

The Cassini Gravitational Wave Experiment

John Armstrong, for the Cassini GWE Team

Outline

- **Experimental method and experiment objectives**
 - **GWs in the low-frequency band**
 - **Signal and noise transfer functions in Doppler experiments**
- **The Cassini experiment and the data set analyzed here**
- **Refinement of the noise model**
 - **Principal noises and their magnitudes**
 - **Correlation between Doppler and WVR data and the AMC calibration**
 - **Local and average spectra, nonstationarity of the data, and systematic effects**
- **Pilot analysis**
 - **Periodic and quasi-periodic waves**
 - **Burst waves, "all-sky" survey**
 - **Upper limit to GW background at $f \approx 0.0001$ Hz**
- **How to do better than Cassini with Doppler tracking**
- **Concluding comments (& GWE2 & GWE3)**

Bruno Bertotti*

Frank Estabrook

Luciano Iess*

Massimo Tinto

Hugo Wahlquist*

and hundreds of engineers working in the DSN and the Cassini project...

*** Cassini Gravitational Wave Experiment Investigator**

Some Jargon

DSN	"Deep Space Network", the NASA/JPL tracking antennas in California, Spain, and Australia
DSS	Deep Space Station – one of the DSN antennas, identified by number (e.g. DSS25)
X-band	radio frequency of about 8.4 GHz (Mars Observer)
Ka-band	radio frequency of about 32 GHz (Cassini)
Allan variance	a measure of fractional frequency stability, $\Delta f/f$, as a function of integration time
scintillation	variation of the amplitude or phase of a radio signals due to scattering by a medium (solar wind, ionosphere, troposphere) between the source and the receiver
AMC	Advanced Media Calibration system (DSS25 only)
FTS	Frequency and Timing System
clock	precision frequency standard

Representative References

Regarding the method and the experiment:

- Estabrook and Wahlquist, *GRG*, 6, 439 (1975)
- Wahlquist *GRG*, 19, 1101 (1987)
- Bertotti *et al. Astron. Astrophys* 296, 13 (1995)
- Tinto *Phys Rev D* 53, 5354 (1996)
- Tinto and Armstrong *Phys Rev D*, 58 042002 (1998)

Regarding the noises:

- Armstrong, Woo, and Estabrook *Ap. J.* 230, 570 (1979)
- Bertotti *et al. Astron. Astrophys*, 269, 608 (1993)
- Armstrong *Radio Sci.* 33, 1727 (1998)
- Iess *et al. CQG* 16, 1487 (1999)
- Tinto *CQG*, 19, 1767 (2002)
- Abbate *et al. SPIE* (in press 2002)

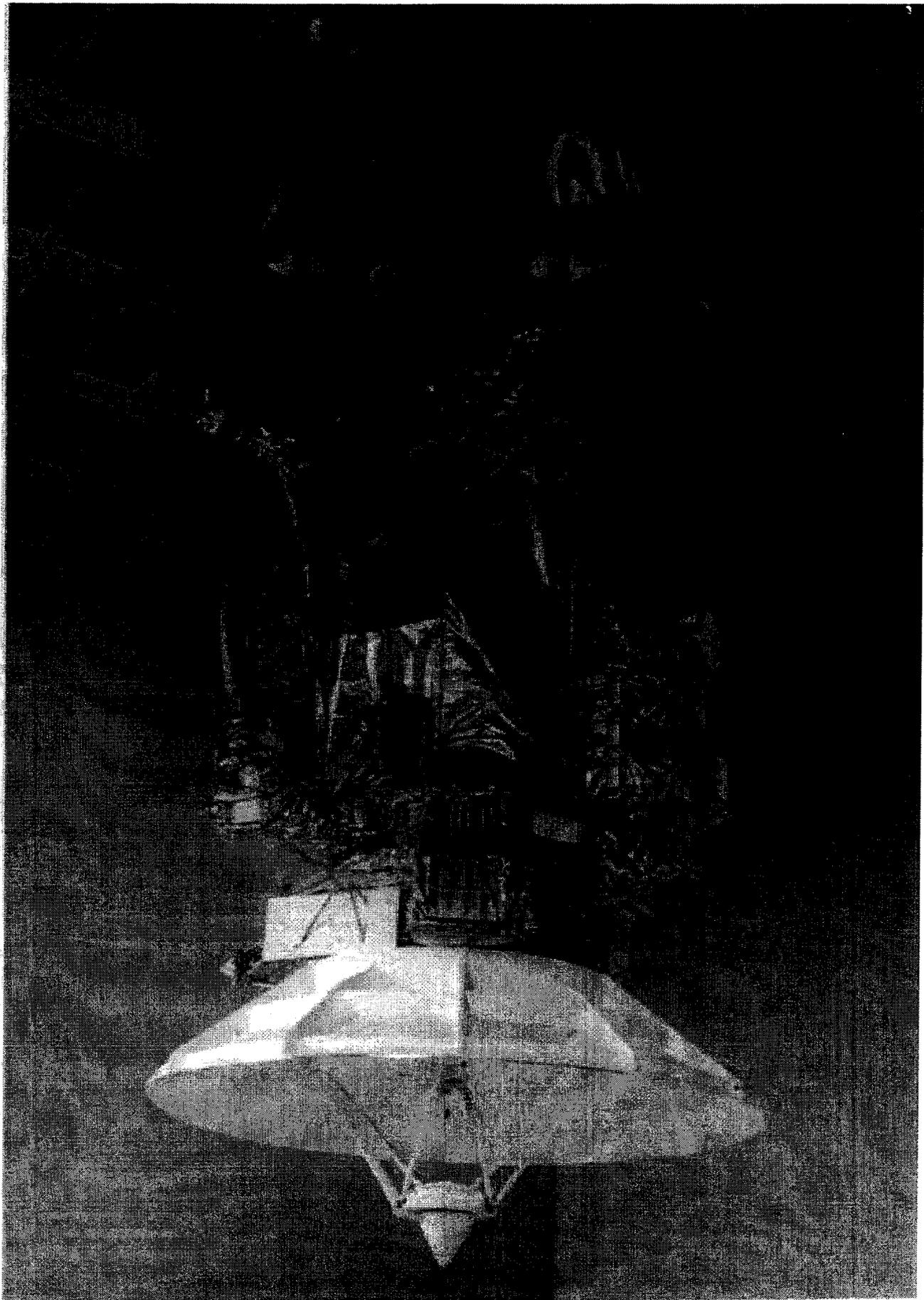
Web-based lectures and notes:

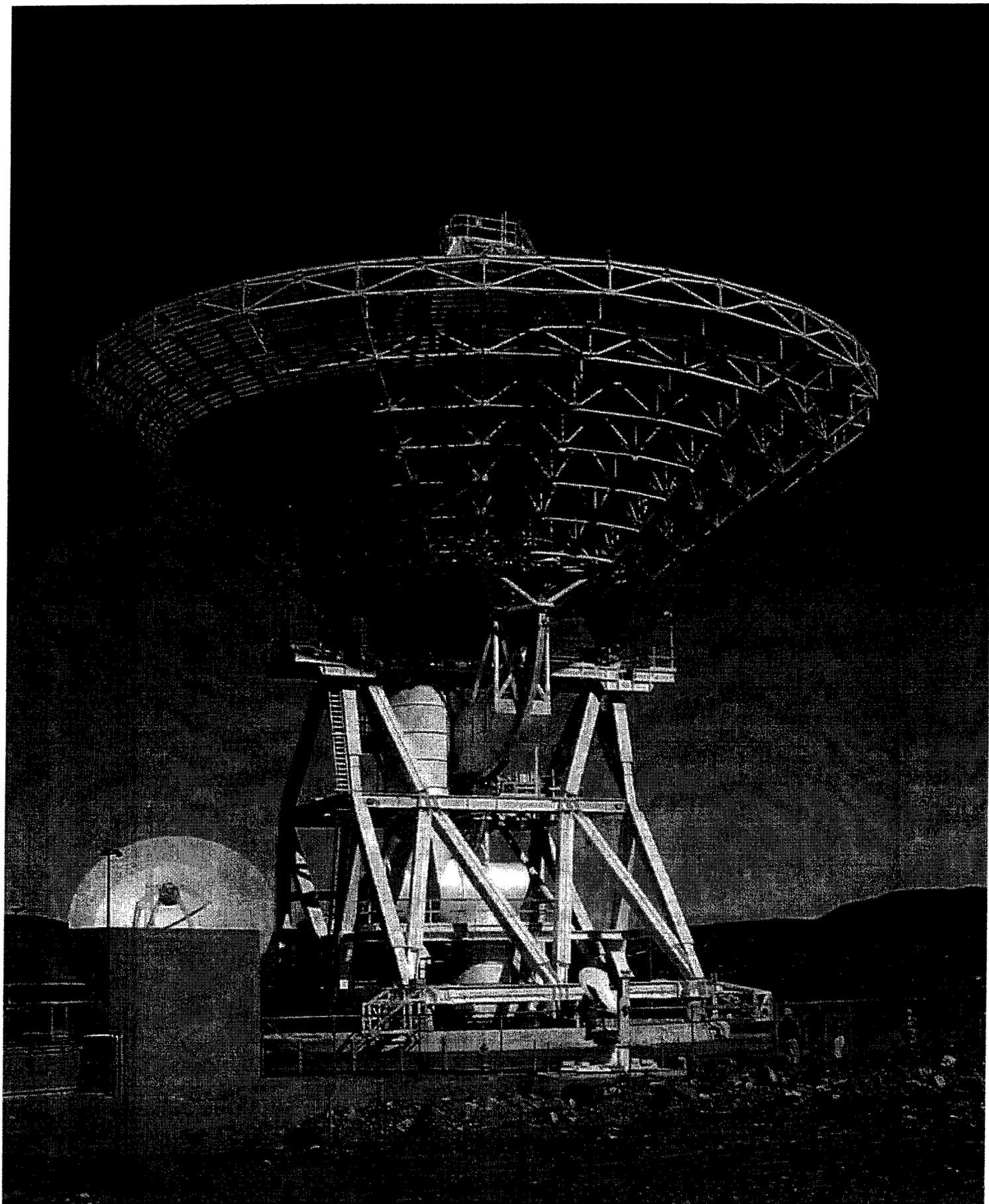
<http://elmer.tapir.caltech.edu/ph237/week15/week15.html>

http://www.cco.caltech.edu/~cajagwr/scripts/doppler_tracking.html

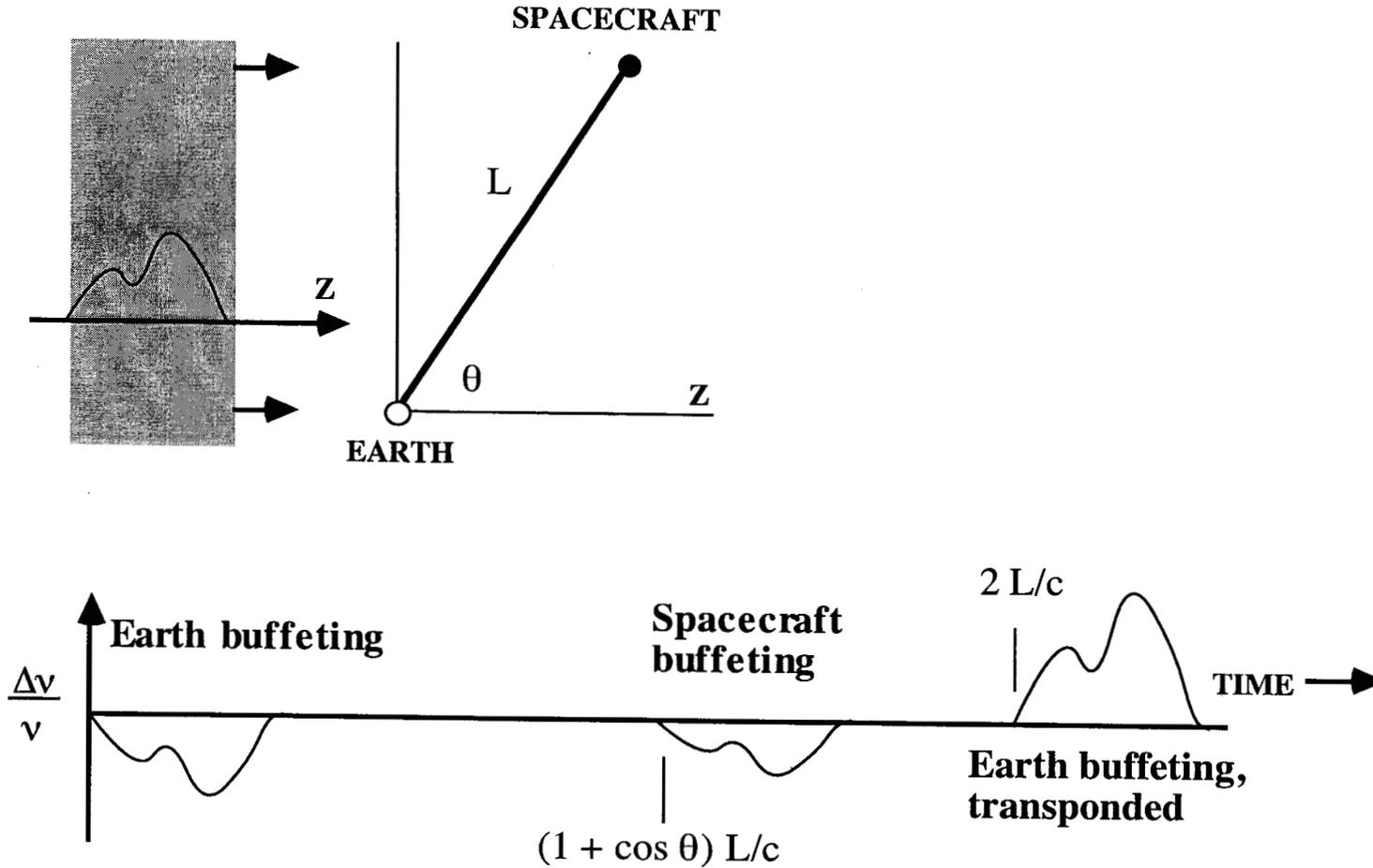
Disclaimer

- **We are analyzing the data cooperatively but so far largely independently — my colleagues and I have not compared results in detail yet!**
- **This should be regarded as provisional and a status report on aspects of the GWE1 data analysis**



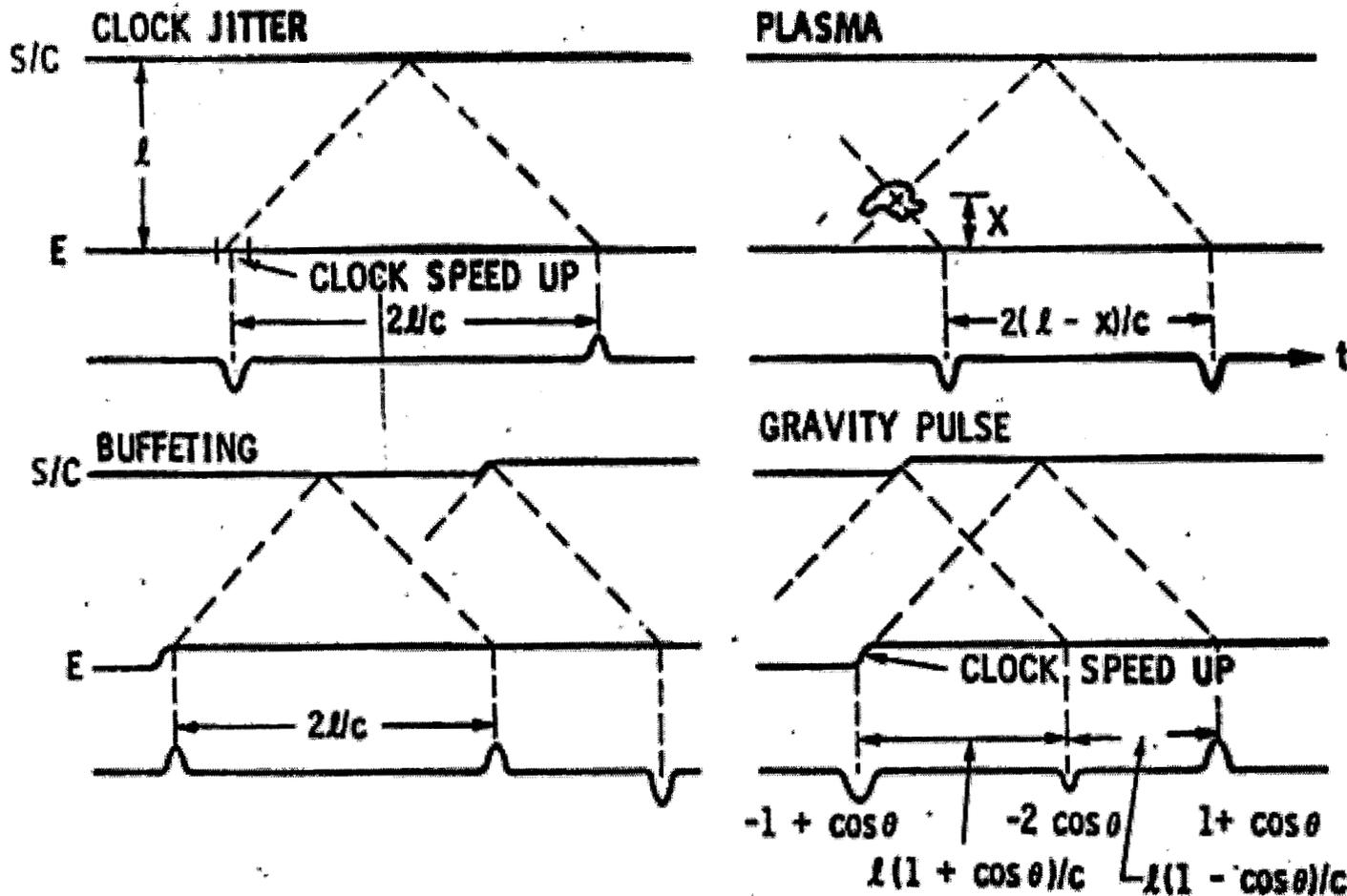


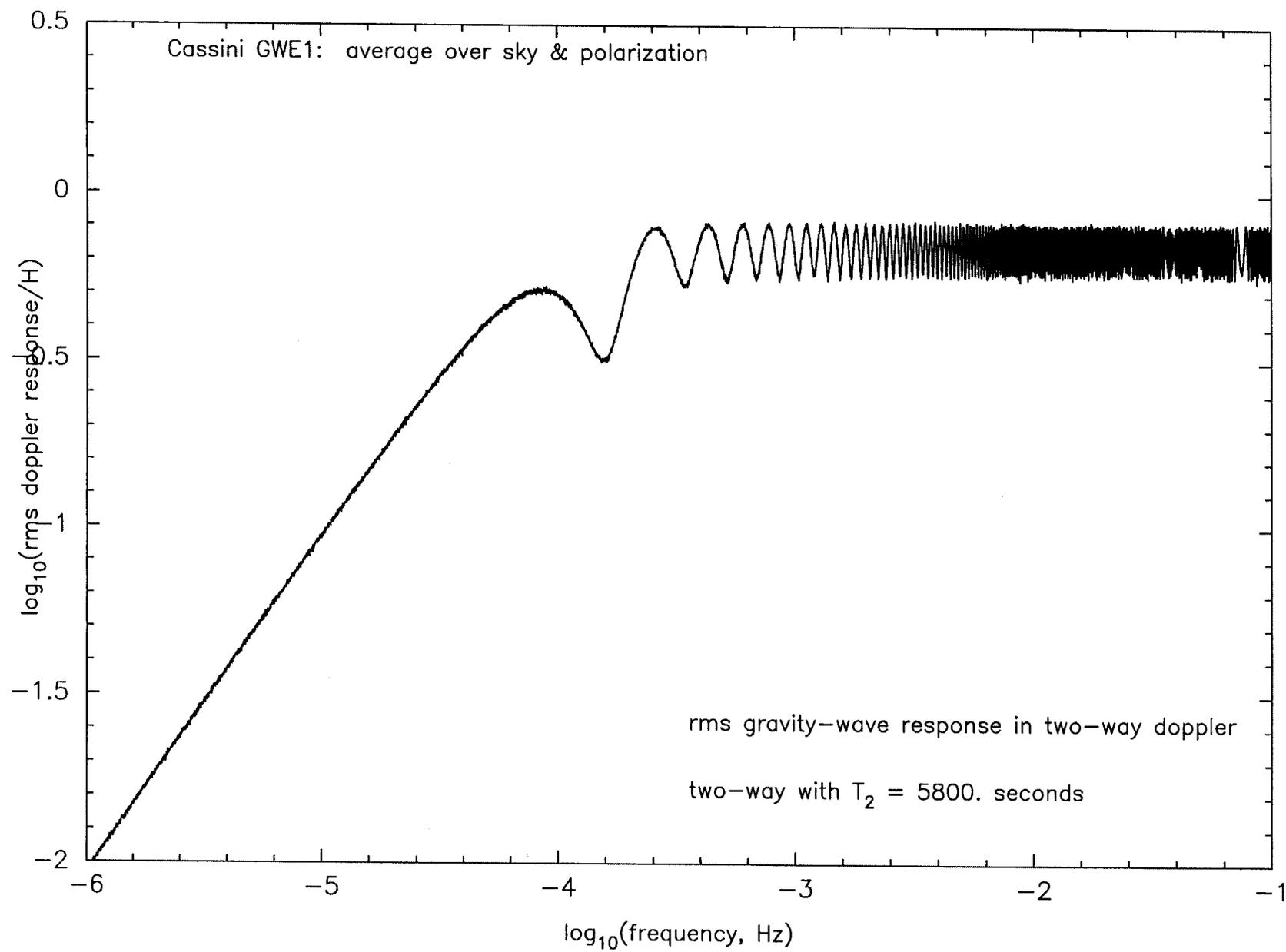
Response of Spacecraft Doppler Tracking to Gravitational Wave

