Andrzej Krolak

Title: Lisa sensitivity

We present a procedure for computing LISA's sinsitivity to gravittaional radiation. Our method is general in that
(1) the LISA arm length do not have to be equal, and
(2) time-delays in propagation of GW signals and laser light across the apparatus are explicit.
The multiple Doppler readouts available on LISA permit simultaneous formation of several interferometric observables. All thes observables are independent of laser frequency fluctuations and have diffferent couplings to gravitational waves and the various LISA instrumental noises. We have identified a triplet of interferometric combinations that show optimally combined sensitivty.

Laser Interferometer Space Antenna (LISA)

The LISA Sensitivity

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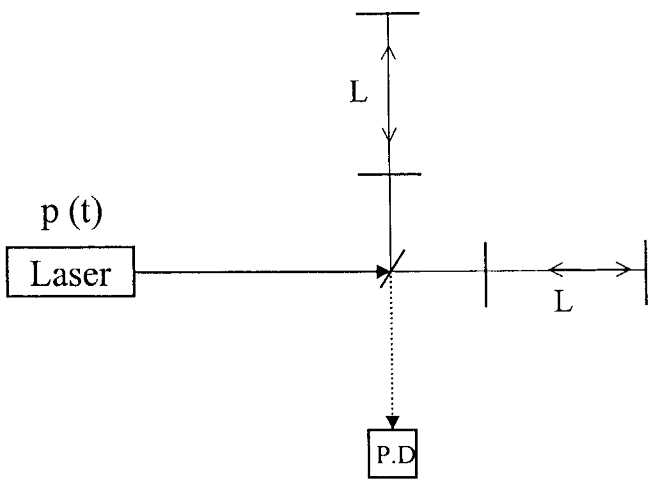
Earth vs. Space-Based Interferometers

- Earth-based interferometers have arm lengths essentially equal. This is in order to directly remove laser frequency fluctuations at the photodetector, where the two beams interfere.
- They operate in the long-wavelength limit ($L \ll \lambda$).
- By contrast, LISA will have arm lengths significantly different ($\Delta L/L \sim 10^{-2}$), with $L = 5 \times 10^6$ km.
- Over much of its sensitivity frequency-band, it will **<u>not</u>** operate in the long-wavelength regime.
- Time-of-flight delays in the response to the wave, and travel times along the beams in the detector must be allowed for, in order to derive a correct theory of the detector response.

Statement of The Problem

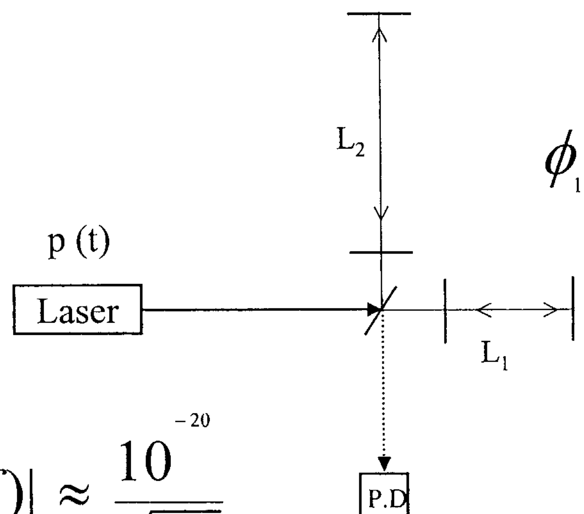
p (t) = Laser phase fluctuations



$$\frac{1}{2\pi\nu_0}\frac{dp(t)}{dt}=\left[\frac{\Delta\nu(t)}{\nu_0}\right]_{Laser}\equiv C(t)$$

