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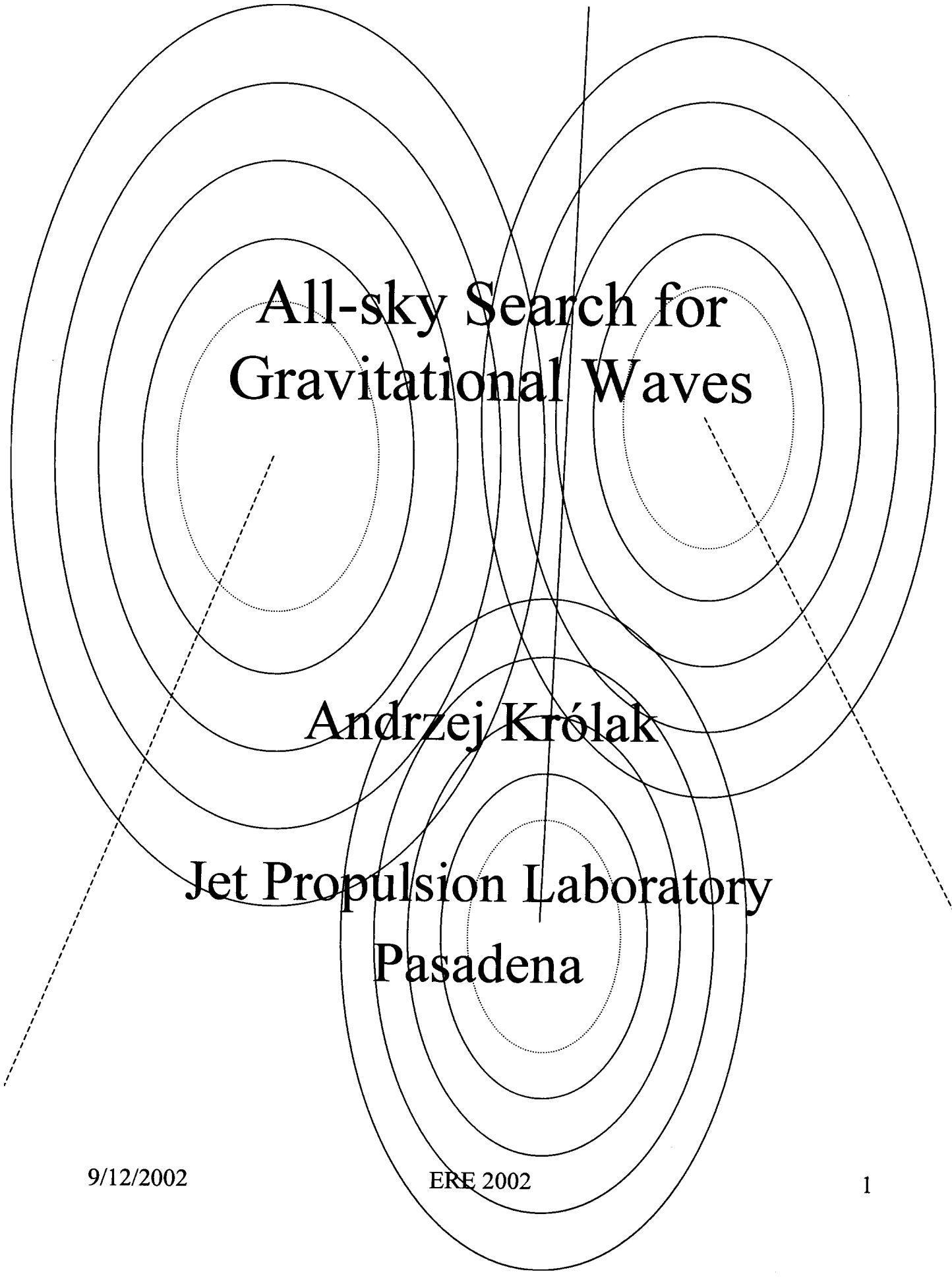
Title: All-sky search for gravitational waves

Abstract: We present a set of general tools to search for multiparameter gravitational-wave signals buried in the noise of a detector. Our analysis is based on the maximum likelihood detection method that is related to the matched-filtering technique which is the optimal method in the sense that it maximizes the probability of detection of the signal. We apply our techniques to perform an all-sky search for gravitational-wave signals from spinning neutron stars in the data of the EXPLORER bar detector. We present the results of our search.