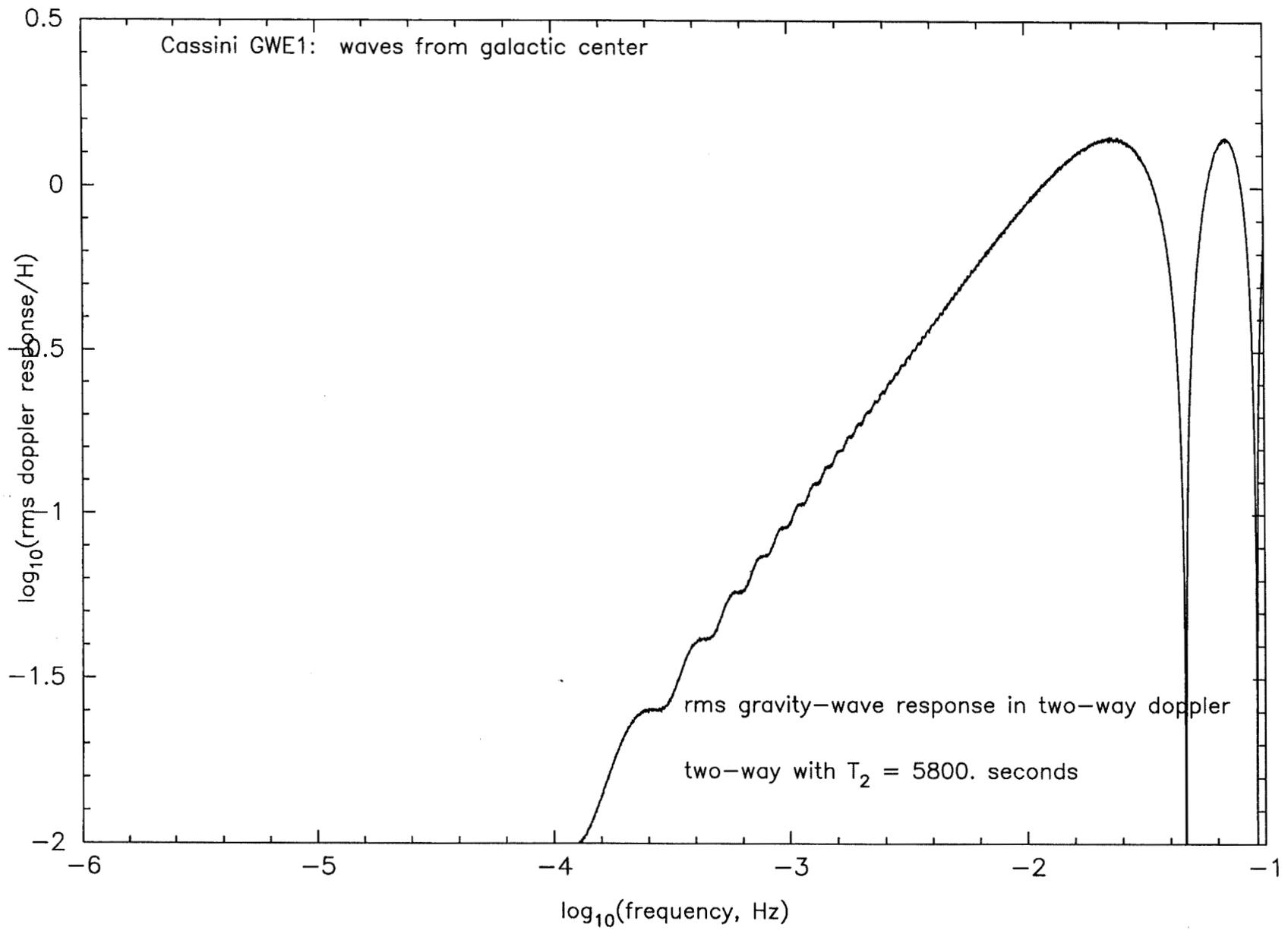


Estabrook and Wahlquist, *Gen. Rel. Grav.* 6, 439 (1975)

DOPPLER SIGNALS CORRESPONDING TO DIFFERENT TYPES OF DISTURBANCE IN THE COMMUNICATION LINK







Model of Doppler Time Series

$$y(t) = \Delta f/f_0 = \begin{array}{ll} \text{gravity waves} & + \text{unmodeled spacecraft motion} \\ + \text{propagation noise} & + \text{antenna mechanical noise} \\ + \text{clock noise} & + \text{thermal noise} \\ + \text{systematic effects} & \end{array}$$

$$\begin{aligned} &= \mathbf{g}(t) * \left\{ \left[\frac{(\mu - 1)}{2} \right] \delta(t) - \mu \delta\left[t - (1 + \mu)L/c\right] + \left[\frac{(1 + \mu)}{2} \right] \delta(t - 2L/c) \right\} \\ &+ \text{propagation } (t) * \left\{ \delta(t) + \delta(t - 2L/c) \right\} \\ &+ \text{antenna mechanical } (t) * \left\{ \delta(t) + \delta(t - 2L/c) \right\} \\ &+ \text{frequency standard } (t) * \left\{ \delta(t) - \delta(t - 2L/c) \right\} \\ &+ \text{thermal } (t) \\ &+ \text{systematic effects} \end{aligned}$$

where: $\mathbf{g}(t) = (1 - \mu^2)^{-1} \left\{ \mathbf{n} \cdot \left[\mathbf{h}_+(t)\mathbf{e}_t + \mathbf{h}_x(t)\mathbf{e}_x \right] \cdot \mathbf{n} \right\}$

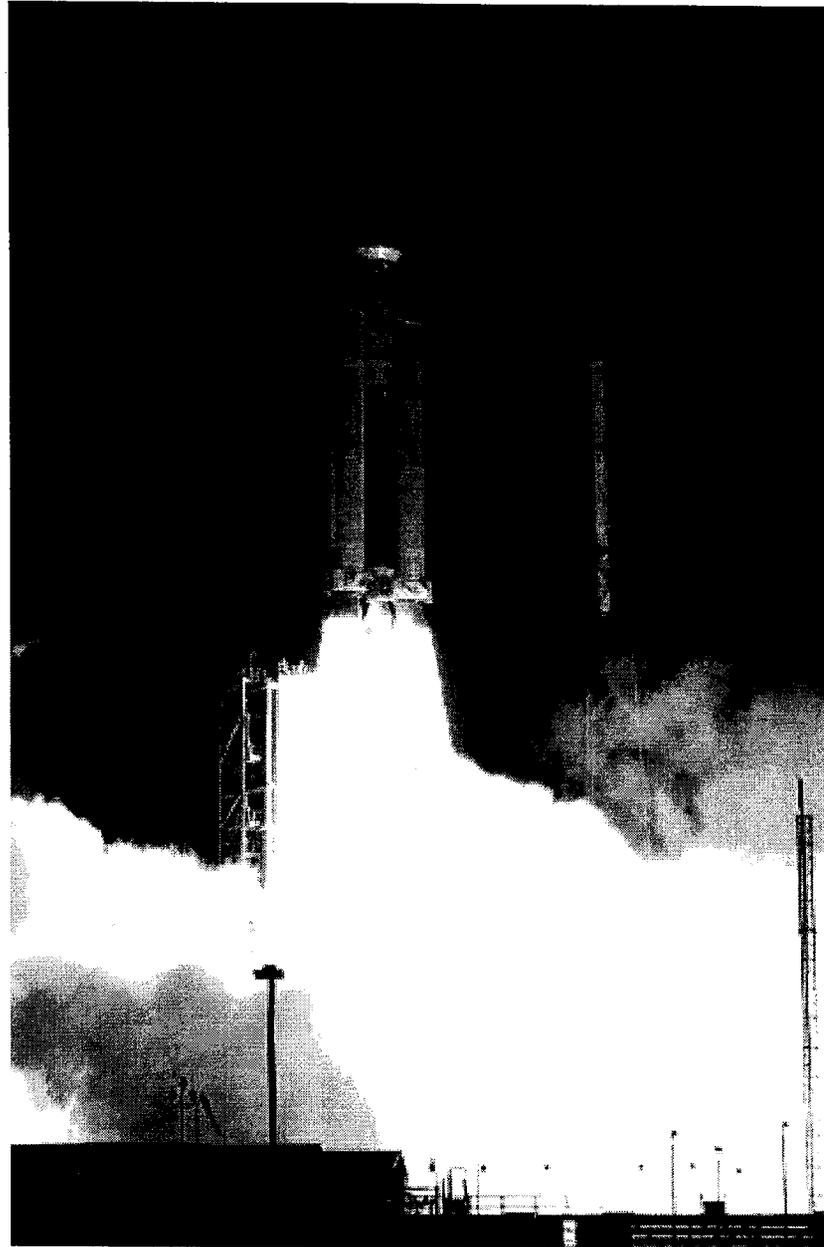
$L = \text{earth-s/c distance}; \mu = \mathbf{k} \cdot \mathbf{n}; * = \text{convolution}$

HOW TO DO A DOPPLER TRACKING EXPERIMENT

- **Need two separated test masses—the earth and a spacecraft in cruise as operationally quiet as possible (need to be far from perturbing masses and need to minimize unmodeled motion of the spacecraft)**
- **Spacecraft should be as close to anti-solar direction as practical (minimize charged particle scintillation due to solar wind)**
- **Spacecraft-earth separation should be large (maximize band of Fourier frequencies to which the experiment is sensitive)**
- **Highly-stable Doppler system to measure relative velocity of the earth and spacecraft (excellent frequency standard; careful signal distribution, etc.)**
- **Ground system and spacecraft telemetry (correct for or veto data based on known systematics of the apparatus)**
- **Good weather and media calibration data**

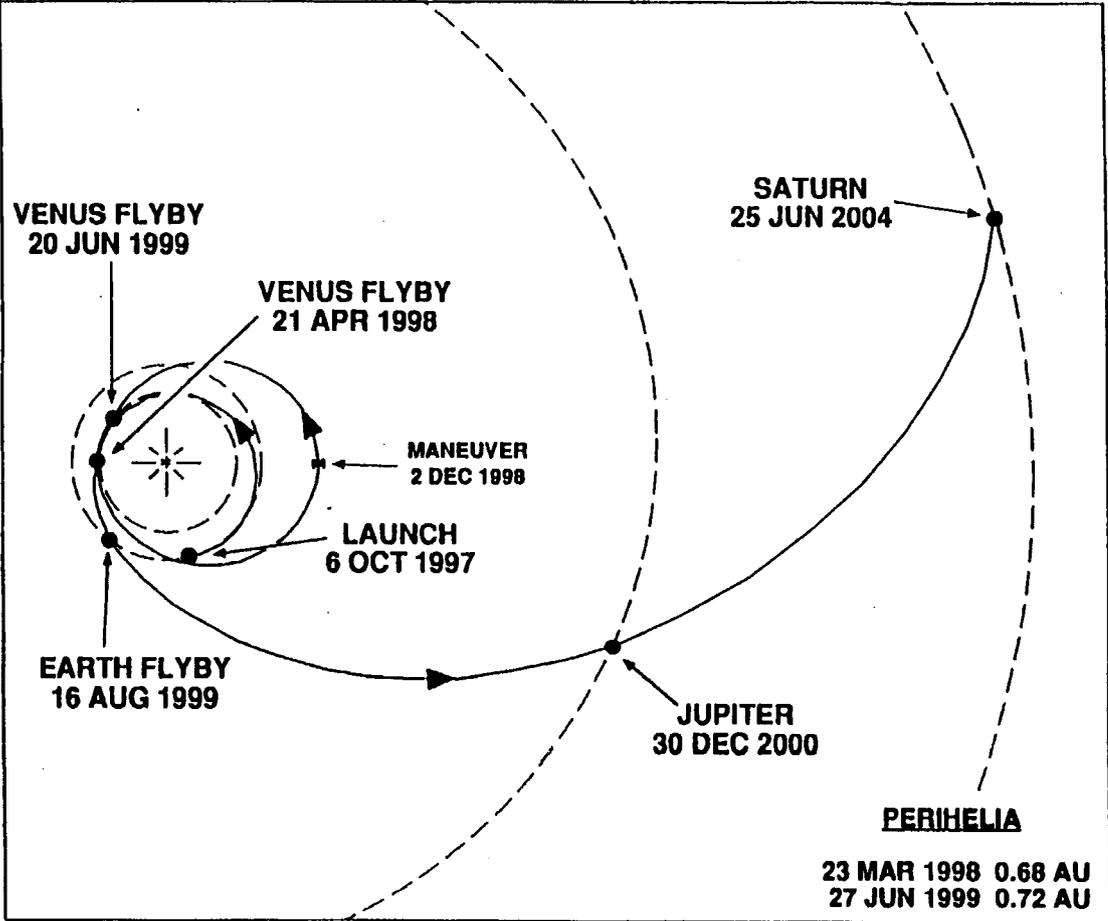
OBSERVATIONS TO DATE

1980	Voyager	Hellings et al. (1981) (few passes; bursts)
1981	Pioneer 10	Anderson et al. (1984) (3 passes, long T_2 ; no GW from Geminga)
1983	Pioneer 11	Armstrong, Estabrook, Wahlquist (1987) (broadband search for periodic waves)
1988	Pioneer 10	Anderson et al. (1993) (10 days; chirps and coalescing binaries)
1992	Ulysses	Bertotti et al. (1995) (1 month; sinusoids and chirps)
1993	MO/GLL/ULS	jGWE collaboration (19 days; X-band on MO; only LF coincidence experiment)
1994-5	Galileo	Estabrook et al. (40 days; long T_2)
1997	Mars Global Surveyor	Armstrong et al. (3 weeks; X-band)