$$\phi_1(t)=h_1(t)+p(t-2L_1)+n_1(t)$$

$$\phi_2(t)=h_2(t)+p(t-2L_2)+n_2(t)$$

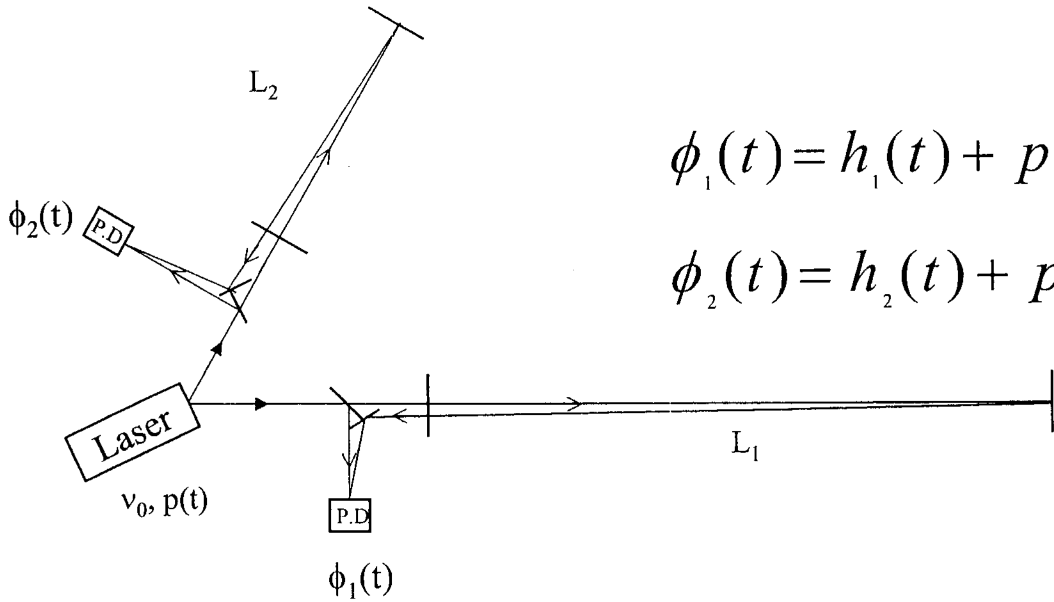


$$\phi_1(t)-\phi_2(t)\Rightarrow p(t-2L_1)-p(t-2L_2)\cong 2\frac{dp}{dt}\varepsilon L_1$$

$$|\tilde{h}(f)|\approx\frac{10^{-20}}{\sqrt{Hz}}$$

$$|\tilde{C}(f)|\approx\frac{10^{-13}}{\sqrt{Hz}}\ ,\ \varepsilon\cong 3\times10^{-2}\Rightarrow\frac{5\times10^{-16}}{\sqrt{Hz}}$$

Unequal-arm Interferometers



$$\phi_1(t) = h_1(t) + p(t - 2L_1) - p(t) + n_1(t)$$

$$\phi_2(t) = h_2(t) + p(t - 2L_2) - p(t) + n_2(t)$$

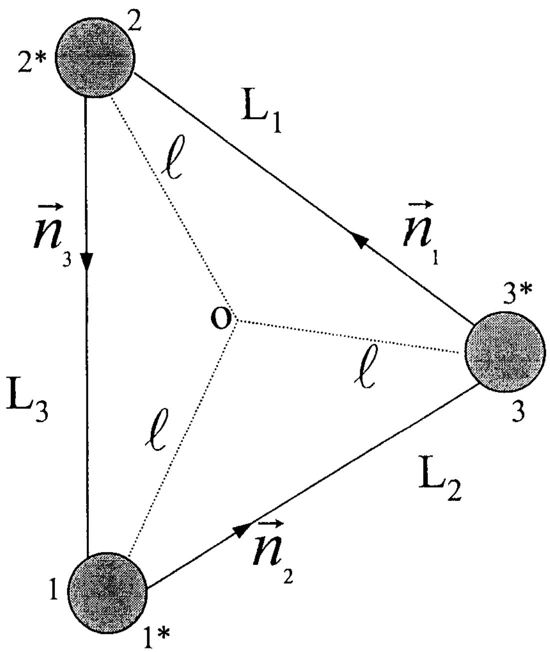
$$\phi_1(t) - \phi_2(t) = h_1(t) - h_2(t) + p(t - 2L_1) - p(t - 2L_2) + n_1(t) - n_2(t)$$

$$\phi_1(t - 2L_2) - \phi_2(t - 2L_1) = h_1(t - 2L_2) - h_2(t - 2L_1) + \\ p(t - 2L_1) - p(t - 2L_2) + n_1(t - 2L_2) - n_2(t - 2L_1)$$

$$X(t) \equiv [\phi_1(t) - \phi_2(t)] - [\phi_1(t - 2L_2) - \phi_2(t - 2L_1)]$$

M. Tinto, & J.W. Armstrong, Phys. Rev. D, **59**, 102003 (1999).

Time-Delay Interferometry (T.D.I.)