All-sky Search for Gravitational Waves

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Methods and tools

1. Response of the detector to a continuous GW signal

$$h(t) = A_1\, a(t)\cos\Phi(t) + A_2\, b(t)\cos\Phi(t) + A_3\, a(t)\sin\Phi(t) + A_4\, b(t)\sin\Phi(t)$$

$$A_i \sim h_0 = 4.2\times10^{-23}\left(\frac{\epsilon}{10^{-5}}\right)\left(\frac{I}{10^{45}\,g\,cm^2}\right)\left(\frac{1kpc}{r}\right)\left(\frac{f}{1kHz}\right)^2$$

$$\Phi(t) = \sum_{k=0}^{s_2} \omega_k \frac{t^{k+1}}{(k+1)!} + \frac{\vec{n}_0\cdot\vec{r}_{SSB}(t)}{c}\sum_{k=0}^{s_2}\omega_k\frac{t^k}{k!}$$

$$\Phi_s(t) = p_0 t + \sum_{k=1}^{s} p_k t^{k+1} + A\cos(\nu_0 t) + B\sin(\nu_0 t)$$

↑ linear model of the phase

2. Maximum likelihood detection

$$x(t) = n(t) + h(t)$$

data — noise — signal

$$\mathcal{F} = \frac{2}{S_h(f_0)\,T_0}\left(\frac{|F_a|^2}{\langle a^2\rangle} + \frac{|F_b|^2}{\langle b^2\rangle}\right)$$

$$F_a := \int_0^{T_0} x(t)\,a(t)\exp[-i\Phi(t)]\,dt \qquad F_b :=$$

In the case of a linear phase model $\mathcal{F}$ is a homogeneous random field

$$C(\tau) = \frac{1}{2}C(0) \leftarrow \text{characteristic correlation hype}$$

$$N_{cells} = \frac{V_{parameter\ space}}{V_{cell}}$$

$$P_F^*(\mathcal{F}_0) = 1 - [1 - P_F(\mathcal{F}_0)]^{N_c}$$

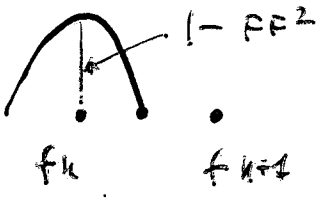
3. Grid of templates in our parameter space

(P_0, P_2, A, B)

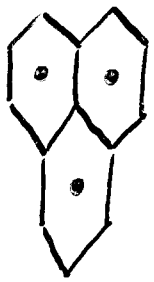
4-dimensional prism the two bases of which are 3-dimensional hexagonal prisms

The size of the prism is constrained by the fitting factor (FF) of the FFT

FF for FFT is ~ 0.63
FF for twice as fine a Fourier grid is ~ 0.90



Hexagonal grid in A-B plane



4. Two-step search procedure

1. Coarse search

2. Fine search ← Nelder-Mead simplex algorithm
(Only where threshold has been crossed)

Analysis of the EXPLORER detector data

Explorer bar detector is run by Italian collaboration called ROG and based in Rome

Explorer has collected years of data with good sensitivity and high duty cycle (e.g. 75% in 1991)

1. Parameters of the search

$f_0 \simeq 922$ Hz $\Delta\nu = 0.76$ Hz $T_0 = 2$ days

Parameters of the filter: frequency, 1 spindown, position in the day (2 parameters)

No. of filters: $\simeq 1.85 \times 10^9$ all-sky, $\tau_{min} := \frac{f_{max}}{2\dot{f}_{max}} = 100$ days