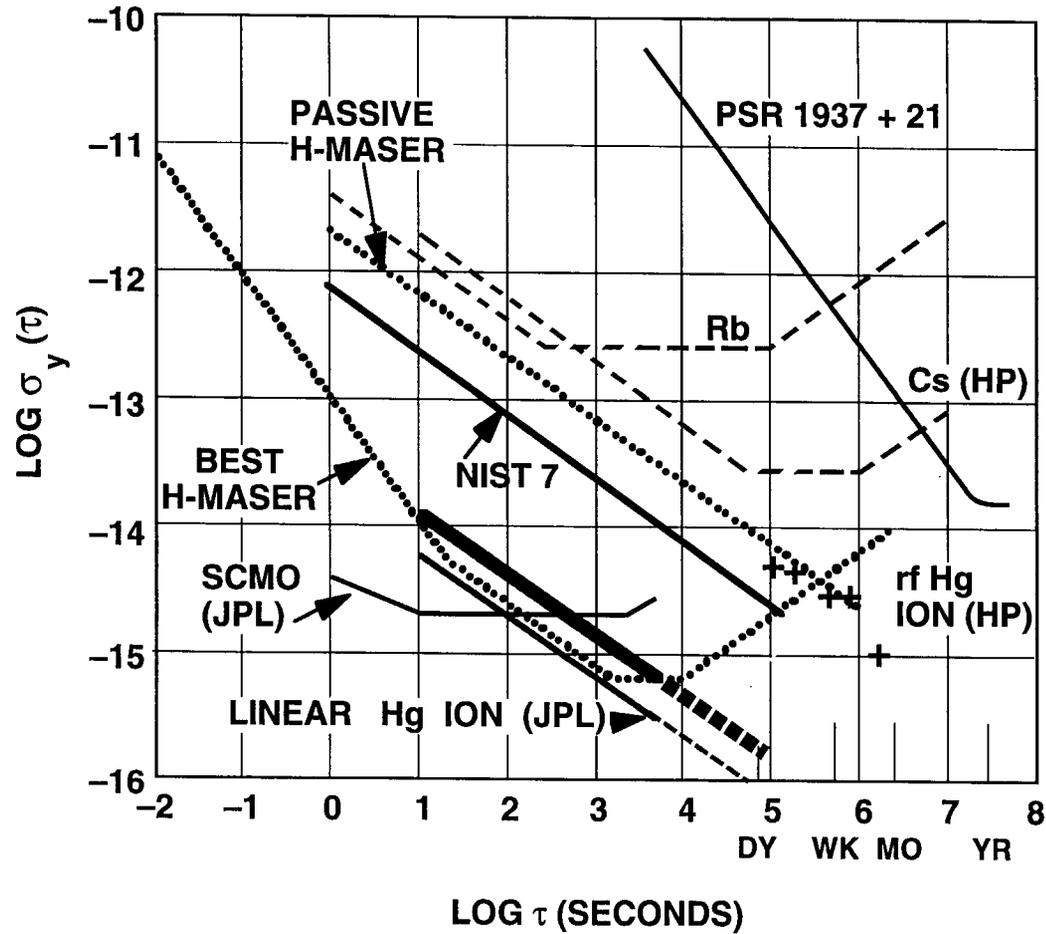
CaJAGWR-36
11/3/00

CASSINI OCT 1997 VVEJGA INTERPLANETARY TRAJECTORY

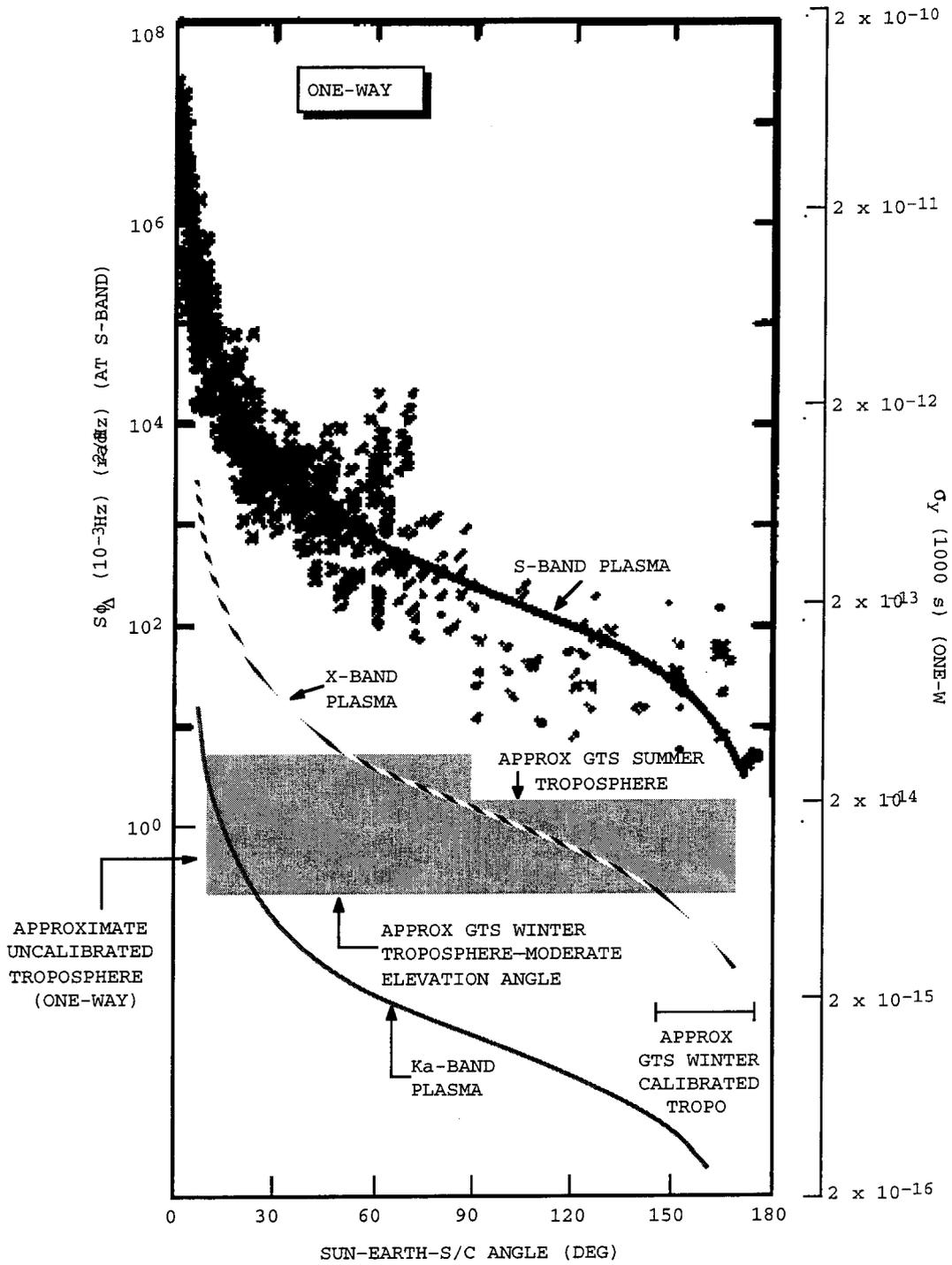


13 JUL 1992

LINEAR ION TRAP STANDARD (LITS) FRACTIONAL FREQUENCY STABILITY



INTERPLANETARY PHASE SCINTILLATION

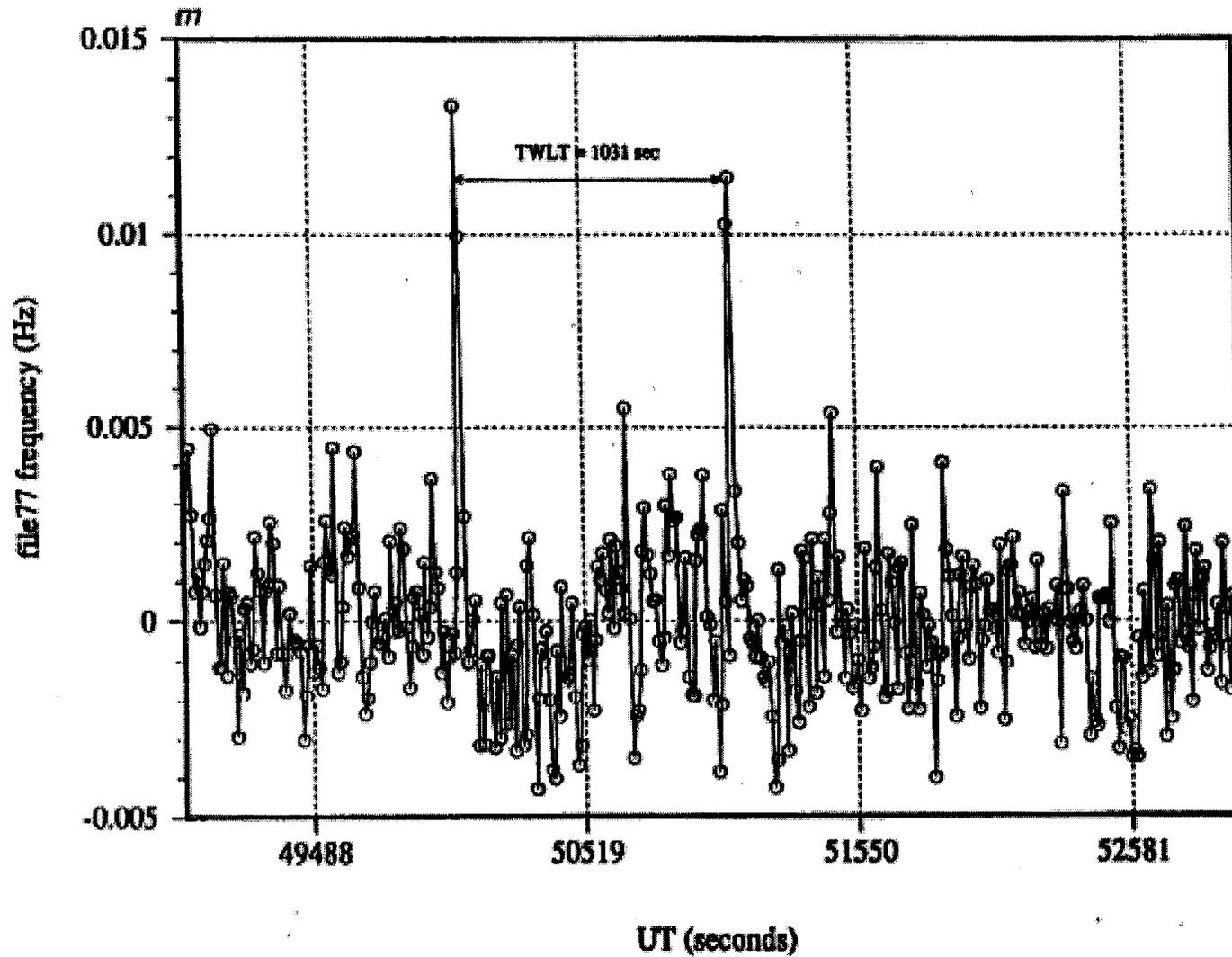


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MAIN NOISES: ANTENNA MECHANICAL STABILITY

- Differential measurements (under controlled conditions) indicate $\sigma_y(1000\text{sec}) \approx 1 \times 10^{-15}$ (Otohi and Franco, *TDA Prog. Report 42-10*, 151 (1992))
- Transfer function in two-way Doppler is $\delta(t) + \delta(t - T_2)$
- Measurements at X-band under operational conditions are confused with tropospheric scintillation and produce only poor limits ($< 1 \times 10^{-14}$ at $\tau = 1000$ sec) (Armstrong *Radio Sci.* 33, 1727 (1998))
- Infrequent large events—almost certainly antenna mechanical—are observed, however (see example)

ooj052_mo_089_1254_65_a (file77)



The Data Set Analyzed Here

- **Original data: open loop I-and-Q samples at 1 kHz (8 DVDs)**
 - **Ka-band 2-way at DSS 25**
 - **X-band 2-way at DSS 65, 45**
- **Phase detect all data, remove orbit from high-elevation data**
- **Residuals inspected manually at time constants of 0.2, 10, 100, 1000 seconds; edited for obvious problems (DSS 25 antenna mechanical events at 60 deg. elevation; FTS VCO distribution problems at SPC 10; high variance due to rain/snow; etc.—19% of data vetoed)**
- **Tropospheric scintillation correction applied to DSS25 data**
- **DSS 25, 65 selected for high elevation (= minimize problems)**
- **Programs used and "rules" defining transformations of the data set are in makefiles and under SCCS, so in principle could be reconstructed from scratch by someone else (practical caveats: mathematica, PGPLOT, fortran compilers and switches, swapping DVDs, disc space, AMC corrections)**
- **Edited data merged into one file, with 10 second time resolution ("edited_superfile", 021002 version)**

Refinement of the Noise Model

$$y(t) = \Delta f(t)/f_0 = \text{gravity waves} \\ + \text{propagation noise} + \text{antenna mechanical noise} \\ + \text{clock noise} \quad + \text{thermal noise} \\ + \text{systematic effects}$$

$$= g(t) * \{[(\mu-1)/2] \delta(t) - \mu \delta[t-(1+\mu)L/c] + [(1+\mu)/2] \delta(t - 2L/c)\} \\ + \text{propagation}(t) * \{\delta(t) + \delta(t - 2L/c)\} \\ + \text{antenna mechanical}(t) * \{\delta(t) + \delta(t - 2L/c)\} \\ + \text{frequency standard}(t) * \{\delta(t) - \delta(t - 2L/c)\} \\ + \text{thermal}(t) \\ + \text{systematic effects}$$

where: $g(t) = (1 - \mu^2)^{-1} \{n \cdot [h_+(t) e_+ + h_x(t) e_x] \cdot n\}$

$L = \text{earth-s/c distance}$

$$\mu = k \cdot n$$

$* = \text{convolution}$

data from 2001_340_0405_82_25_kk_v2.5

amplitude (dB)

0

-10

noncausal-filtered residual (mHz)

5

0

-5

-10

10 SECOND
AVERAGES

7

8

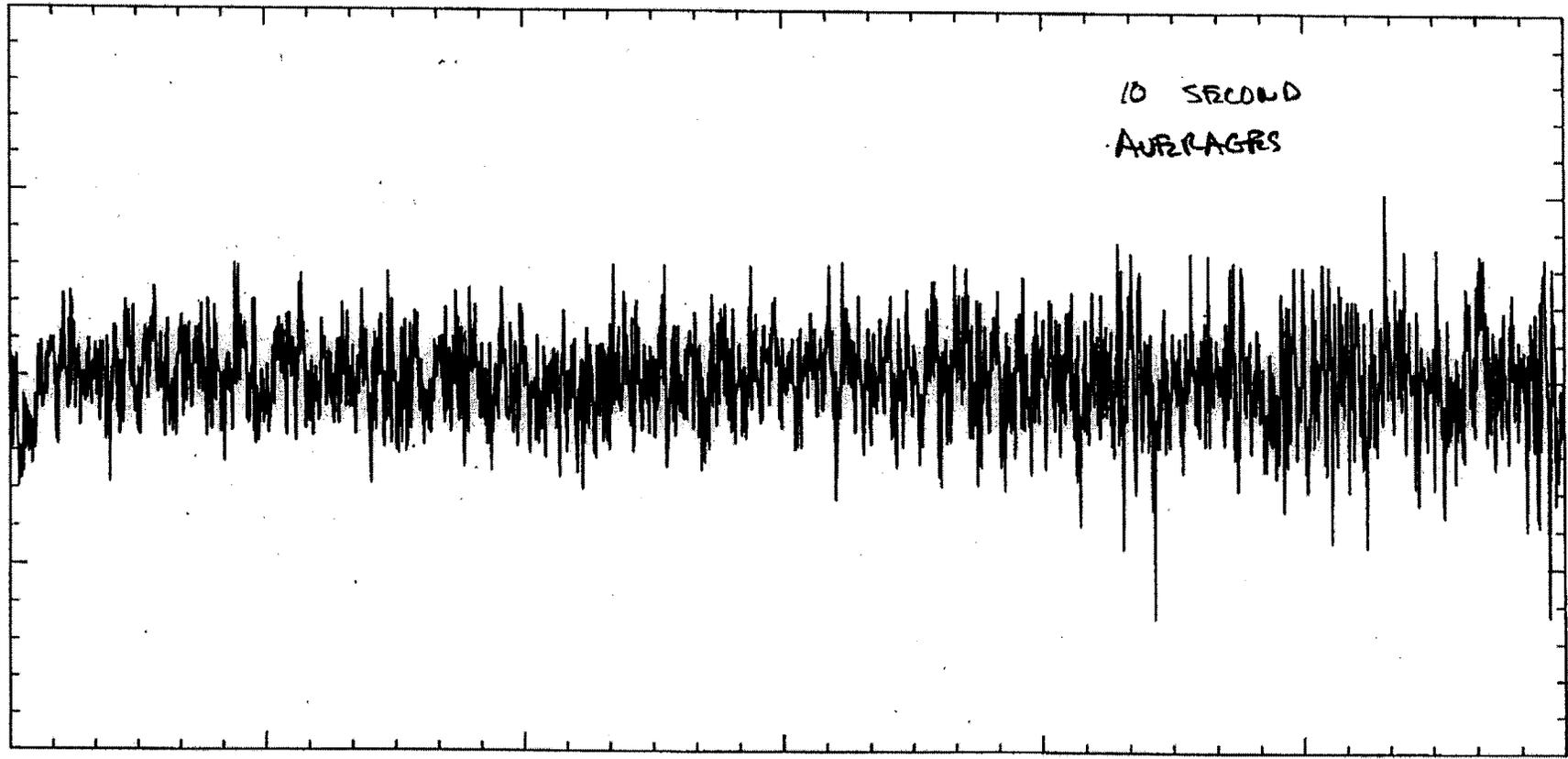
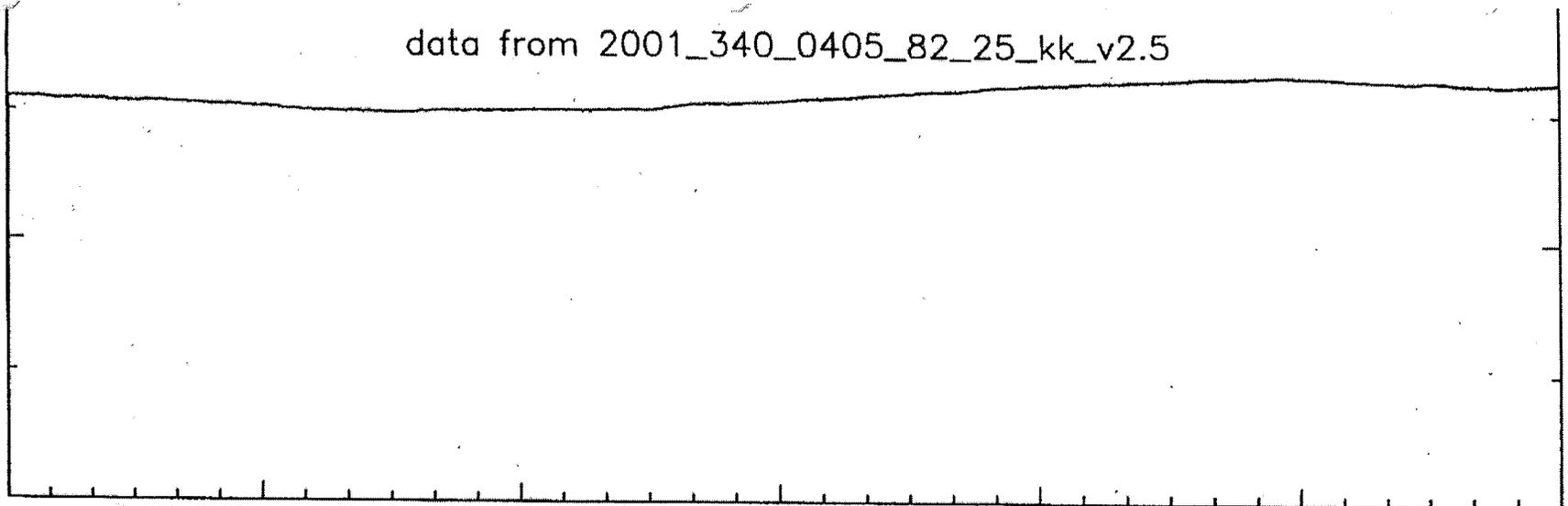
9

10

11

12

13



(mean removal only; ka-band normalization for y)

