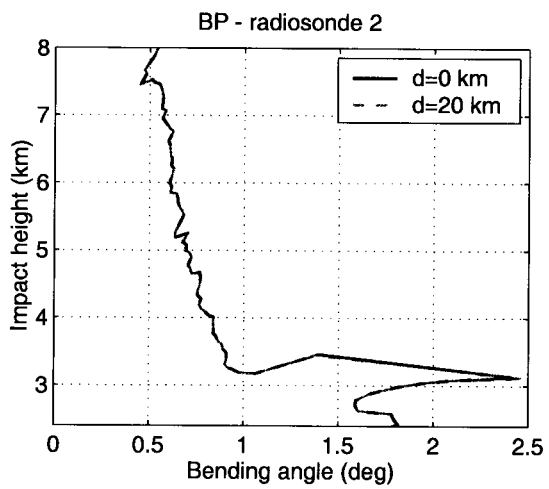
$$\Delta\alpha \approx \frac{d}{D}\Delta\gamma \sim \left(\frac{30\ \mathrm{km}}{3000\ \mathrm{km}}\right) 1^\circ = 0.01^\circ$$

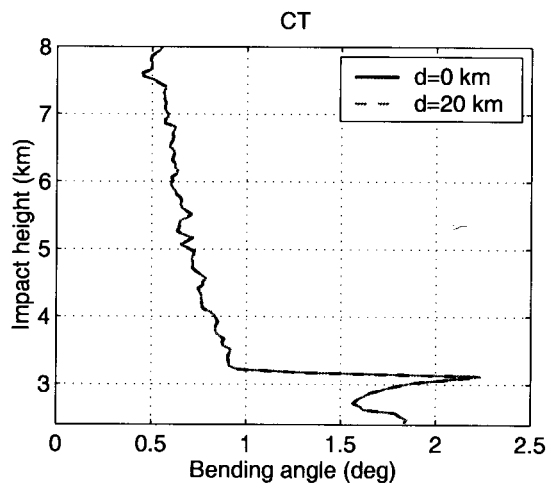
Validation with simulations







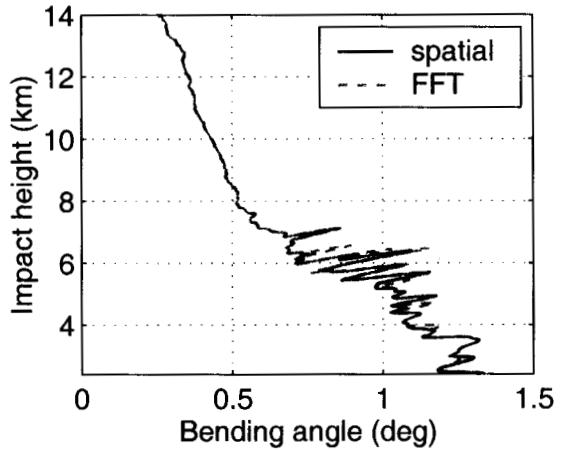


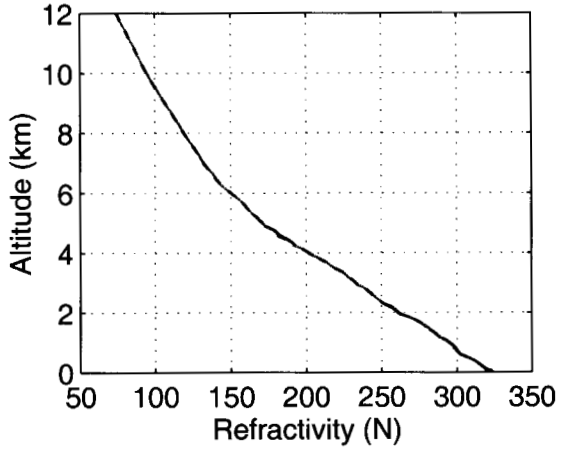


Application to real data

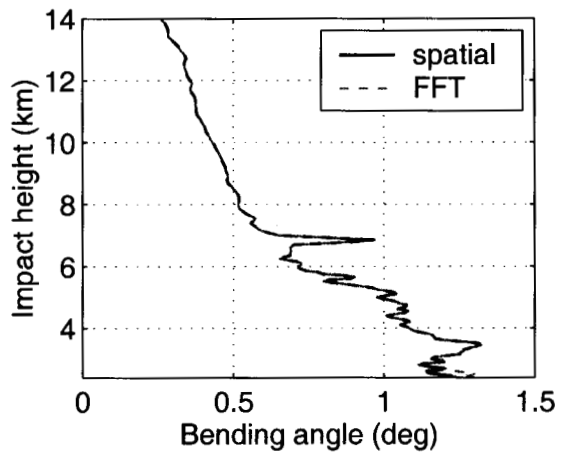
2001-10-10-04:23champ_gps38 (39.3 N, 92.3 W)