Sensitivity of the search:

for SNR = 1 $h_0 = 3.4 \times 10^{-24}$

for SNR = 8.3 $h_0 = 2.8 \times 10^{-23}$

corresponds to 99% confidence of detection

Search threshold: = 6.6

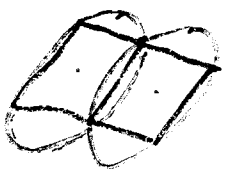
2. Random search

Advantages:

a) Straightforward calculation of false alarm probability: $N_s = \frac{N}{2} \cdot N_T$

b) No need to set up an elaborate grid

c) More effective coverage of parameter space than for systematic search:



d) Effective use of computing resources

3. Results of the search

9×10^5 trials ($P_2 < 0$)

No significant signals found

We have 231 threshold crossings

We expect 228 threshold crossings assuming pure Gaussian white noise

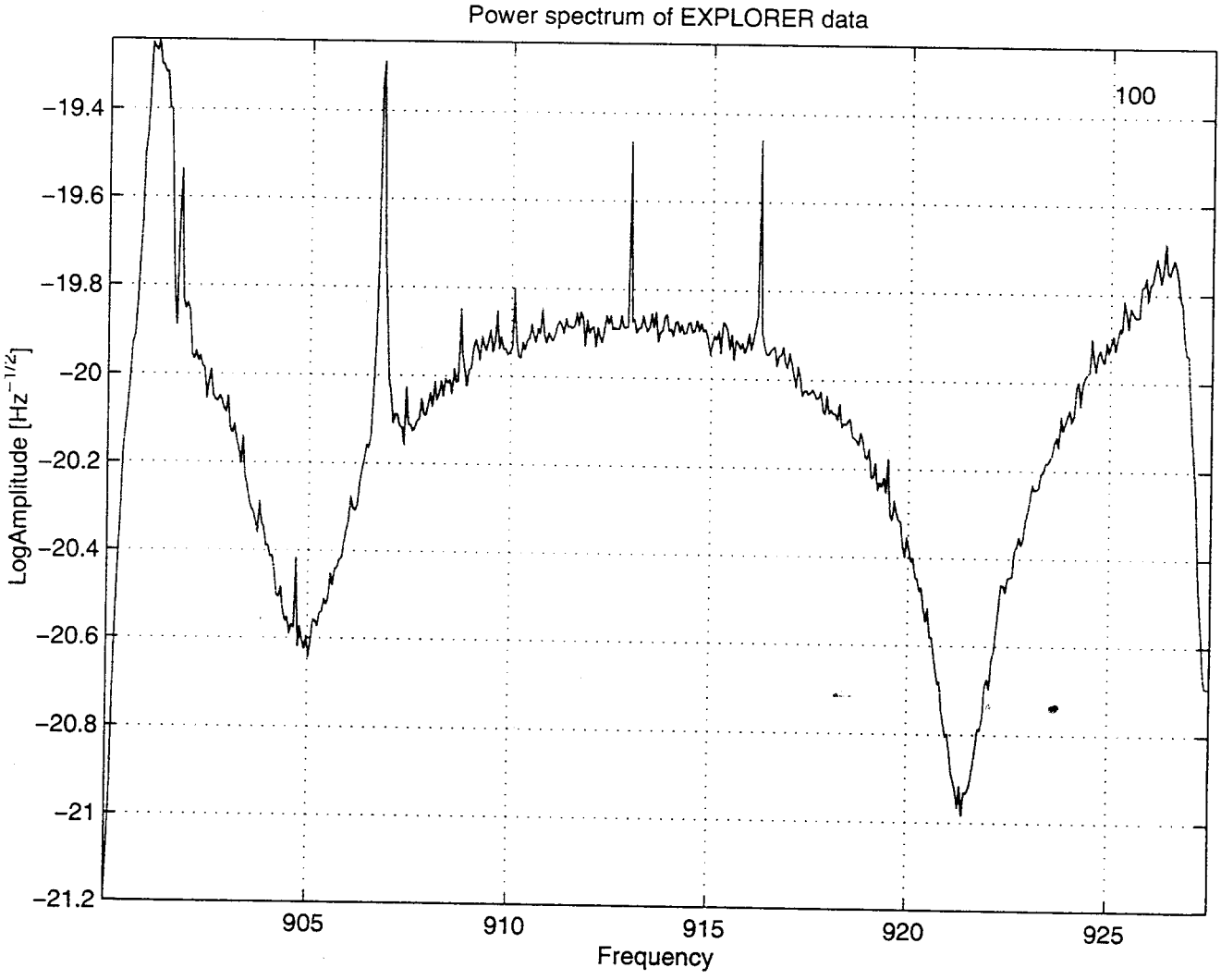
Verification procedure

a) Transformation to astrophysical parameters (f, f_{dot}, x, Γ)