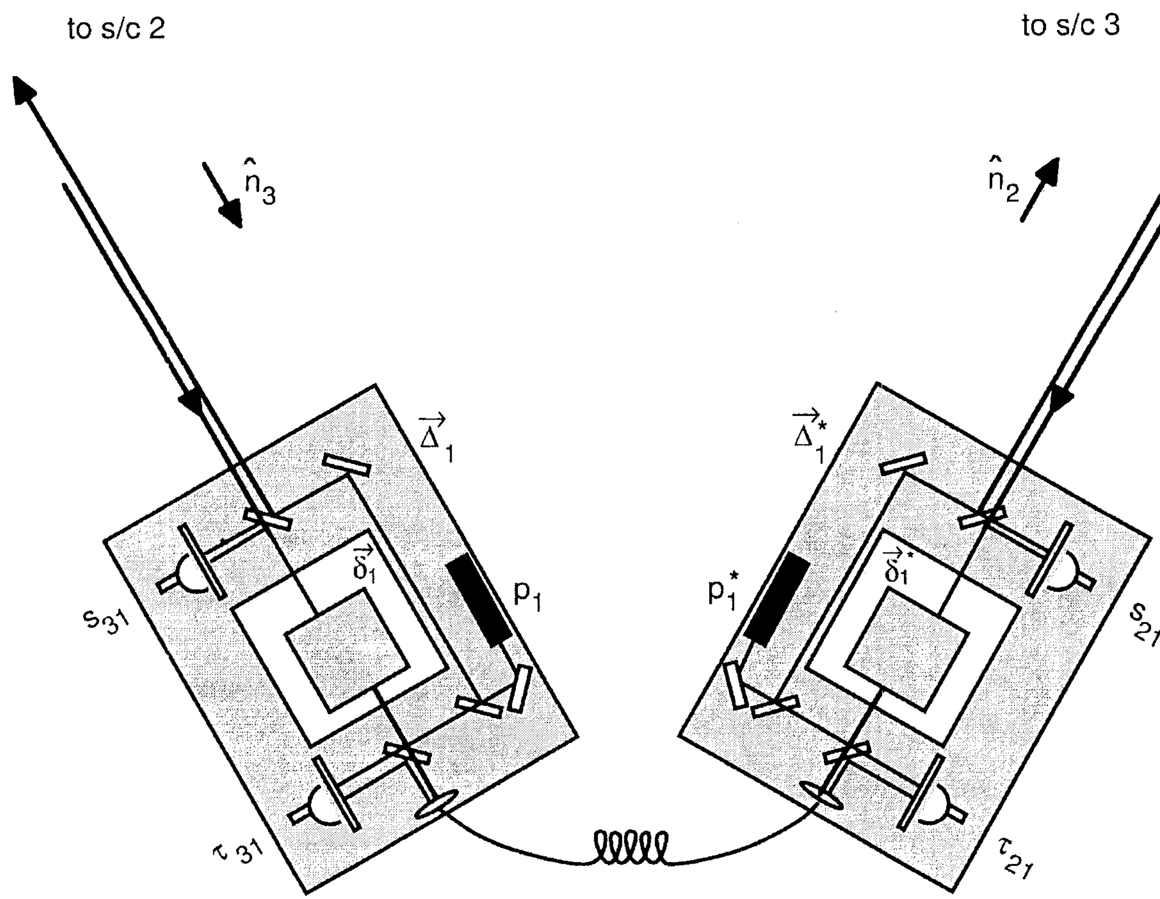


- It is best to think of LISA as a closed array of six one-way delay lines between the test masses.