start time: 340 070142

stop time: 340 125821

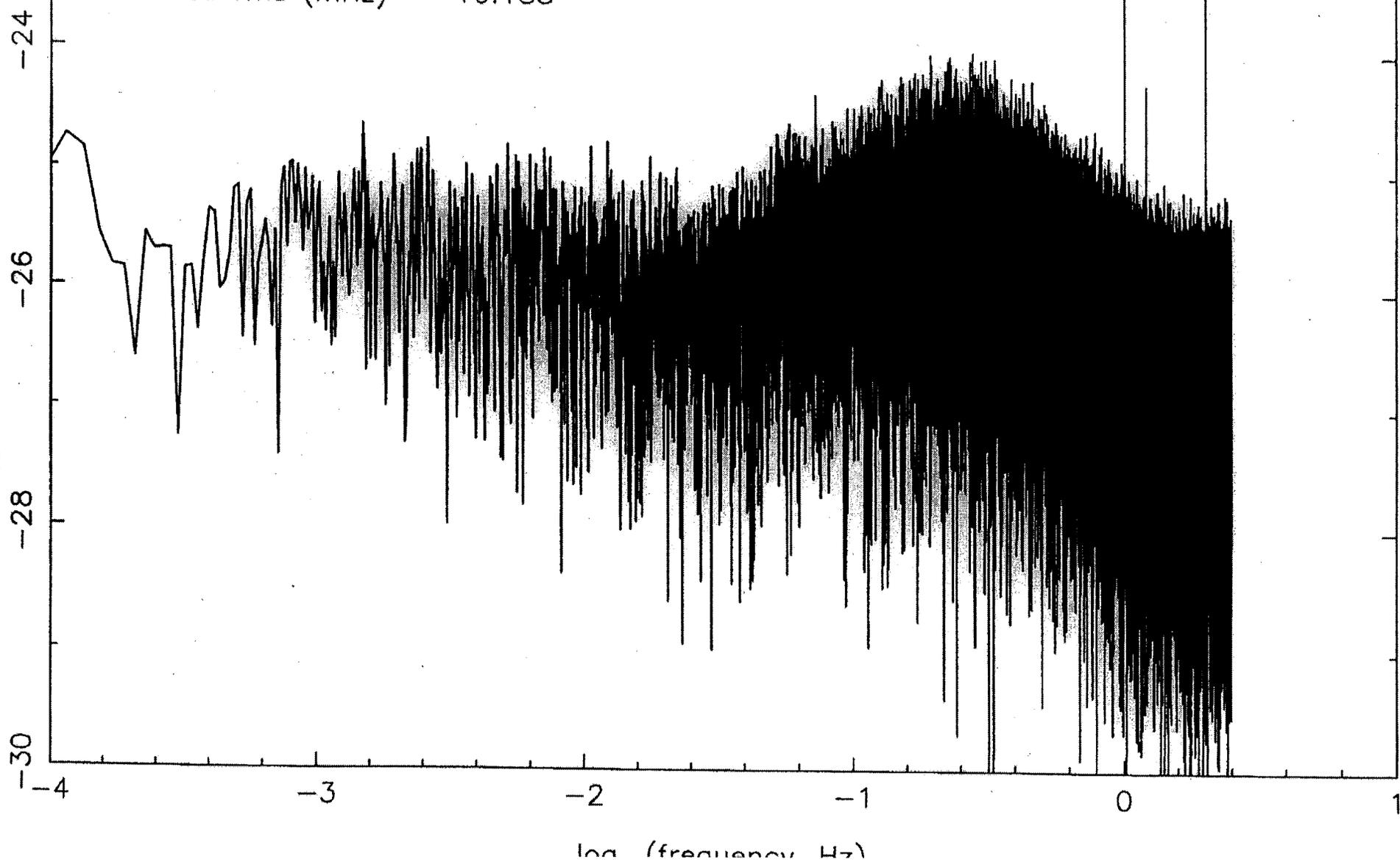
number of points: 107000

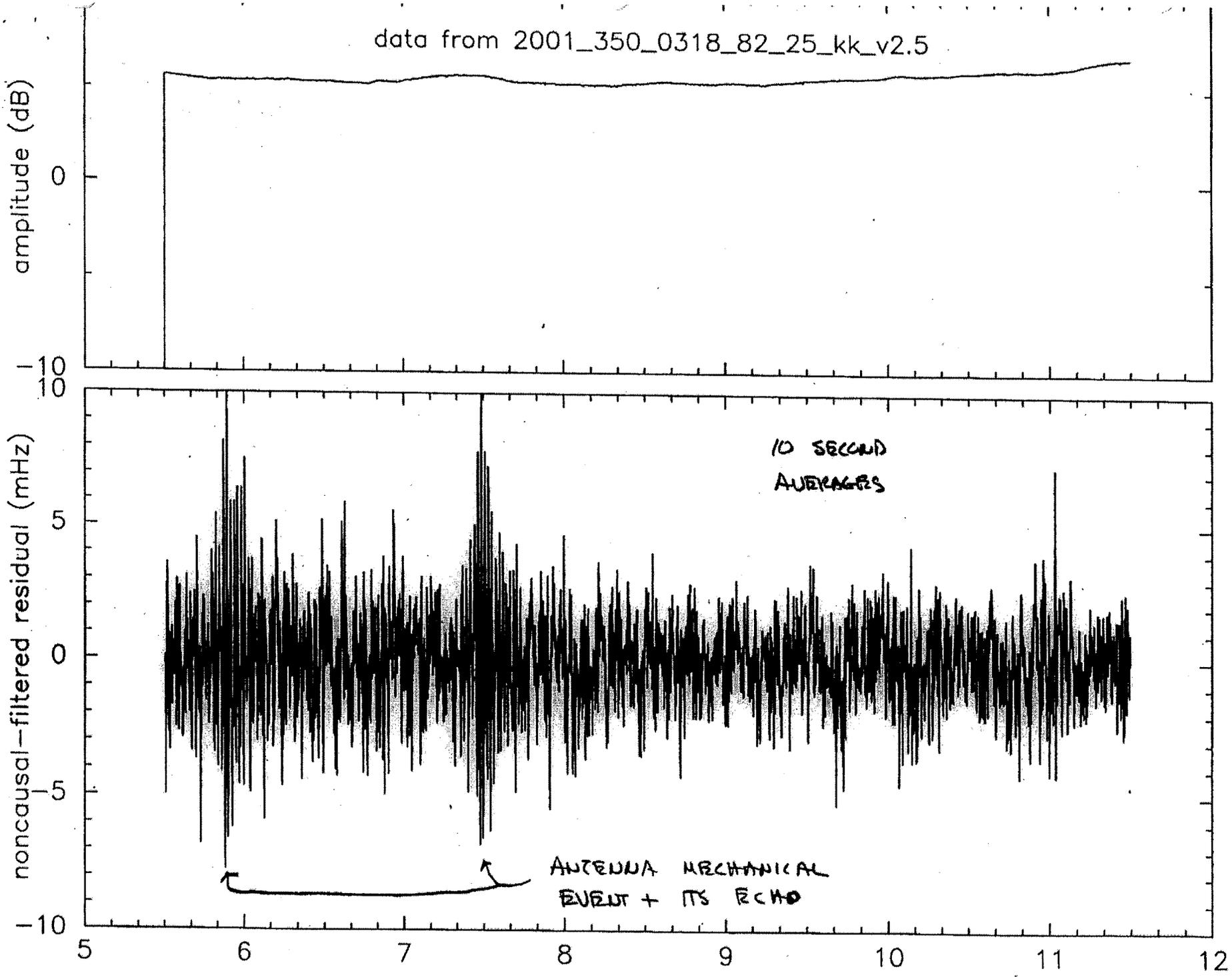
file88 rms (mHz) = 10.188

$\sigma_y(10 \text{ sec}) = 5.4\text{E}-14$

$\sigma_y(100 \text{ sec}) = 1.8\text{E}-14$

$\sigma_y(1000 \text{ sec}) = 5.5\text{E}-15$





(mean removal only; Ka-band normalization for y)

start time: 350 053142

stop time: 350 112821

number of points: 107000

file88 rms (mHz) = 10.566

$\sigma_y(10 \text{ sec}) = 6.8\text{E-}14$

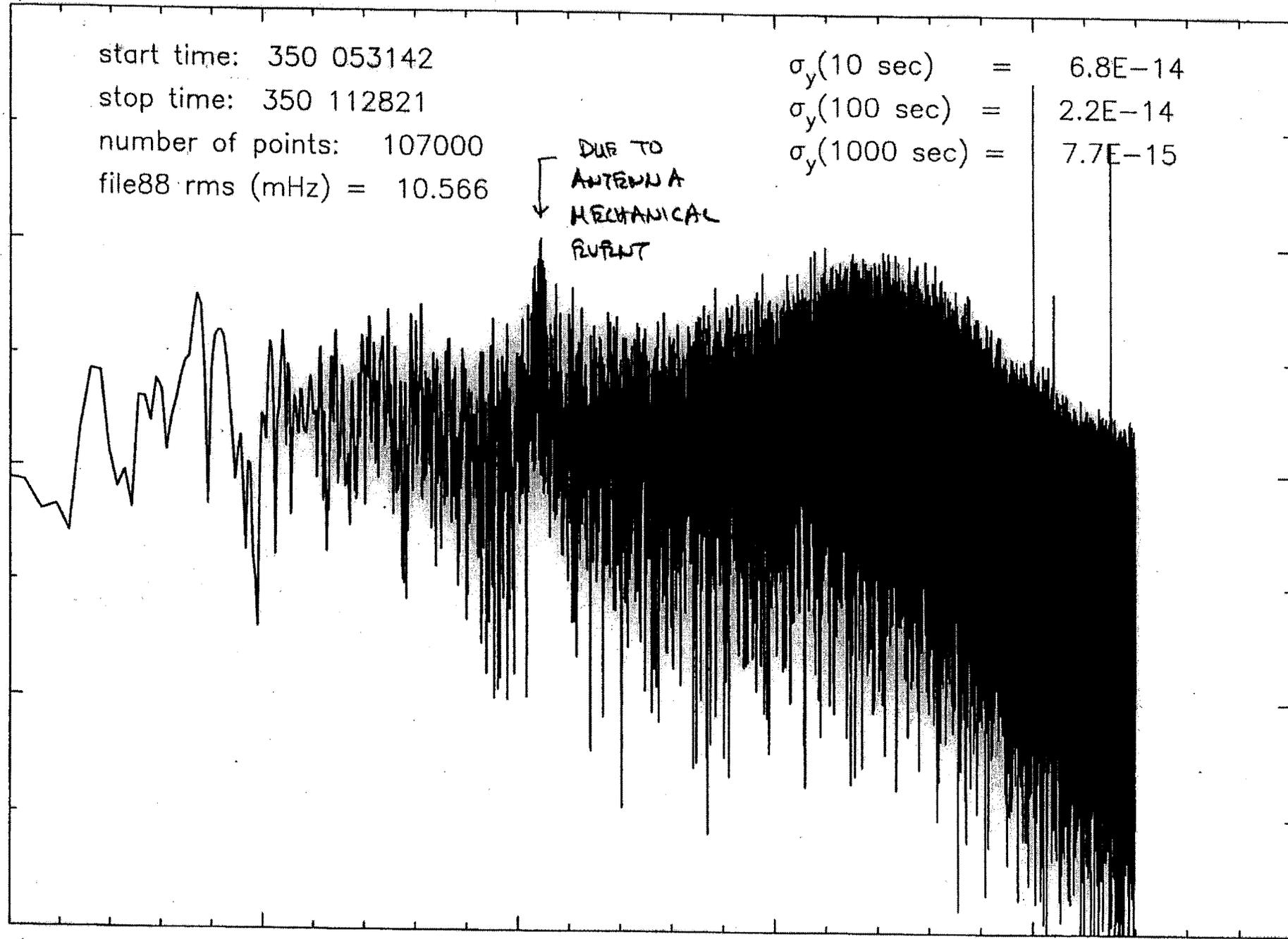
$\sigma_y(100 \text{ sec}) = 2.2\text{E-}14$

$\sigma_y(1000 \text{ sec}) = 7.7\text{E-}15$

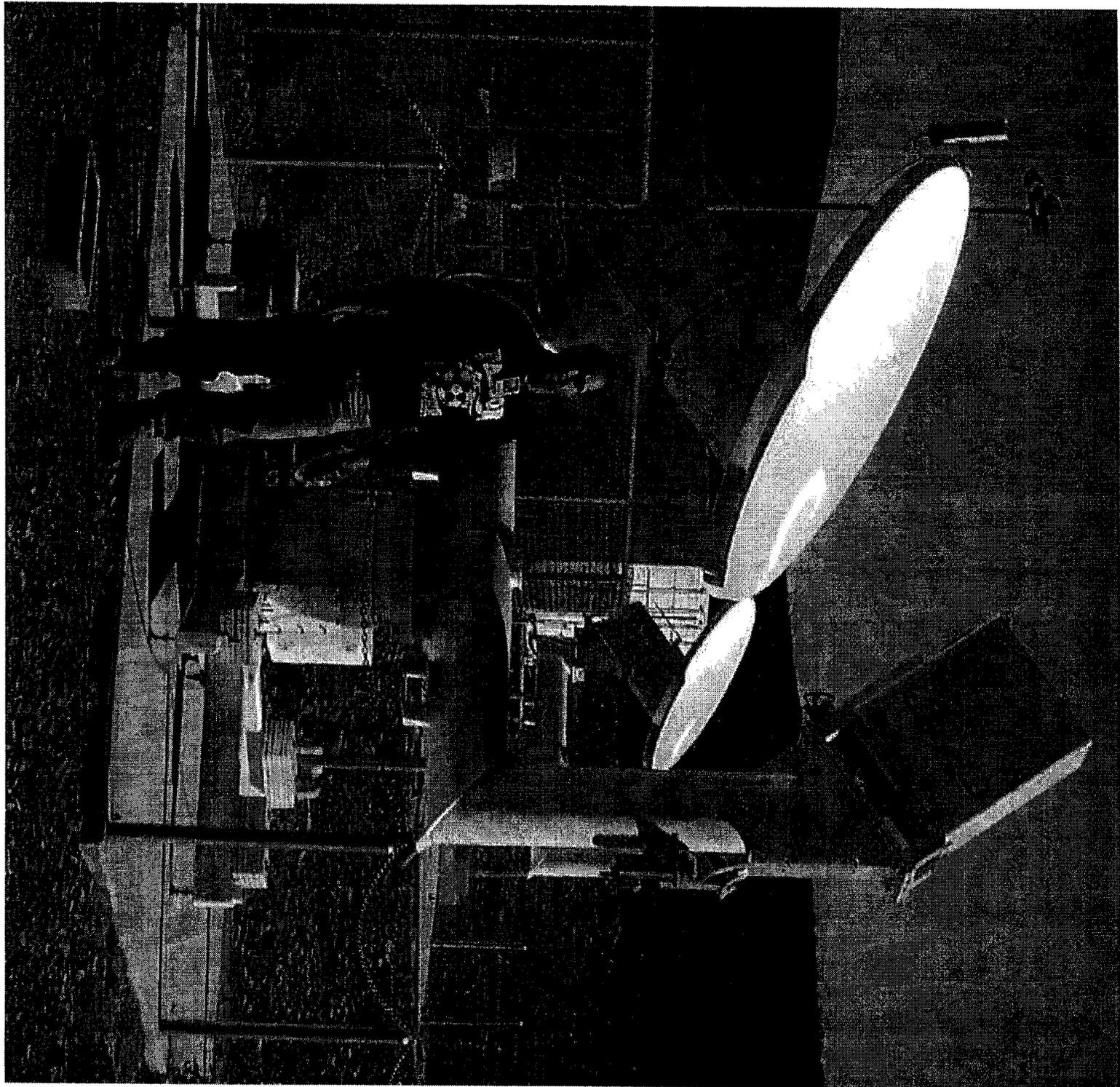
DUE TO
ANTENNA
MECHANICAL
EVENT

10y10 spectral density of y, 11z /

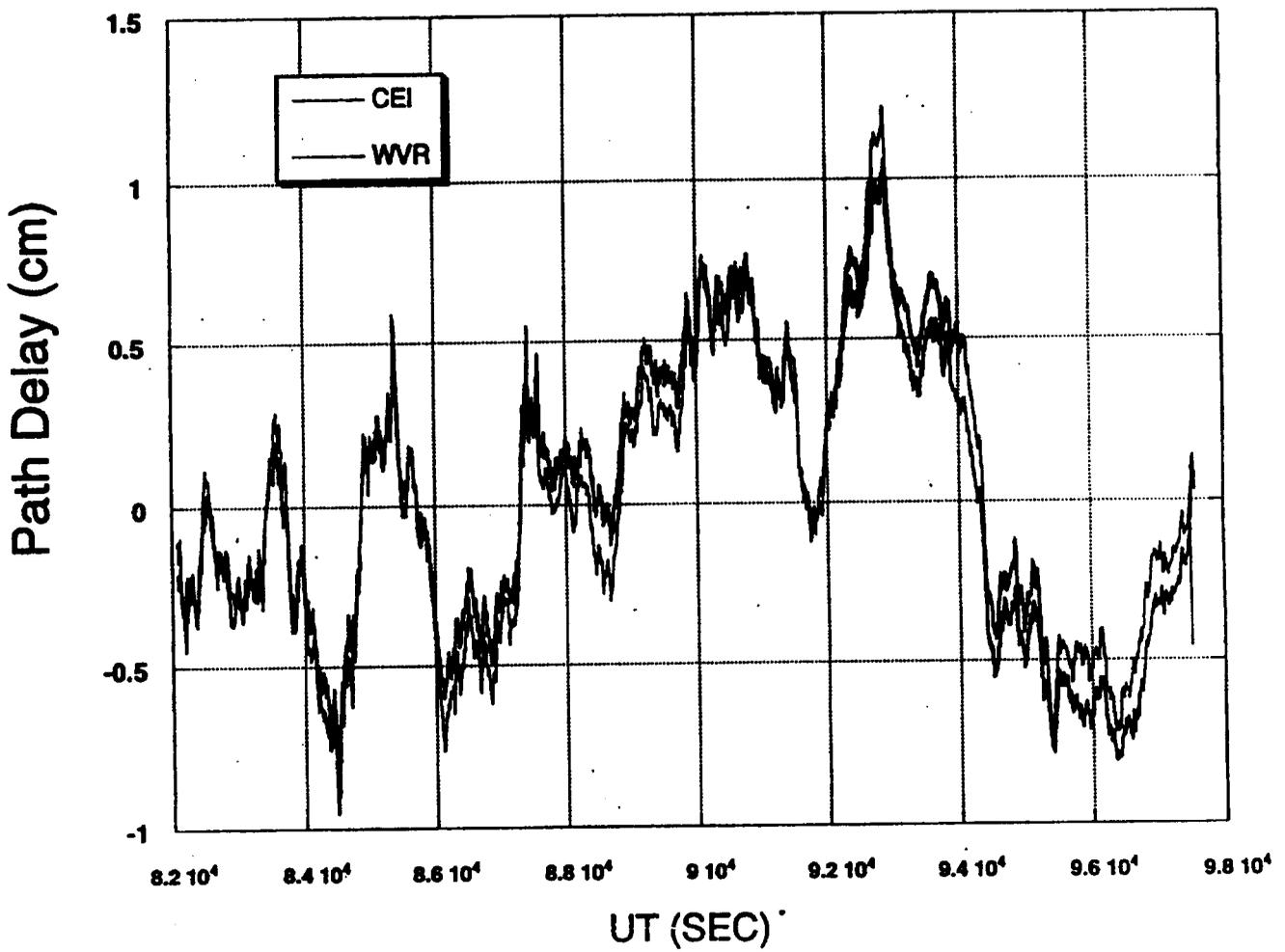
-22
-24
-26
-28
-30



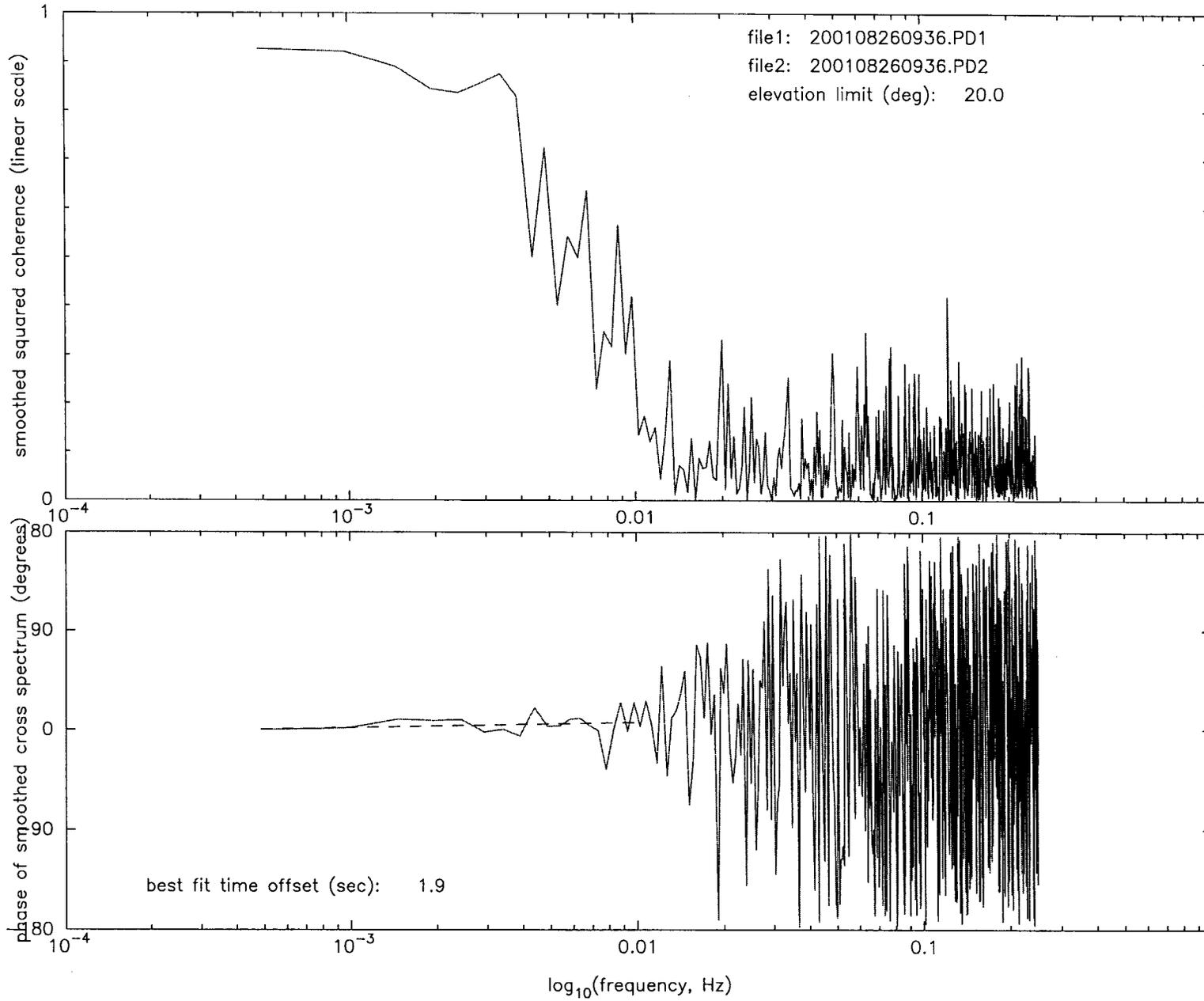
log (frequency Hz)