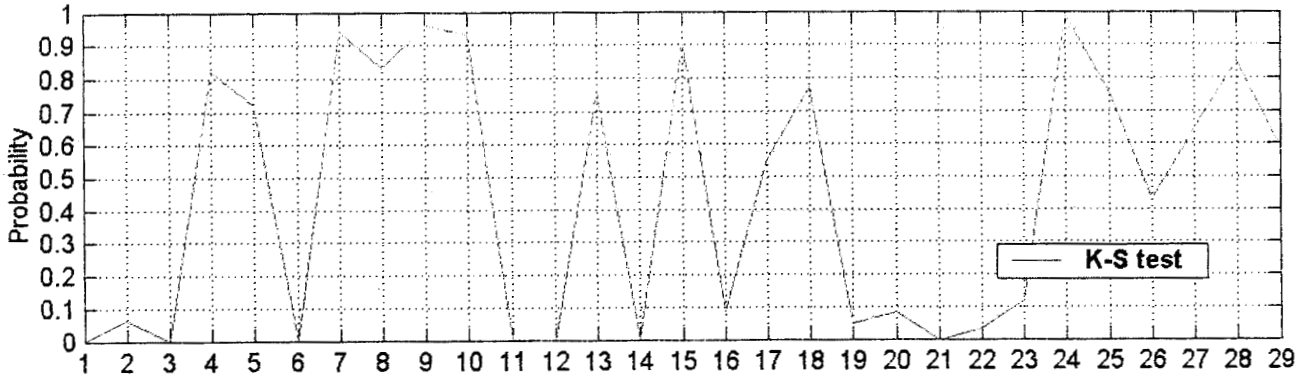
b) Search on a small grid around astrophysical parameters.

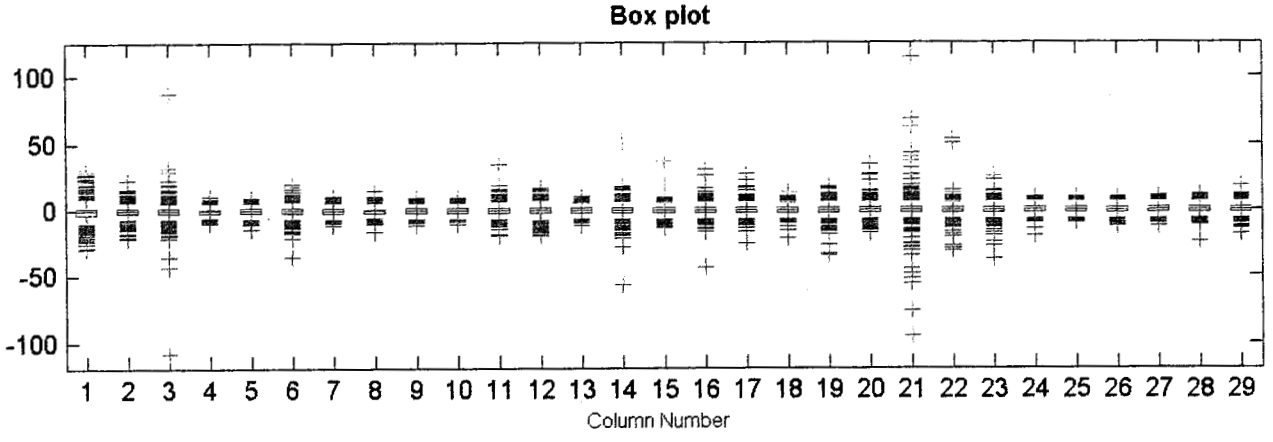
c) Search in 4-day stretch of data including the original one.