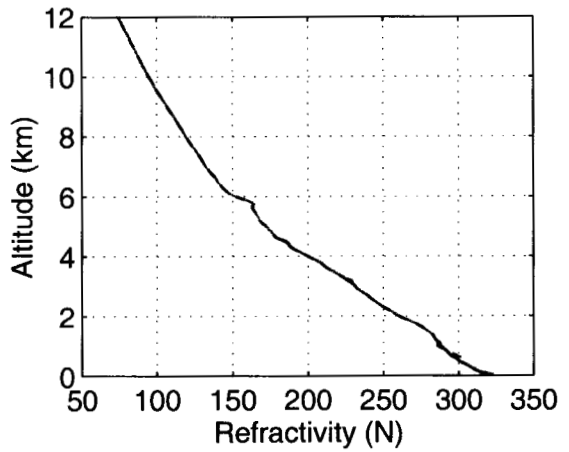
BP





CT





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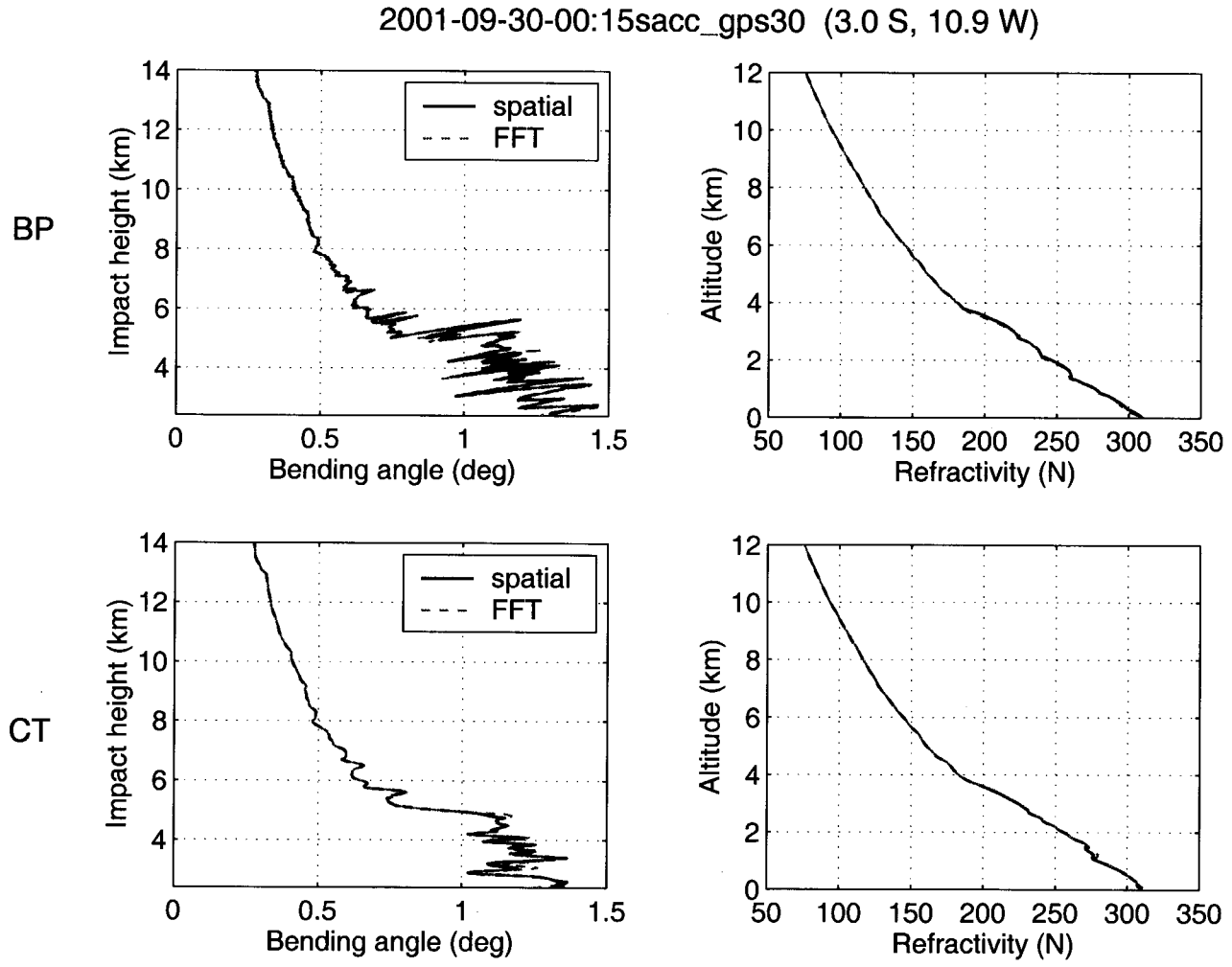

Summary

- **Spectral formulation of BP and CT can be implemented using FFT.**
  - Order of magnitude speed improvement over spatial integration methods.
  - Limited to straight line orbits (SLO).
- **The phase extrapolation method is proposed whereby the phase data can be quickly "propagated" from the LEO to a nearby straight line. The FFT approach can be applied on data residing at the SLO.**
  - ***To go fast, first go SLO(w).***
- **Validation using simulations as well as real occultation data shows that this method gives accurate bending angle and impact parameter.**
- **Reduction of computational cost enables retrieval products to be delivered quickly. This is important as the volume of occultation data is due to increase substantially with COSMIC.**