WVR-CEI Comparison DOY 138, 2000

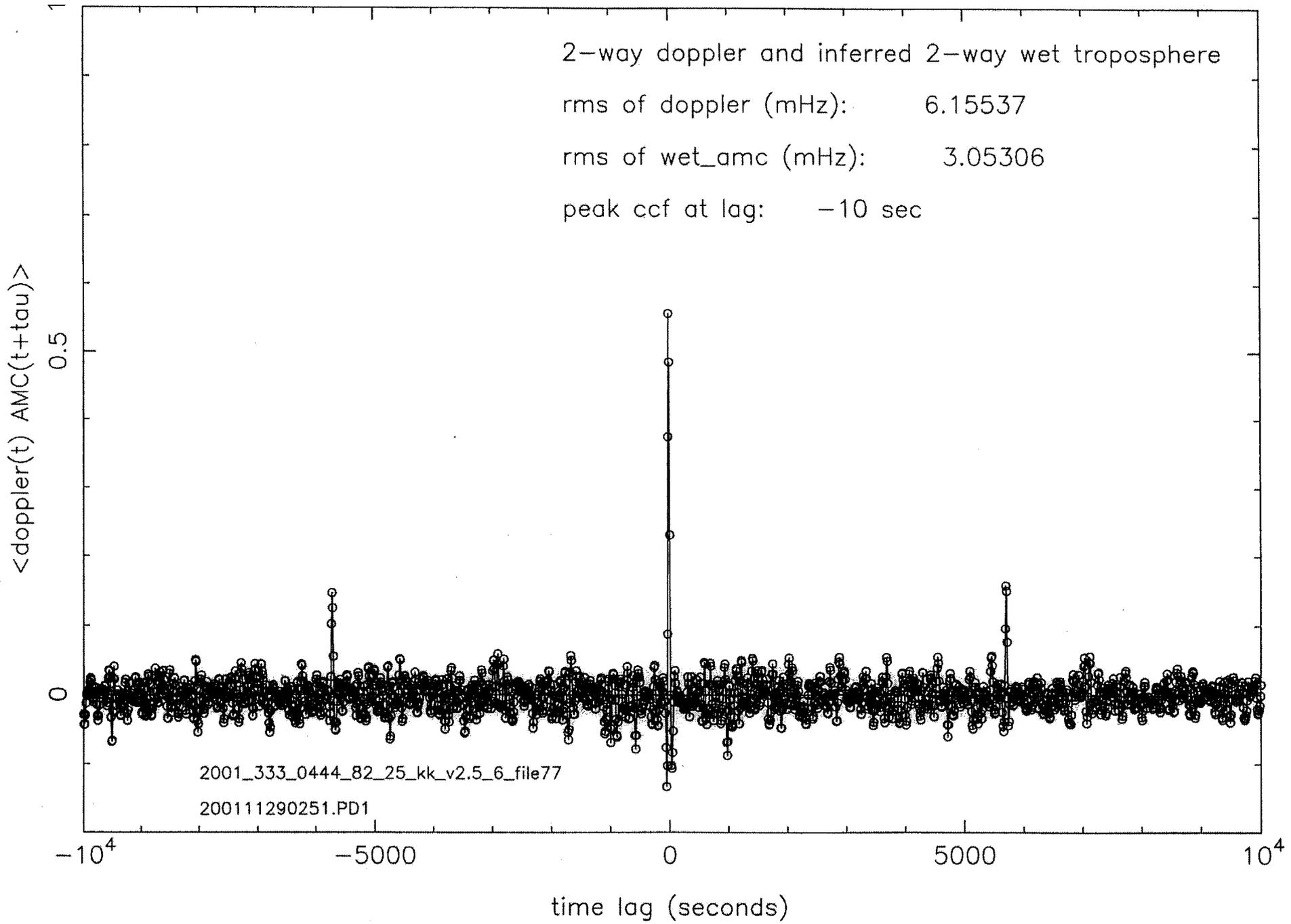


start time: 238 093600



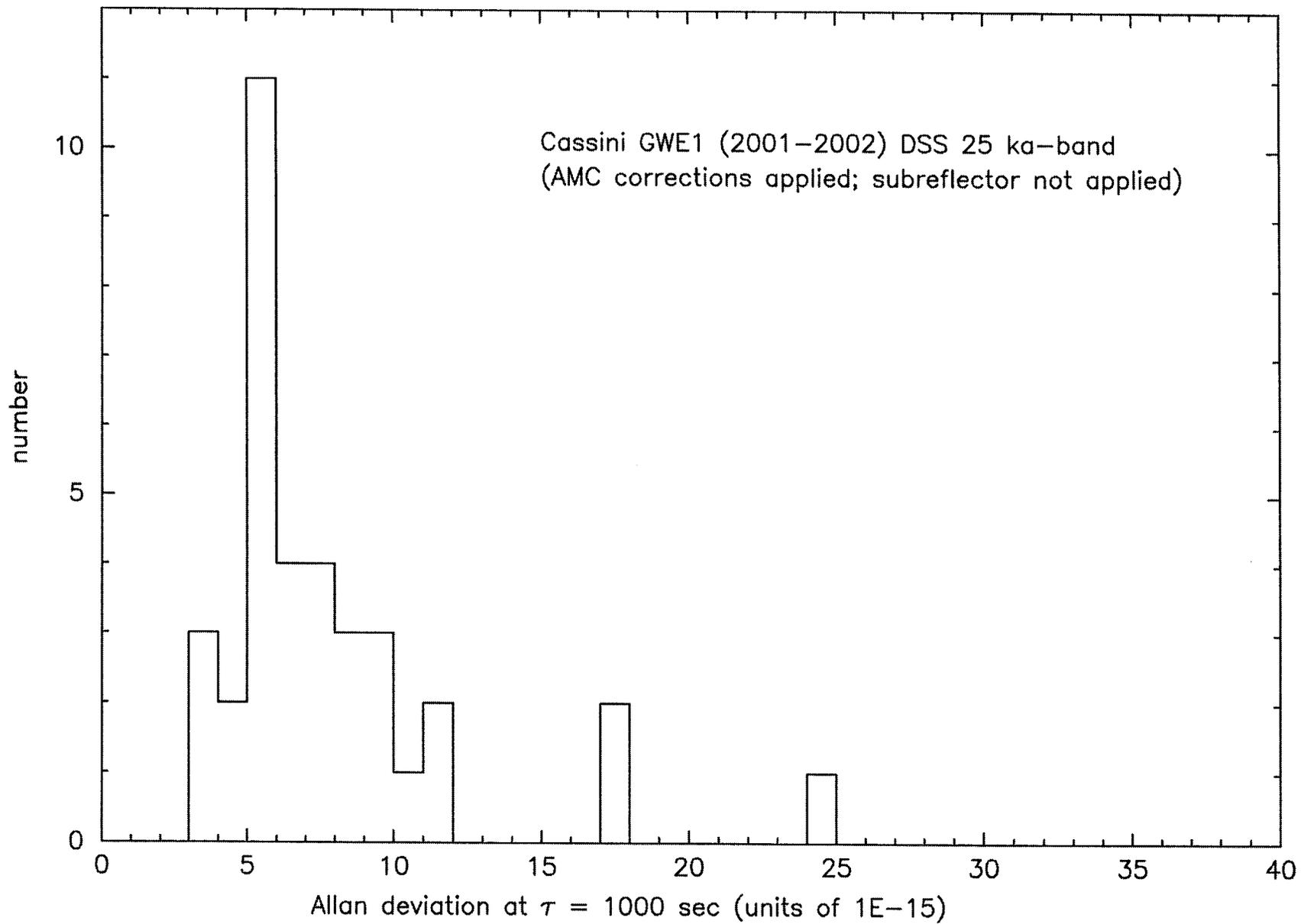
Correlation Between Doppler and WVR Data

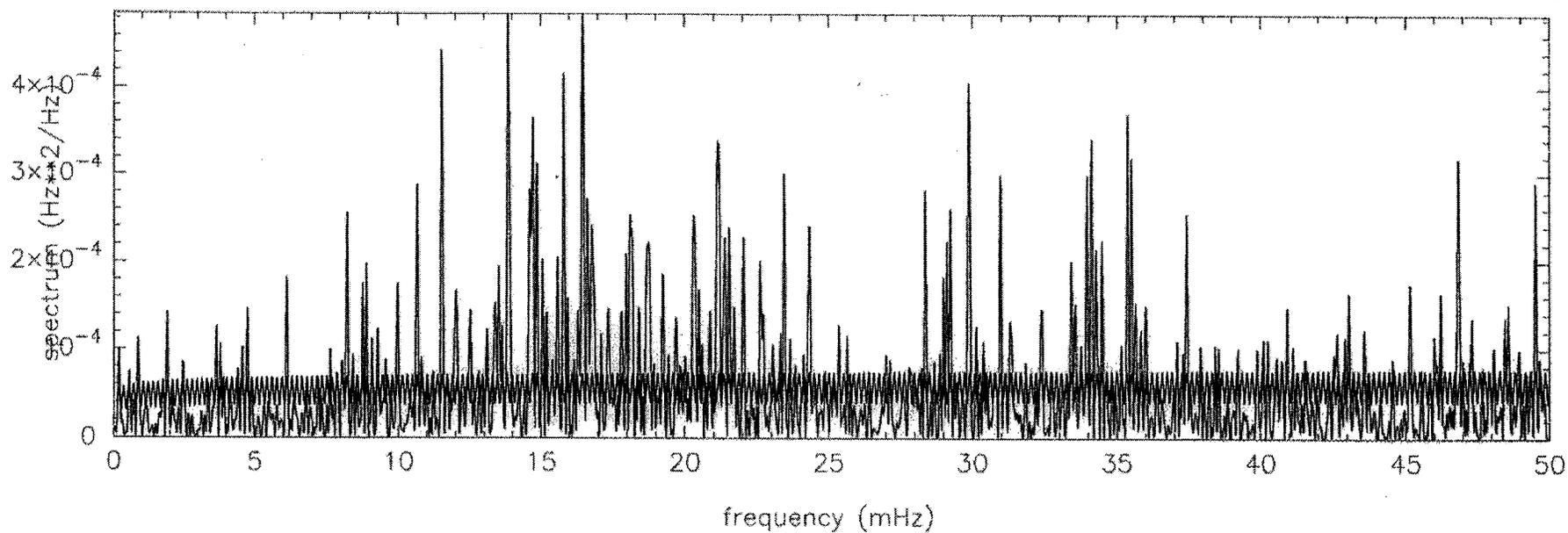
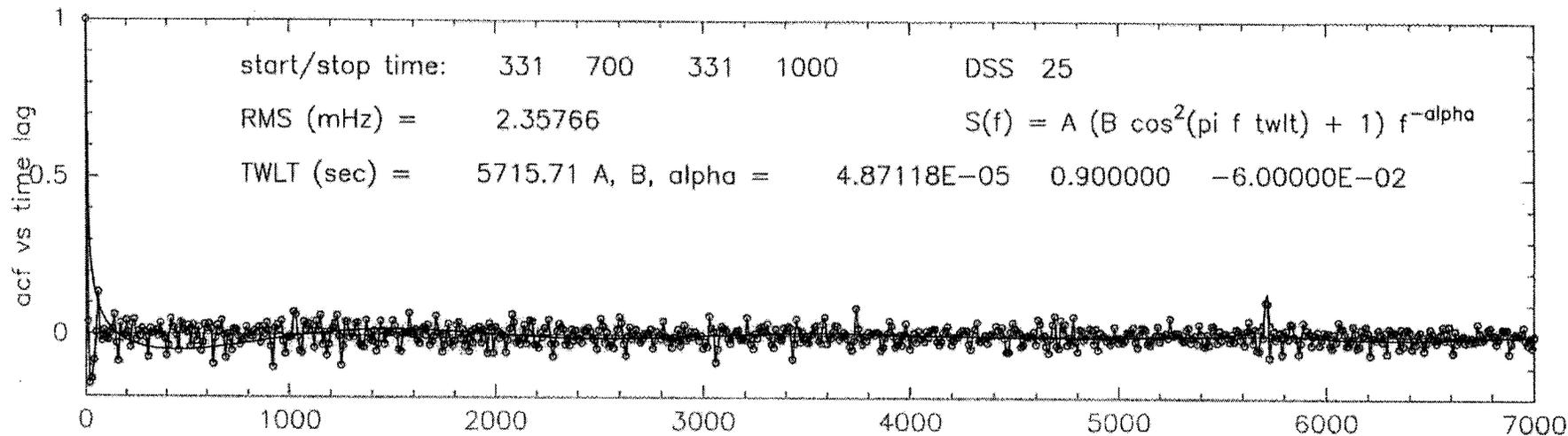
- **AMC data used here are only the WVR data**
- **Coherence of two AMC units usually high at Fourier frequencies of interest**
- **From RSST, and after careful review by S. Keihm, wet path delay at zenith as a function of time processed to 24 sec averages**
- **Differentiated path delay time series to get velocity; multiplied by (f/c) to get frequency fluctuations; mapped to LOS; interpolated to 10 second centers to compare with the Doppler ("superfile" format is 10 sec averages); offset by two-way light time and added to itself to give the signature in the Doppler**
- **Crosscorrelation functions of the Doppler and the inferred wet-component, 10 sec averages, illustrate the extent to which the AMC corrections will improve the Doppler**
- **Examples follow**



AMC Calibration

- **DSS25 Ka-band two-way data were corrected for wet-troposphere by subtracting the AMC correction from the Doppler data**
- **As a test: I allowed a scale factor in the AMC data to see if I got better residuals by subtracting (AMC correction) x (solved-for constant) from the data. Result: scale factors were almost always close to unity, so this was abandoned.**
- **Histogram of Allan deviation at 1000 seconds of high-elevation-selected data after wet-component corrections for 38 DSS25 tracks follows**
- **Discussion**
 - **Allan deviation improvement was large in some cases**
 - **Typical improvements of the high-elevation data are 1.5-2X, roughly consistent with CEI testing experience by Resch *et al.***
 - **Residuals after correction have TWLT correlation structure and spectral level about right for antenna mechanical motion (see example spectra)**
 - **This residual correlation has not been studied in any more detail (i.e., to see if cause is wind, liquid water in the beam, etc.)**
- **AMC worked as advertised**





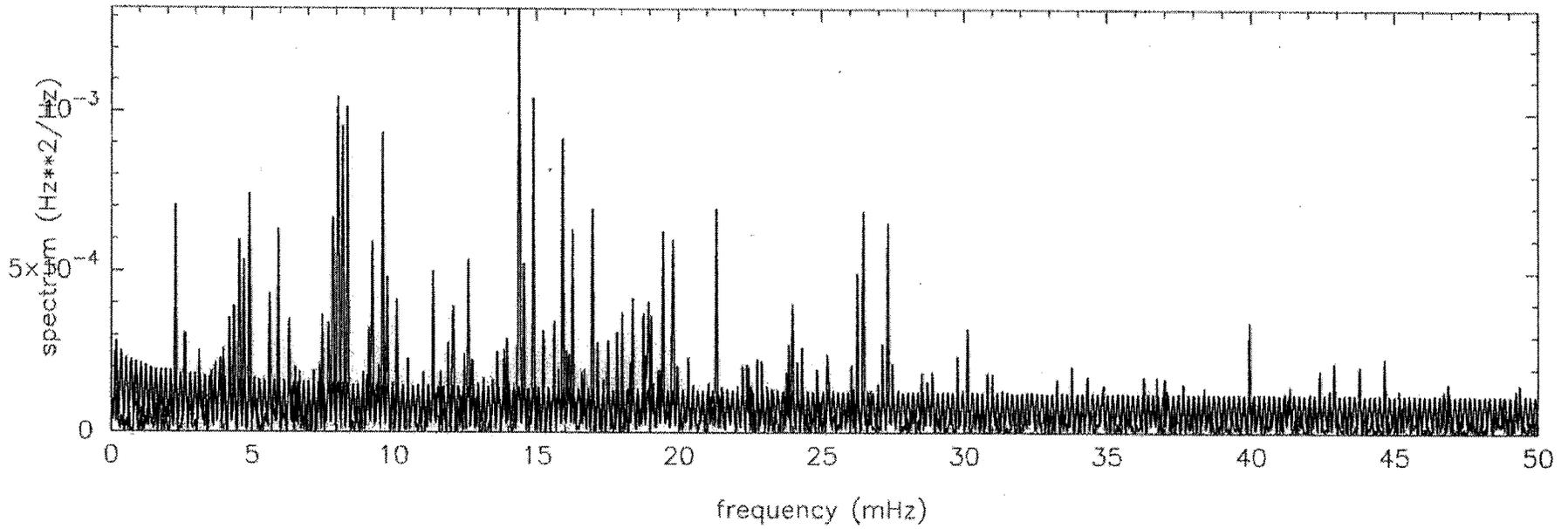
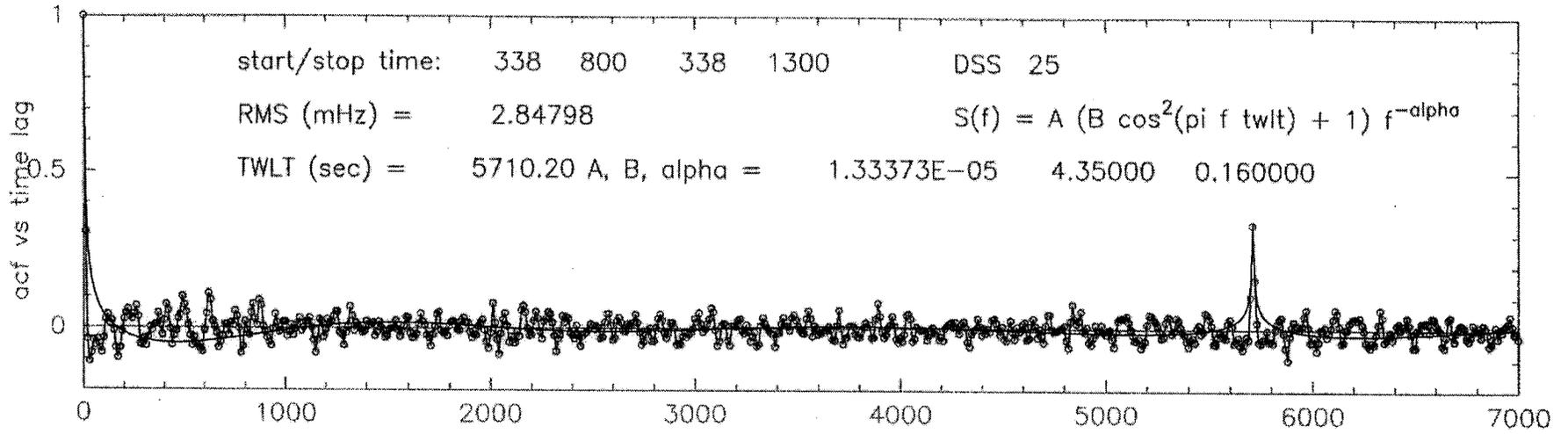


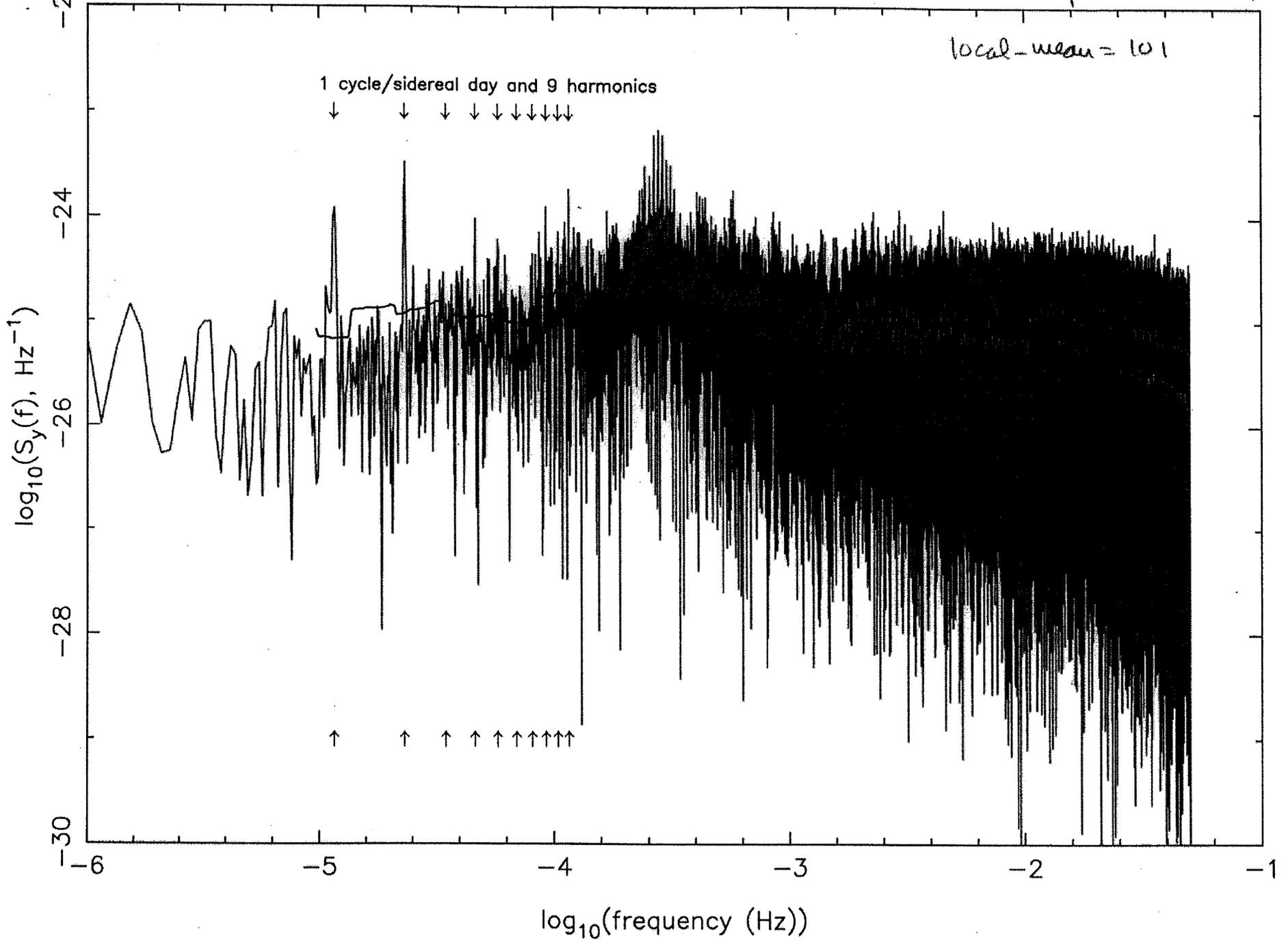
Table 1. Noise budget for the Cassini experiment. The main noise sources are discussed in the text. The noise levels are characterized here by the Allan deviation (square root of Allan variance) at an integration time of 1000 seconds.