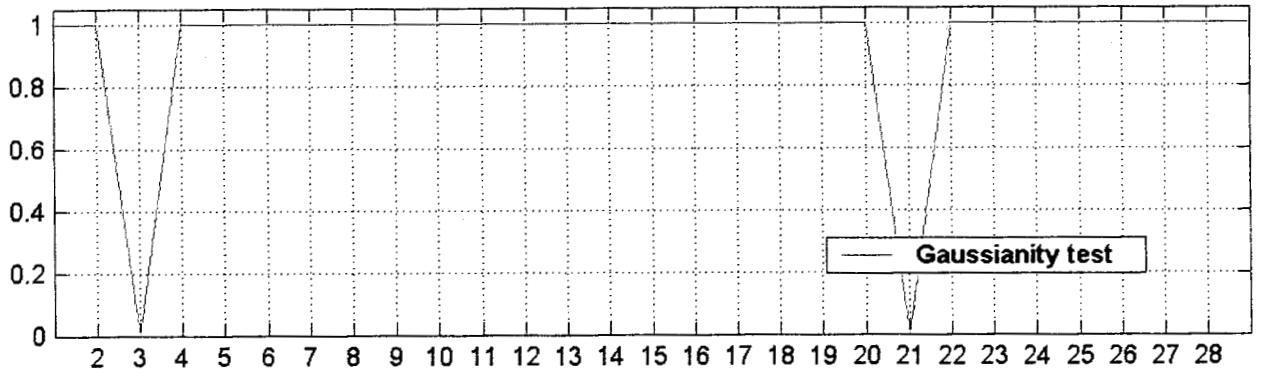
d) Search in a different 2-day stretch of data.

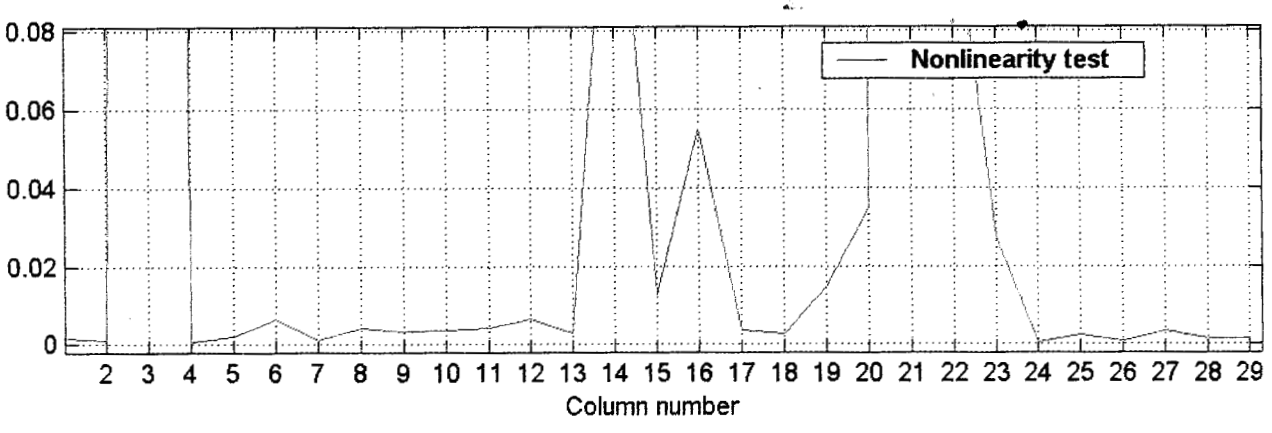

Power spectrum of EXPLORER data
100
-19.4
-19.6
-19.8
-20
-20.2
-20.4
-20.6
-20.8
-21
-21.2
LogAmplitude [Hz$^{-1/2}$]
905
910
915
920
925
Frequency