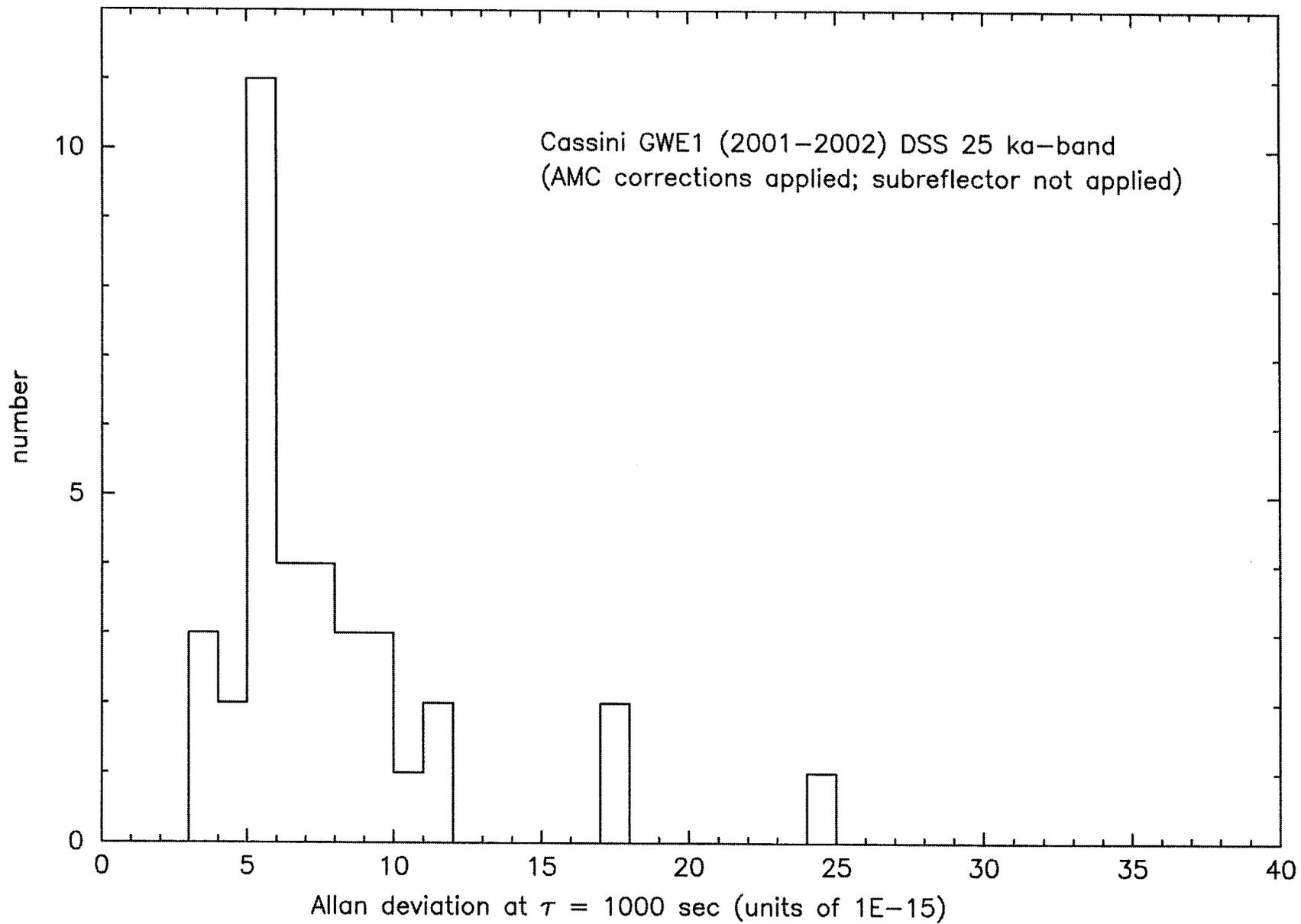
noise source	Allan deviation at 1000 sec (2-way)	comments/references
FTS (including distribution)	$\approx 1\text{E-}15$	<ul style="list-style-type: none"> • frequency and timing system (FTS) is small but fundamental noise source in Doppler experiments
antenna mechanical noise	$< 4\text{E-}15$ (favorable conditions)	<ul style="list-style-type: none"> • DSS25 measured $3.6\text{E-}15$ under static conditions at elevation = 47 degrees (Rochblatt, Richter, and Ootshi 1997) • DSS13 measured $< 3.1\text{E-}15$ under static conditions at elevation = 37 and 46 degrees (Ootshi and Franco 1992) • DSS15, 45, 65 measured $< 1\text{E-}14$ under operational conditions with antenna moving (Armstrong 1998)
ground electronics (exciter, xmitter, rcvr—excludes FTS)	$2.3\text{E-}16$	<ul style="list-style-type: none"> • DSS25 test data with antenna static, 2001 DOY 152-153, taken by S. Abbate
plasma phase scintillation at Ka-band	$< 1\text{E-}15$ for sun-earth-spacecraft angles greater than 160 deg	<ul style="list-style-type: none"> • Armstrong, Woo, and Estabrook (1979) with S-band scaled to Ka-band by wavelength-squared • Armstrong (1998)
spacecraft motion	$2.6\text{E-}16$	<ul style="list-style-type: none"> • Won et al. (2001) • A. Lee (private communication 2001)
KaT noise	$< 1.7\text{E-}15$	<ul style="list-style-type: none"> • prelaunch tests (960611, presented at review 960627) show $\approx 1\text{E-}16$ with -127 dBm signal input level • in-flight determination by L. Iess of $< 1.7\text{E-}15$ [e-mail of 020711] using method of Bertotti, Comoretto, and Iess (1993) to isolate KaT
raw tropospheric scintillation	$< 3\text{E-}15$ to $30\text{E-}15$ (Goldstone winter; highly variable)	<ul style="list-style-type: none"> • Keihm (1995) [year-long WVR observations at Goldstone] • Armstrong and Sramek (1982) [$< 1\text{E-}15$ to $8\text{E-}15$ for elevation angles > 20 degrees at the VLA]
tropospheric scintillation after AMC calibration	$< 1.5\text{E-}15$ (favorable conditions) $\approx 3.2\text{E-}15$ (median conditions observed during AMC/CEI tests)	<ul style="list-style-type: none"> • Resch et al. (2002), comparing AMC and connected element interferometer (CEI) data; median improvement after applying AMC corrections was factor of 2.8

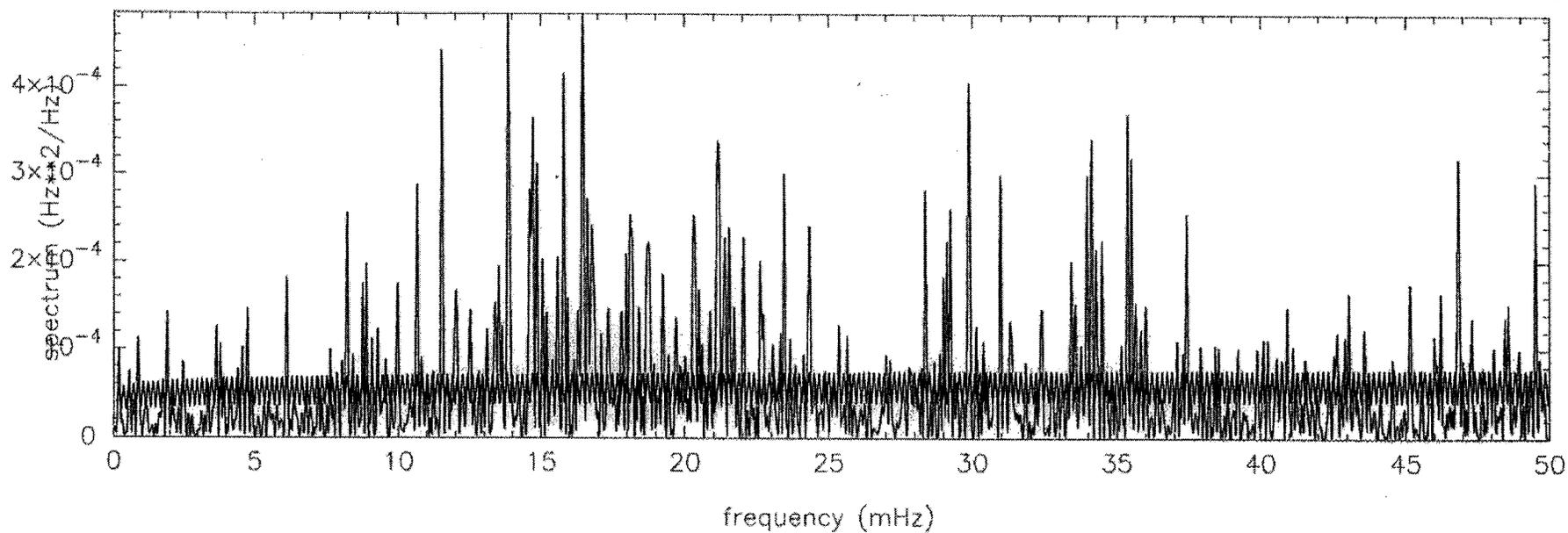
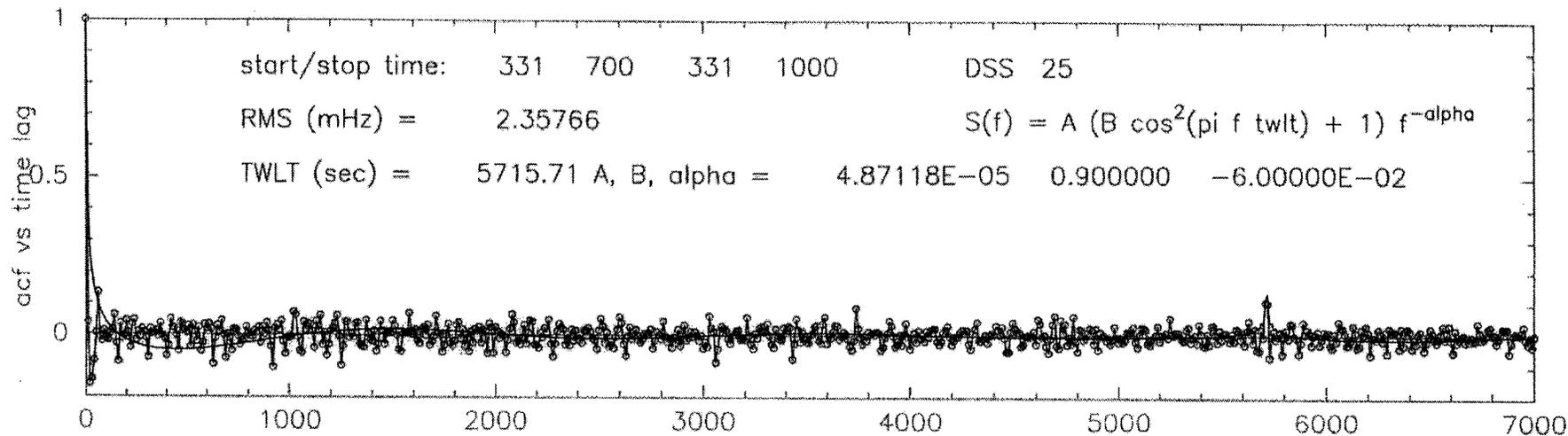
Cassini GWE1 (2001-2002)

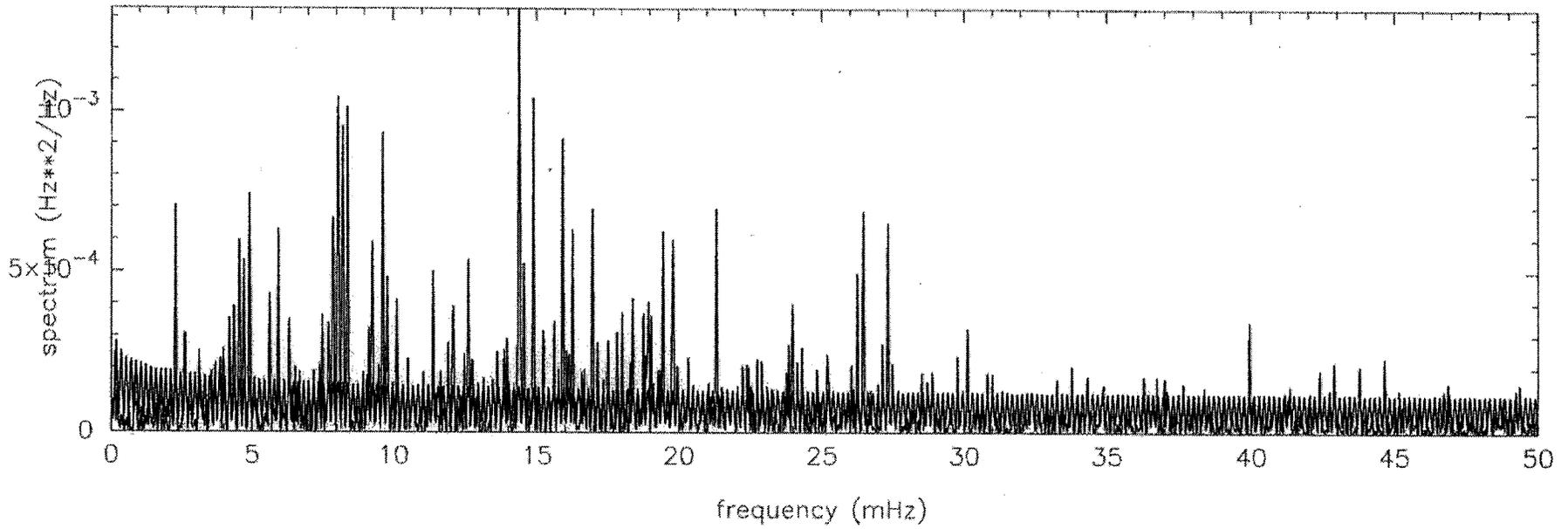
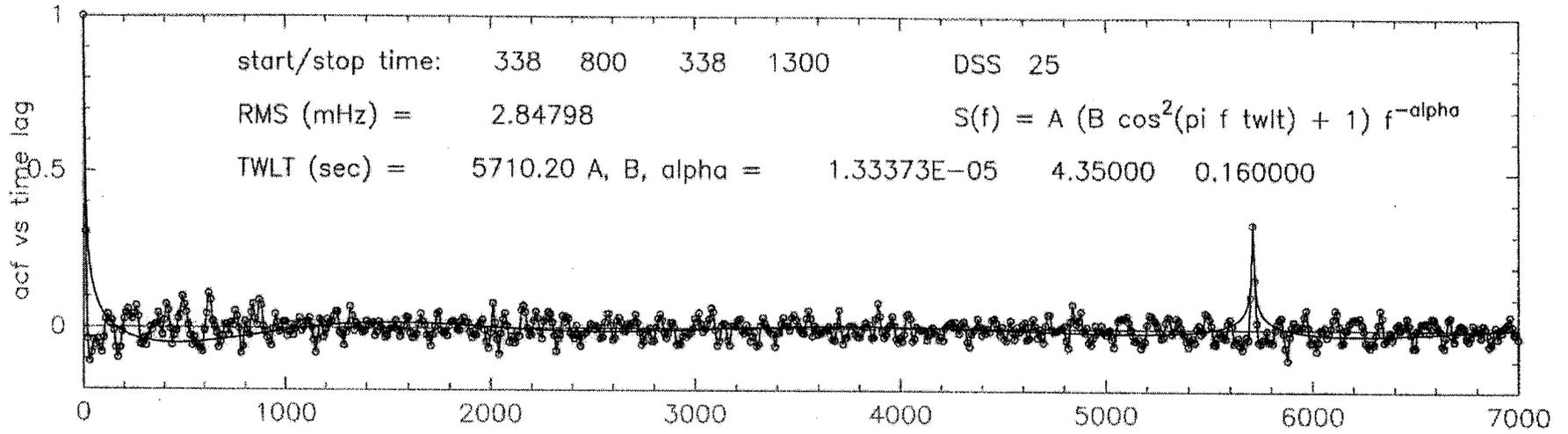
edited - super file (021002)

local-mean = 101





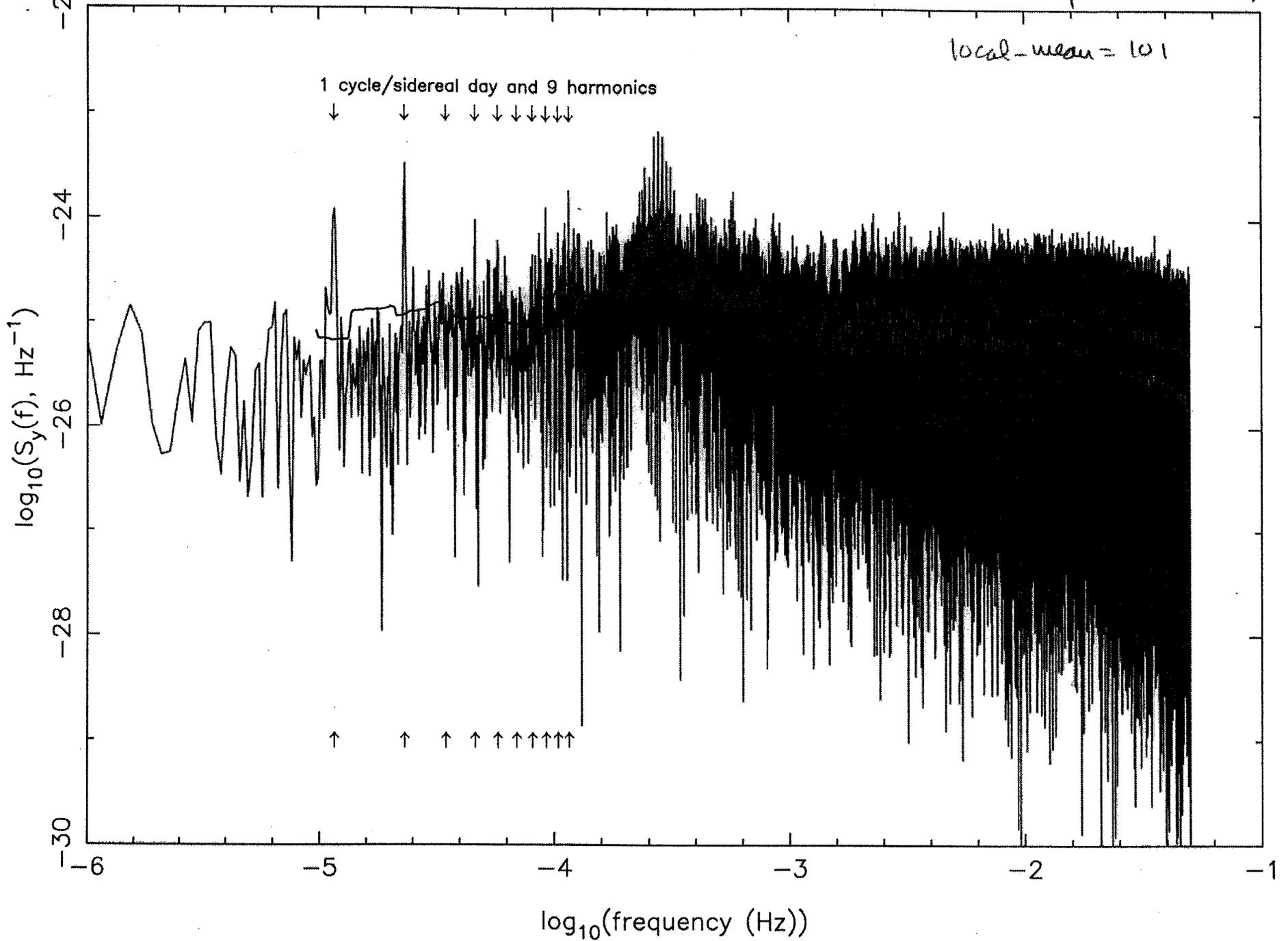




Cassini GWE1 (2001-2002)

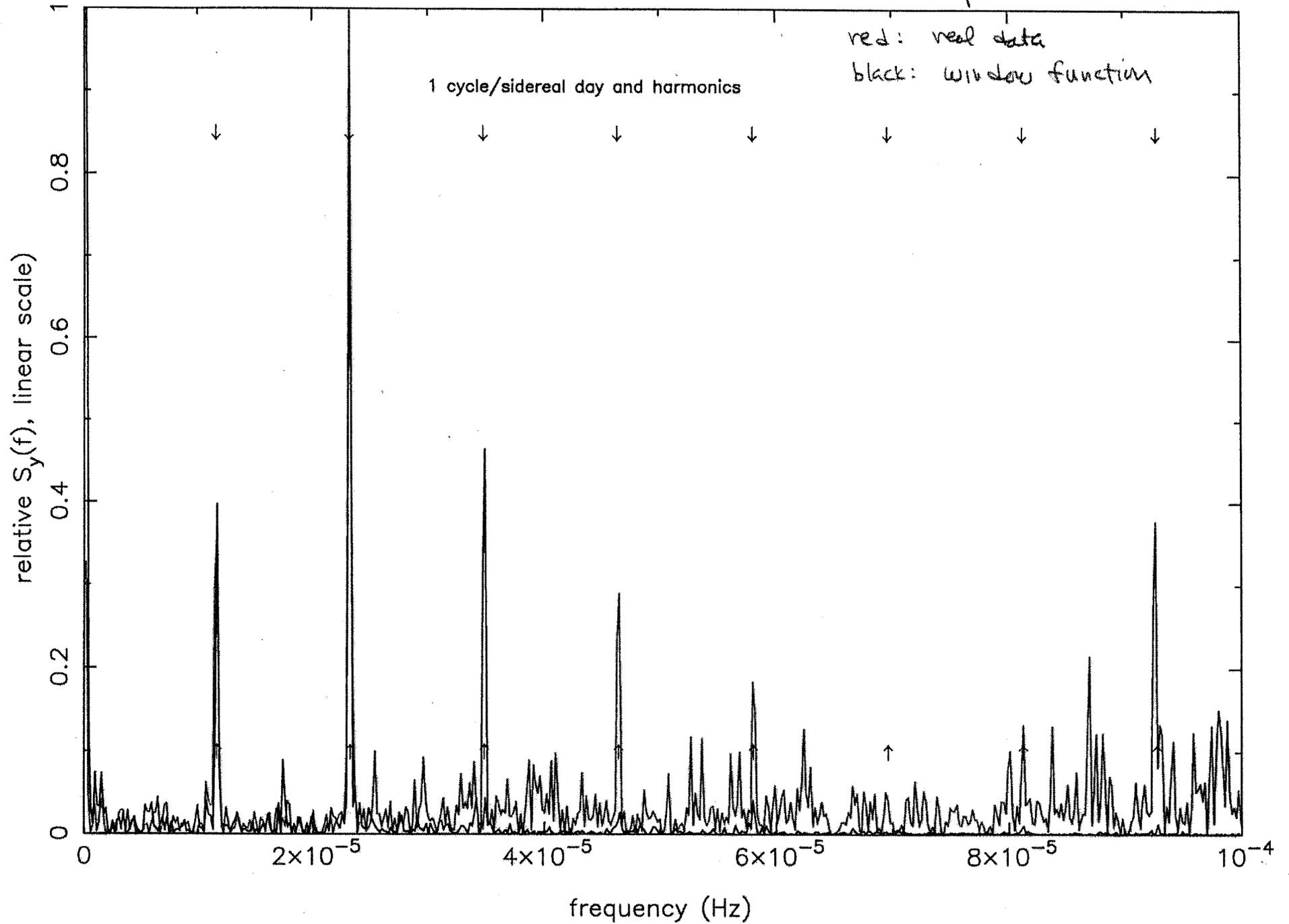
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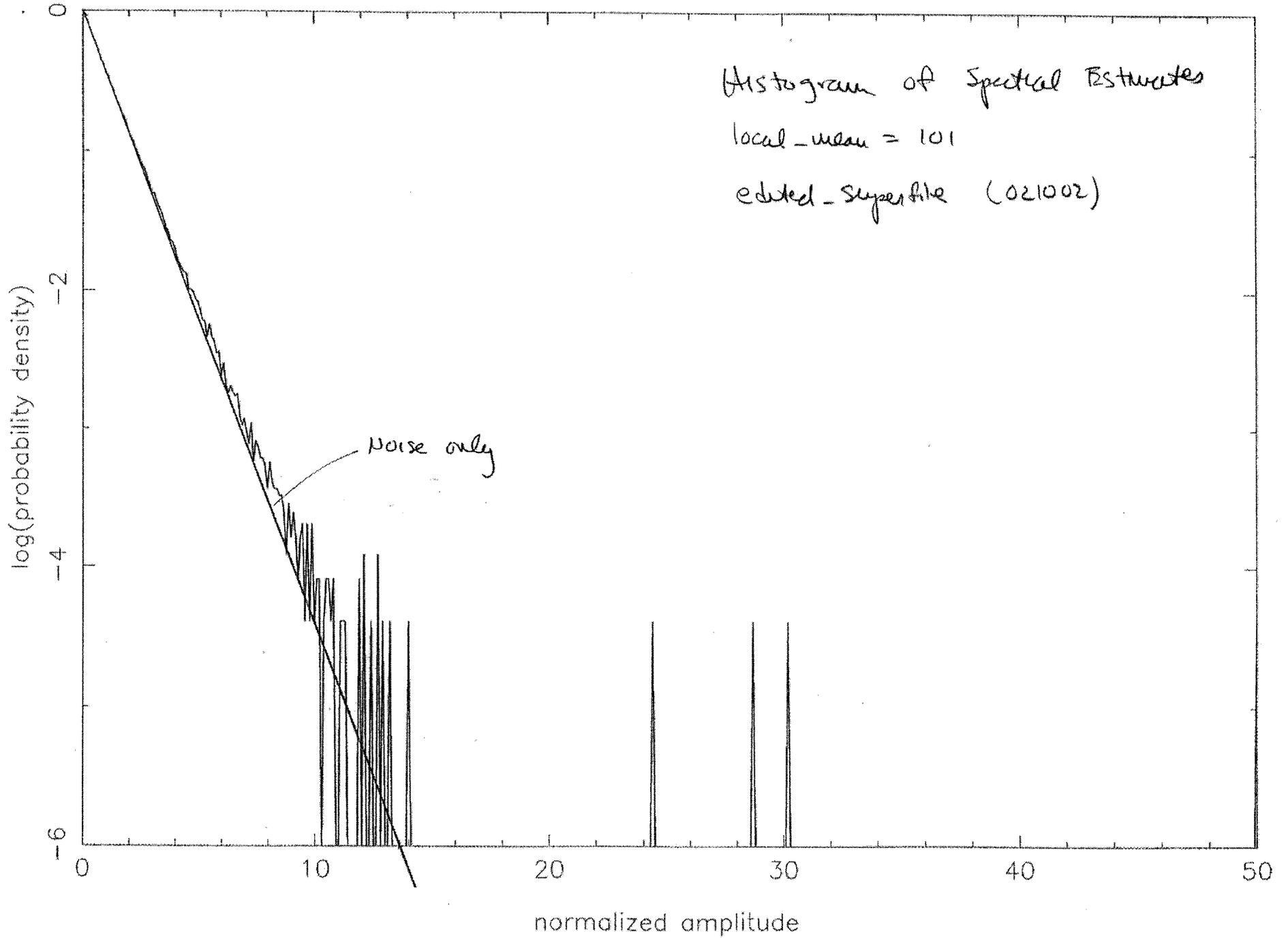


Cassini GWE1 (2001-2002)

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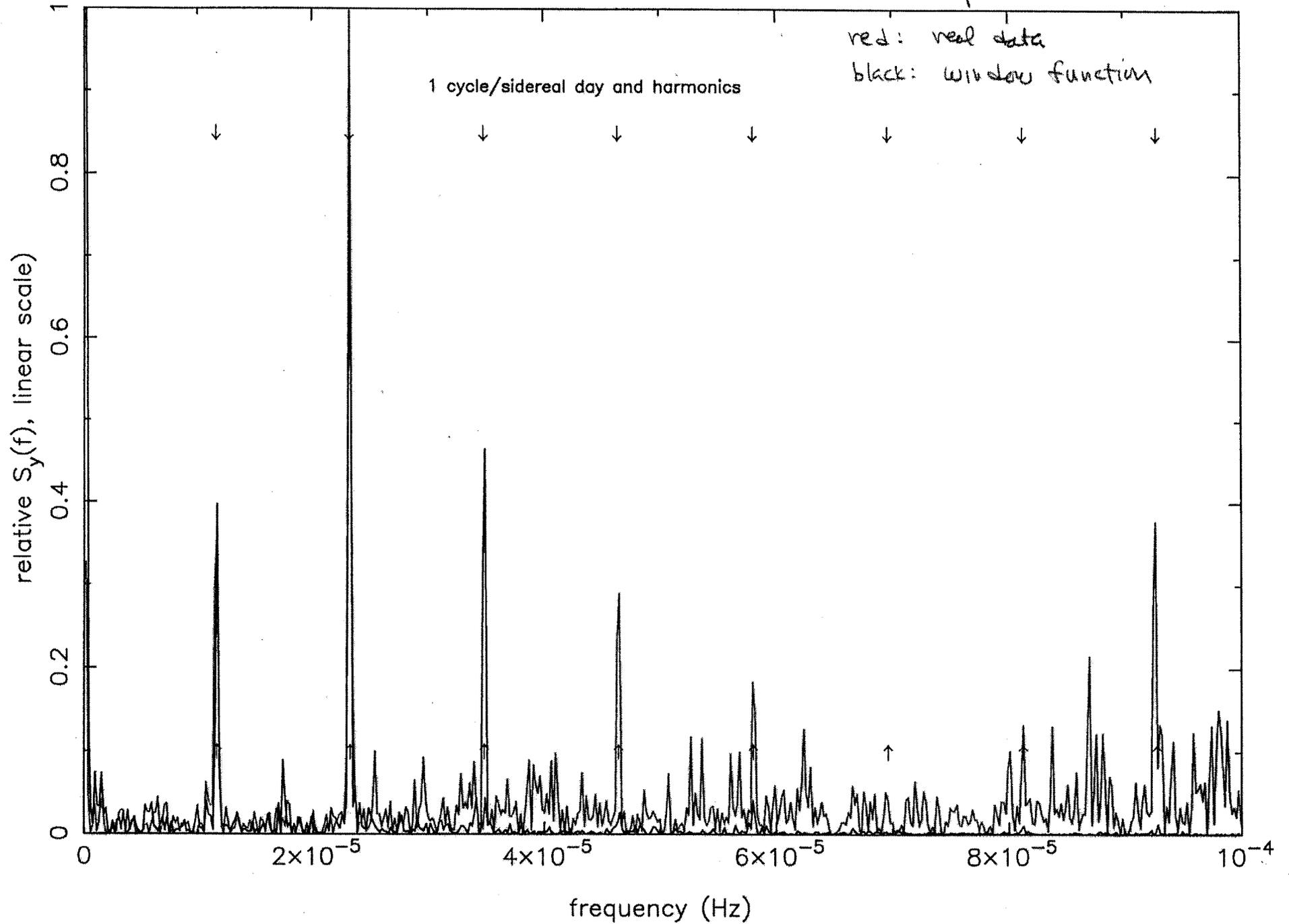


Cassini GWE1 (2001-2002)

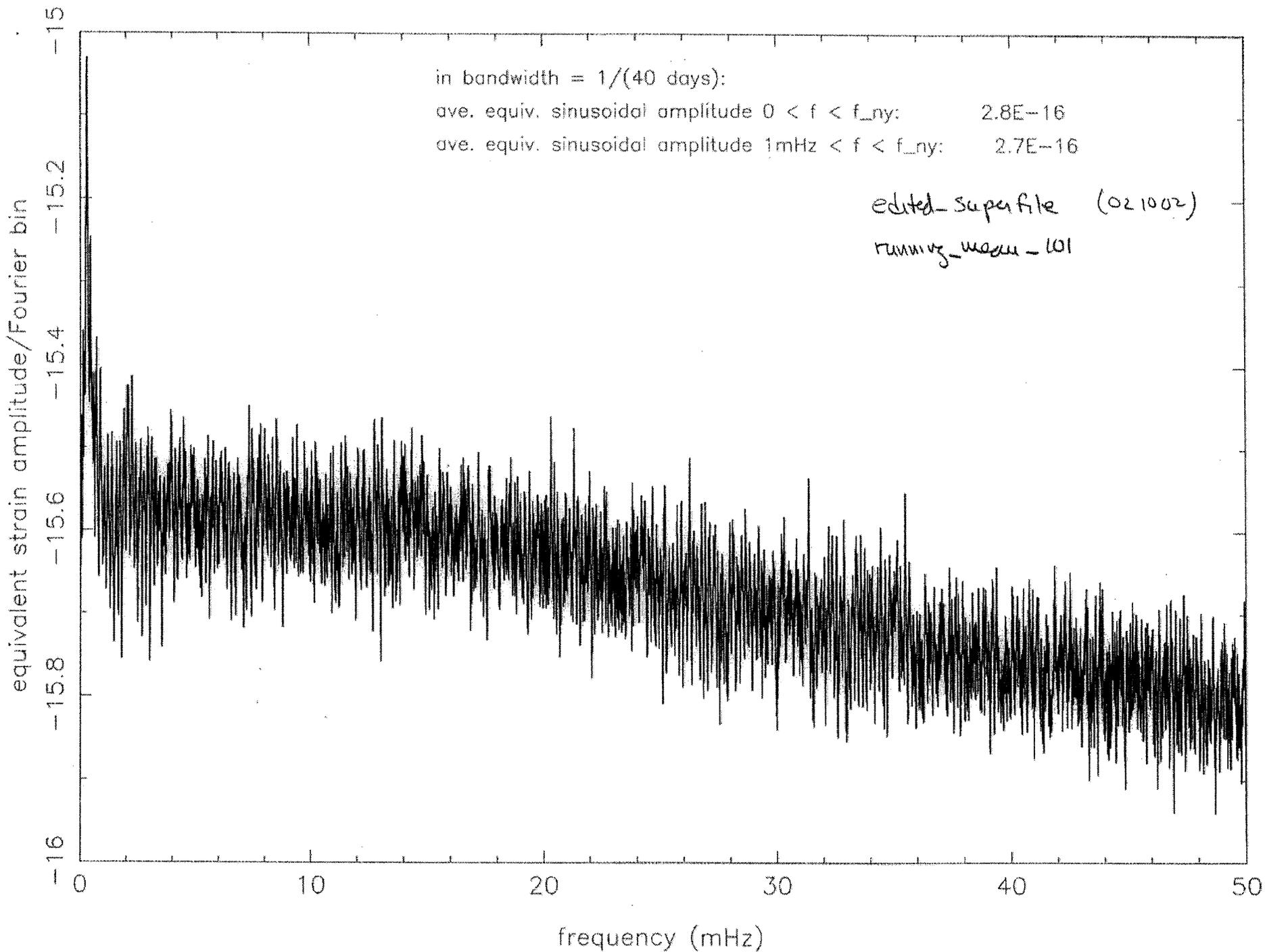


Cassini GWE1 (2001-2002)

edited_superfile (021002)



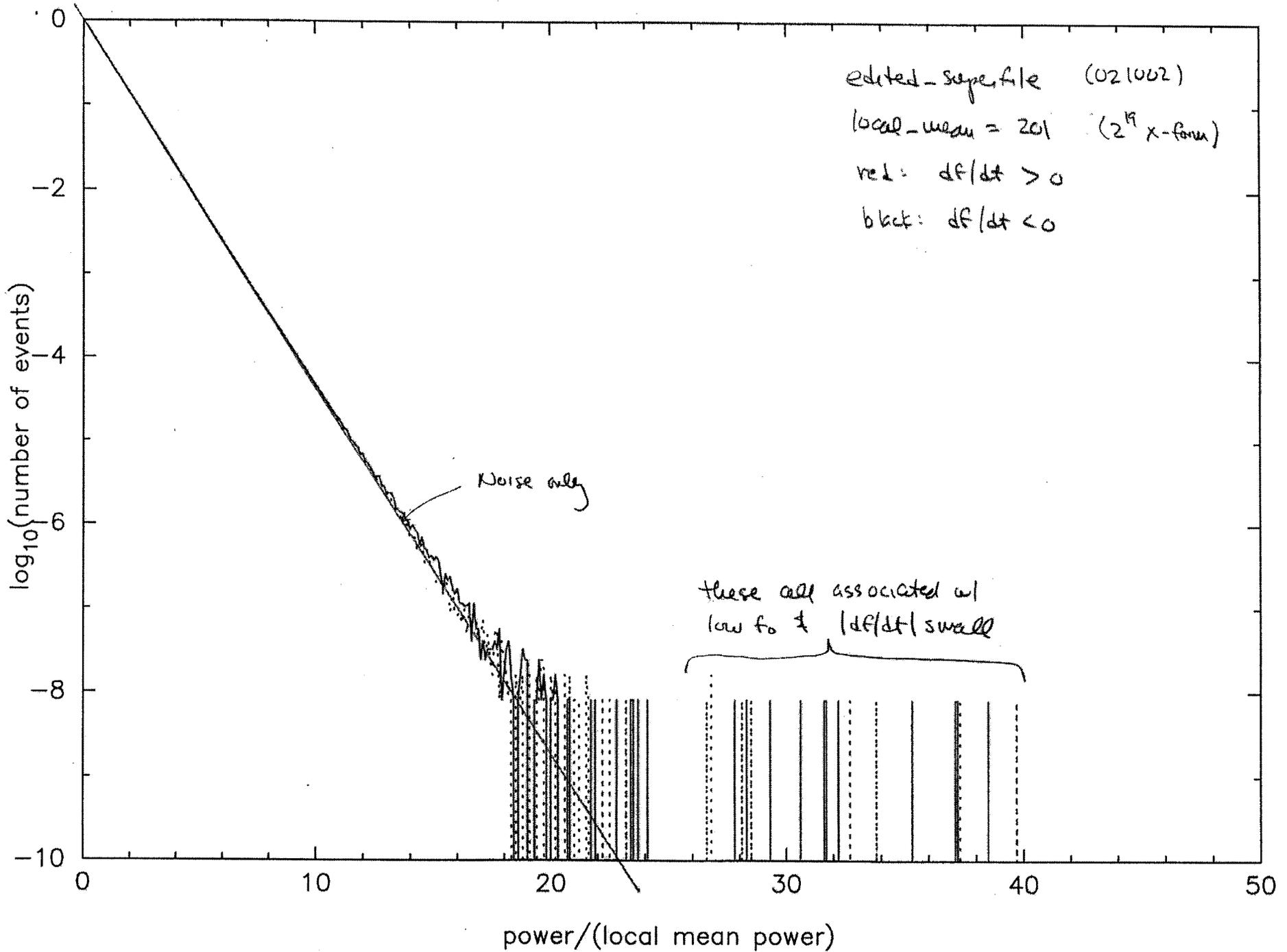
Cassini GWE1 (2001-2002)