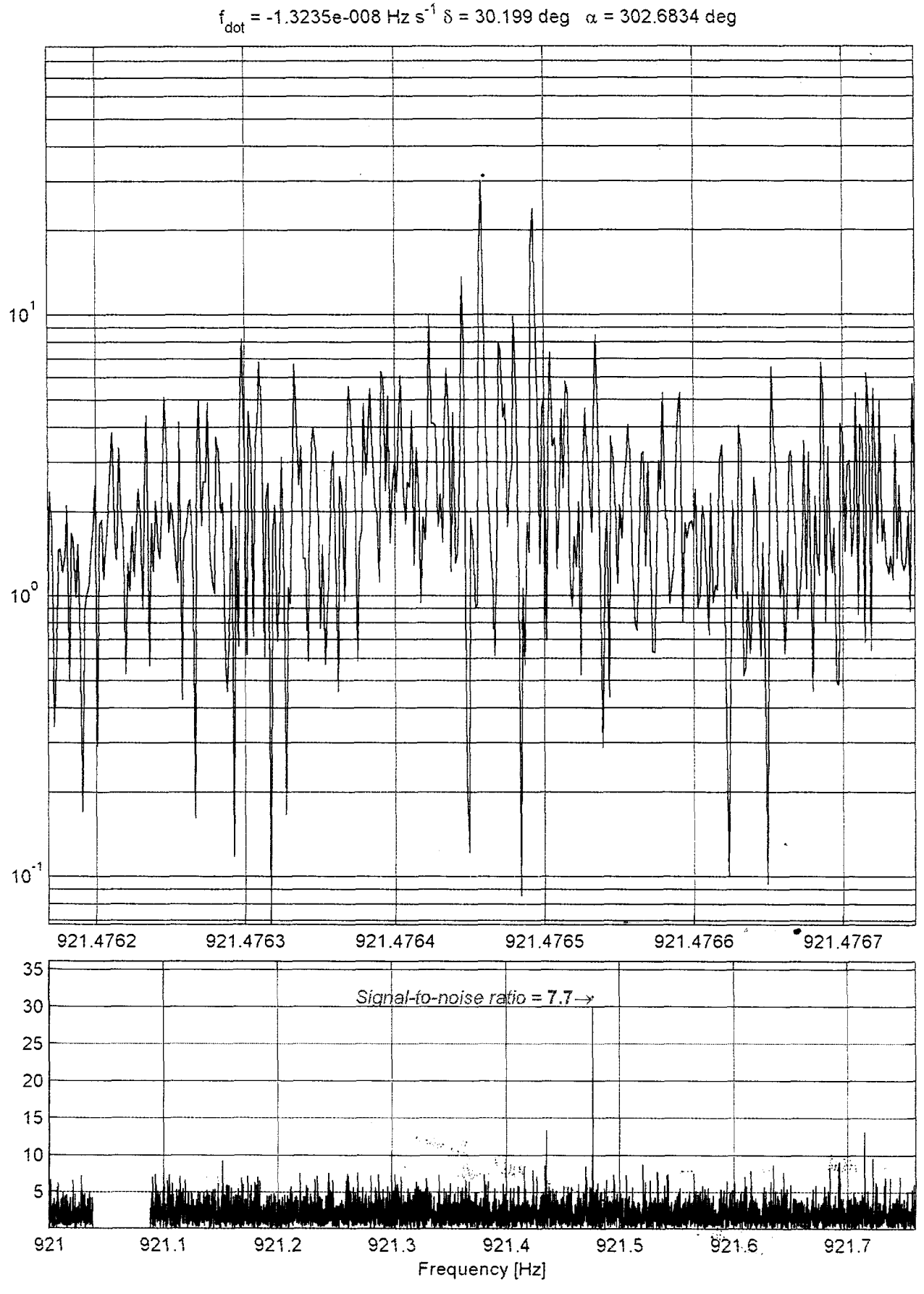
13-day data run (Nov. 1991) 216 point sections










f_{dot} = -1.3235e-008 Hz s^{-1} δ = 30.199 deg α = 302.6834 deg
10^1
10^0
10^{-1}
921.4762
921.4763
921.4764
921.4765
921.4766
921.4767
Signal-to-noise ratio = 7.7→
35
30
25
20
15
10
5
921
921.1
921.2
921.3
921.4
921.5
921.6
921.7
Frequency [Hz]