Steps in Suboptimal Linear Chirp Analysis

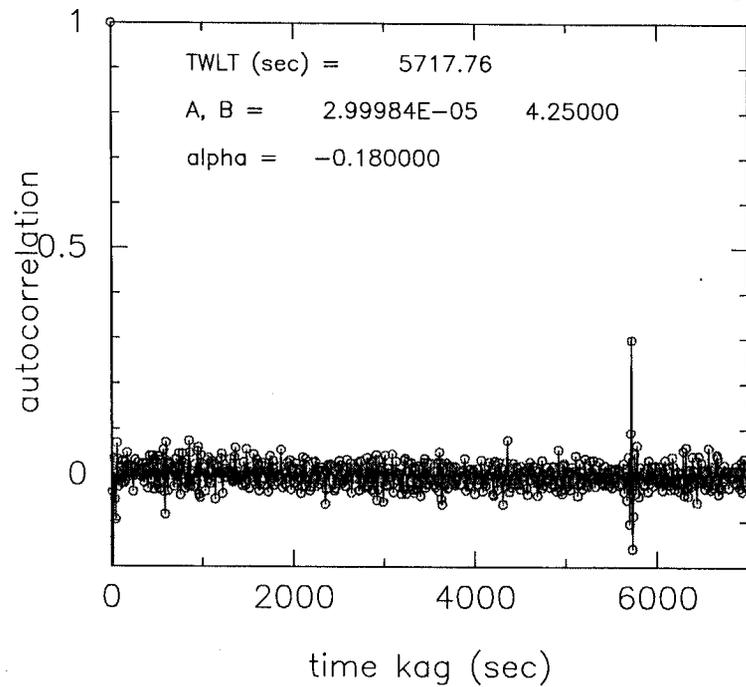
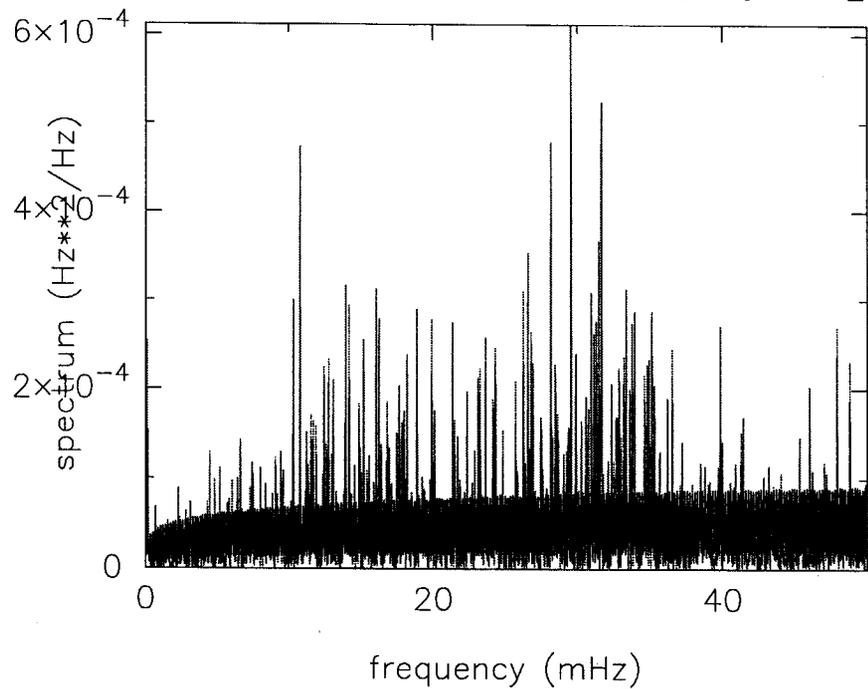
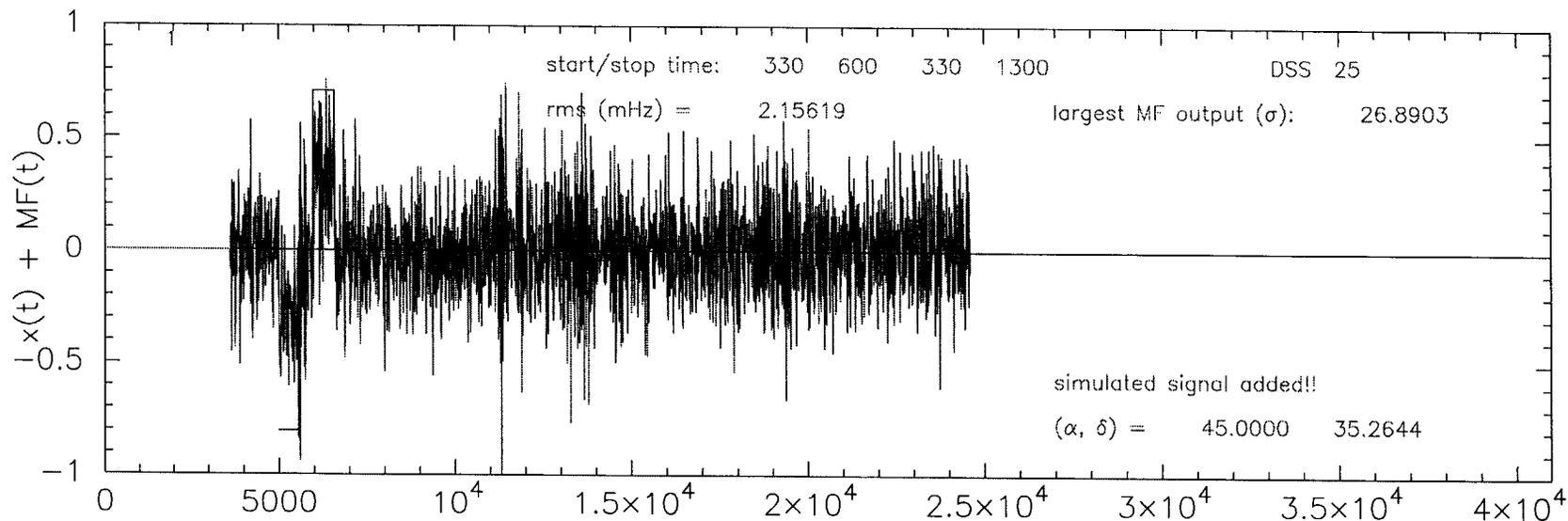
- **Objective:** identify candidates that, with proper processing for direction and $T_2(t)$ could give interestingly large outputs
- See e.g. Tinto and Armstrong ApJ 372, 545 (1991). Assume classical coalescing binary in astrophysically clean system with start frequency f_0 , duration of observation T , and Newtonian time to coalescence τ ; if $f_0 < 128 \tau^2 / (33 T^3)$ signal will look like a linear chirp
- Neglect slowly varying amplitude modulation and multiply time series by $\exp(-i \pi \beta t^2)$, where $\beta = df/dt$ is the assumed frequency derivative, to dechirp the time series
 - True linear chirp becomes, approximately, 3 lines, closely spaced
 - Subtleties when $f_0 \sim (T/T_2^2)$
- Spectrum of dechirped time series \rightarrow look for lines that are large compared with local mean power (determining the local mean power has subtleties, too)
- For free, you get both positive and negative frequency derivatives. Negative df/dt presumably non-physical, but useful as test
- Can redo the analysis restricting f_0 is some range (to avoid spectral lines that are near 1 cycle/day and harmonics)
- Tests: simulated noise only time series w/ and w/o gaps, original time series with scrambled Fourier phases, etc.
- Analysis here: 5000 df/dt 's ranging from 0 to 10 $\mu\text{Hz}/\text{day}$, perhaps 25% of these uncorrelated

suboptimum linear chirp, Cassini GWE1



Steps in Burst Processing

- Assume the time series is locally stationary
- Estimate the spectrum, $S(f)$, locally: I assume that $S(f) = A (B \cos^2(\pi f T_2) + 1) f^\alpha$ and estimate A, B, α for each of 117 intervals in the 40 days
- Assume a waveform, $h(t)$
- For each of the 20 vertices of a dodecahedron inscribed on the celestial sphere, produce $y(t)$ from $h(t)$, T_2 , and (α, δ) of the dodecahedron vertex
- From $y(t)$ and $S(f)$, produce the matching function $q(t)$, $FT(q) = FT(y)/S(f)$
- Crosscorrelate $q(t)$ with the data \rightarrow matched filter outputs against the real data
- As a check of the program and the noise statistics, for each of the time intervals over which the statistics are assumed to be stationary, generate 100 realizations of a stationary random process having ensemble average spectrum the same as $S(f)$ for that interval and with the same data gaps as the real data. Crosscorrelate $q(t)$ with these simulated time series \rightarrow simulated matched filter outputs
- Produce histograms of $z = (\text{matched filter output})/(\text{expected noise only rms output})$ for both the real and simulated data sets
- Flag outputs that have $|z| > 6$ (i.e., 6σ or larger) and overplot $y(t)$ on the data for these hits
- Produce grand histogram over all 20 dodecahedron directions for this $h(t)$
- Repeat for new $h(t)$



A LIMIT TO GW BACKGROUND

$$\Omega = \frac{\rho}{\rho_c} \quad \rho_c = \frac{3H_0^2}{8\pi G} \approx 1 \times 10^{-29} \frac{\text{gr}}{\text{cm}^3} \quad \text{if } H_0 = 80 \text{ km/sec/Mpc}$$

$$\rho = \left(\frac{243}{208}\right) \pi^3 \frac{f^4}{G} \int df \frac{S_y(f)}{4\pi^2 f^2} = \left(\frac{243}{208}\right) \frac{f_c^3 S_y(f_c)}{4G}$$

$$\text{@ } f_c = 10^{-4} \text{ Hz} \quad S_y \approx 10^{-25} \text{ Hz}^{-1}$$

$$\Omega \lesssim 0.04 \quad \text{@ } f_c \approx 10^{-4} \text{ Hz}$$

How to Do Better than Cassini with Doppler Tracking

Problems are:

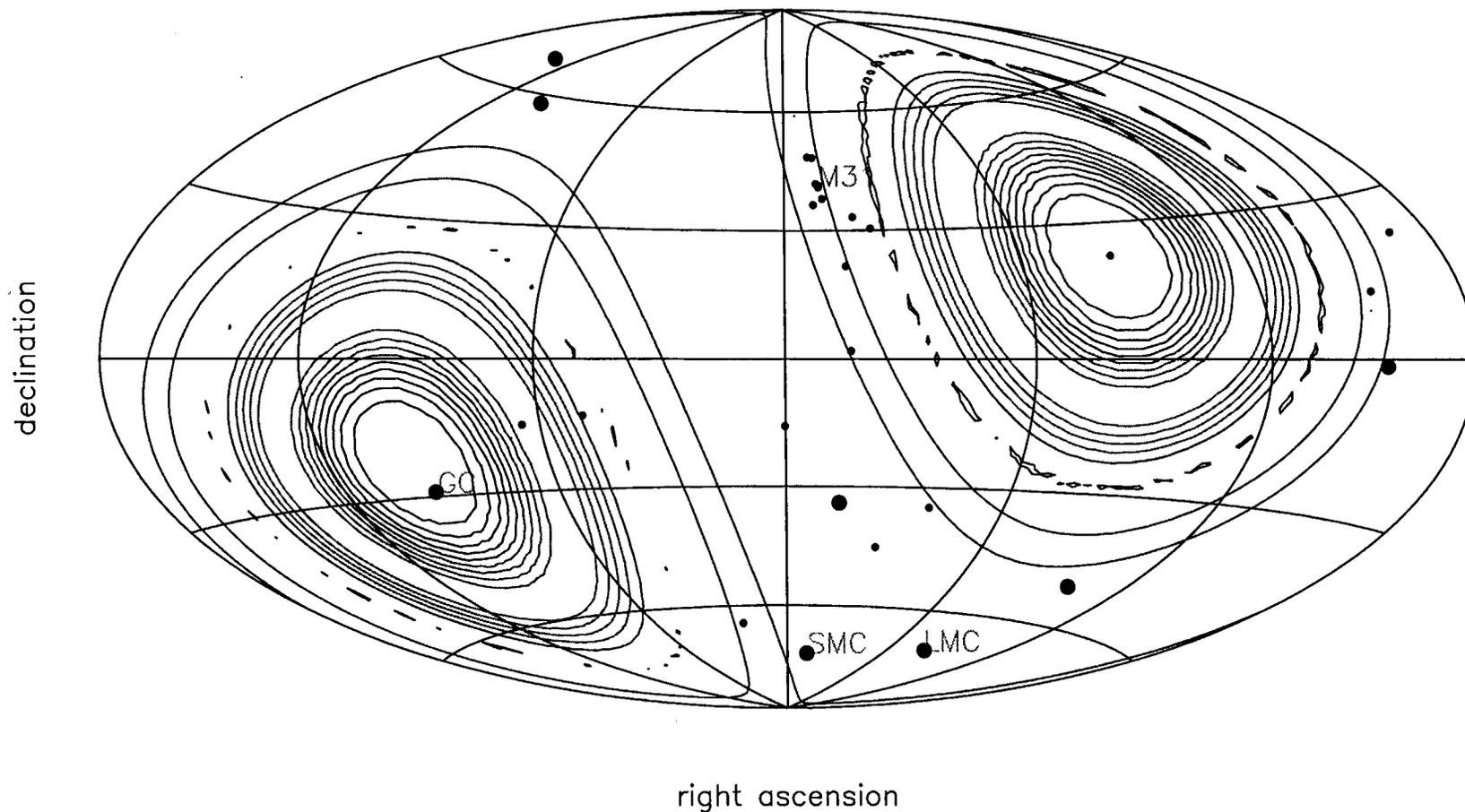
- tropospheric scintillation
- antenna mechanical noise
- plasma scintillation
- frequency standard noise
- spacecraft position noise

Possible Fixes

- better calibrations and/or Estabrook/Hellings idea
- look in nulls of transfer function (?)
- chop beam between sky and a very stable mechanical reference? (R. Spero)
- monitor M1 and subreflector better?
- higher radio frequency and/or Cassini-style multi-frequency links
- 30X better clocks are "straightforward"
- already good ($\sim 3 \times 10^{-16}$); improve with very careful design (?)

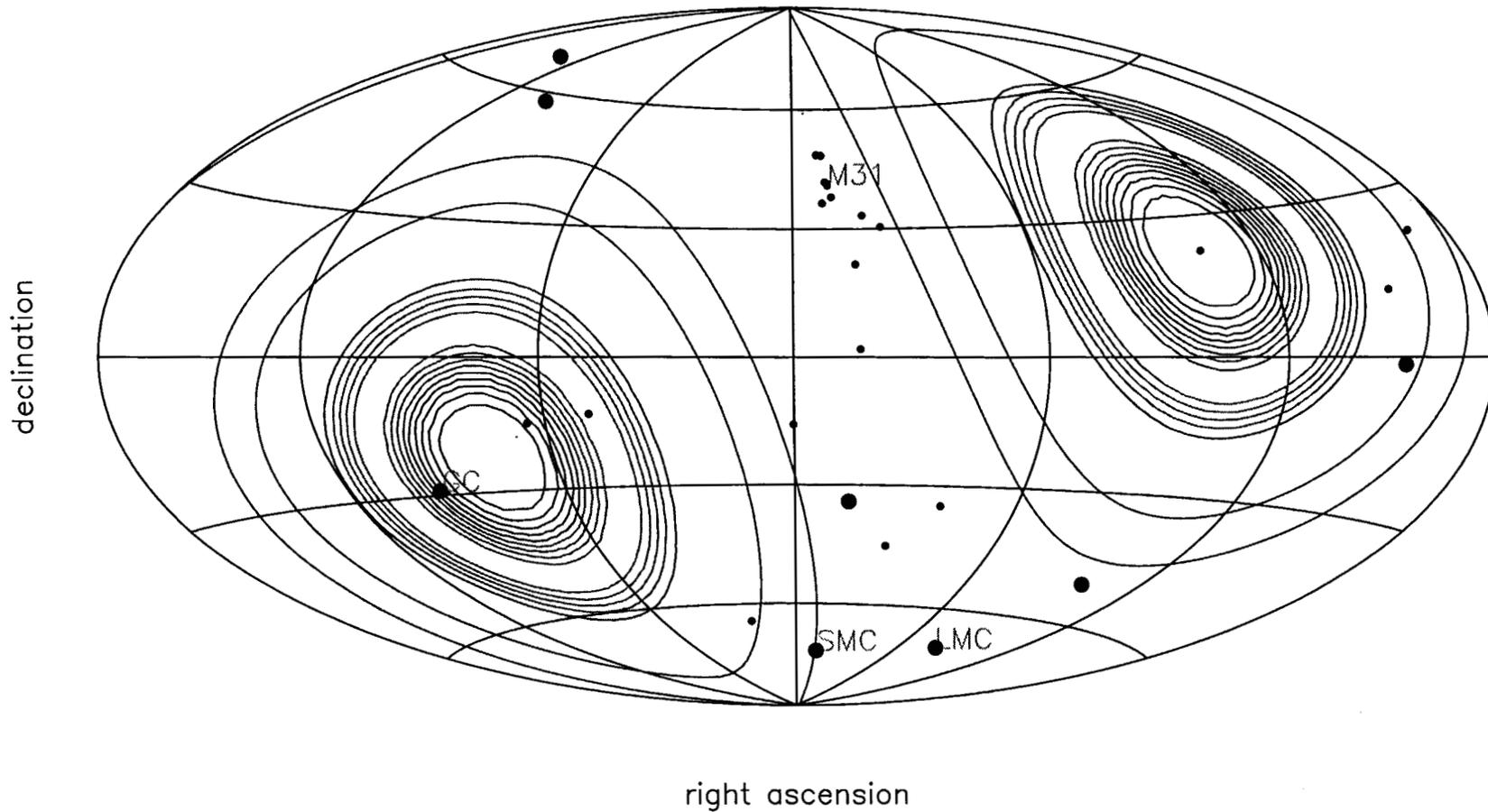
Perhaps a 10-fold improvement over Cassini is possible using spacecraft Doppler tracking from an Earth-based station. Improvements beyond this are unlikely with Earth-based Doppler tracking technique.

relative energy response for Cassini 2001 December 16
circular-pol: $\sin(2 \pi (0.001 \text{ Hz}) t) \cdot \exp(-t/1000 \text{ sec})$



hammer-aitoff equal-area projection (center of plot is RA = 0, dec = 0)

relative energy response for Cassini 2004 January 4
circular-pol: $\sin(2 \pi (0.001 \text{ Hz}) t) \cdot \exp(-t/1000 \text{ sec})$



hammer-aitoff equal-area projection (center of plot is RA = 0, dec = 0)

Concluding Comments

- **GWE1 worked at close to expected level of sensitivity**
 - **Ka-band --> negligible plasma effects at opposition**
 - **AMC --> data calibrated down to a non-dispersive TWLT correlated process (almost certainly residual antenna mechanical noise)**
 - **Median DSS 25 Allan deviation $\sim 7X$ better than best previous experiment**
 - **Best sensitivity 3×10^{-15} (at $\tau = 1000$ sec) consistent with pre-experiment instrumental goals**
- **Editing out only the "obvious" problems gave a data set that is relatively clean of systematics compared with previous experiments**
- **Only a pilot study so far, but using the edited_superfile (021002) data set:**
 - **$\Omega < 0.04$ for $f \approx 0.0001$ Hz (can probably be improved using DSS25 data only)**
 - **Burst search: sensitivity depends on waveform and direction, $\sim 5 \times 10^{-15}$ in midband (some 7σ outliers using square waveform—nothing definitive)**
 - **Chirps in progress, sensitivity $\sim 3 \times 10^{-16}$ averaged over (0.0001-0.05 Hz) band**
- **Geometric coupling to sky will be different in GWE2, GWE3**