- This approach allows us to reconstruct the unequal-arm Michelson interferometer, as well as new interferometric combinations, which offer advantages in hardware design, in robustness to failures of single links, and in redundancy of data.

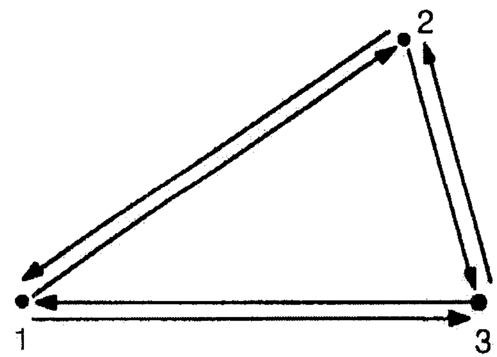
M. Tinto: *Phys. Rev. D,* **53**, 5354 (1996); *Phys. Rev. D,* **58**, 102001 (1998)
J.W. Armstrong, F.B. Estabrook, and M. Tinto: *Ap. J.,* **527**, 814 (1999).

Time-delay Interferometry & The Drag-free Configuration



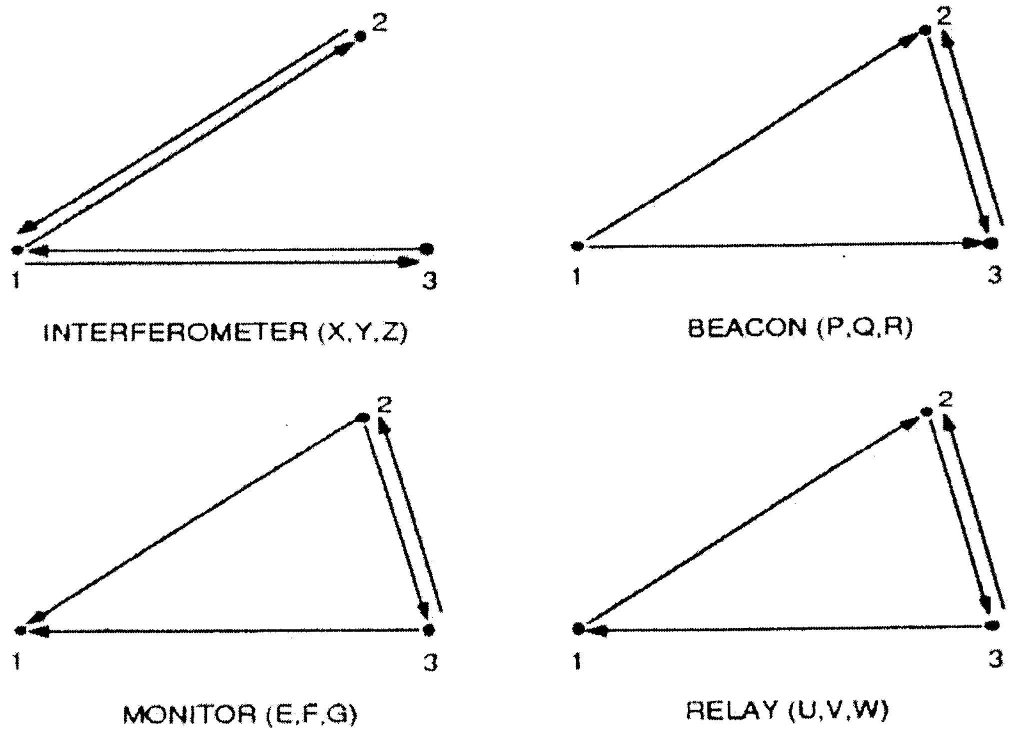
F.B. Estabrook, M. Tinto, and J.W. Armstrong: *Phys. Rev. D,* **62**, 042002 (2000)
M. Tinto, F.B. Estabrook, and J.W. Armstrong: *Phys. Rev. D*, **65**, 082003, (2002)

Six-Pulse Data Combinations



$\alpha, \beta, \gamma, \zeta$

Eight-Pulse Data Combinations

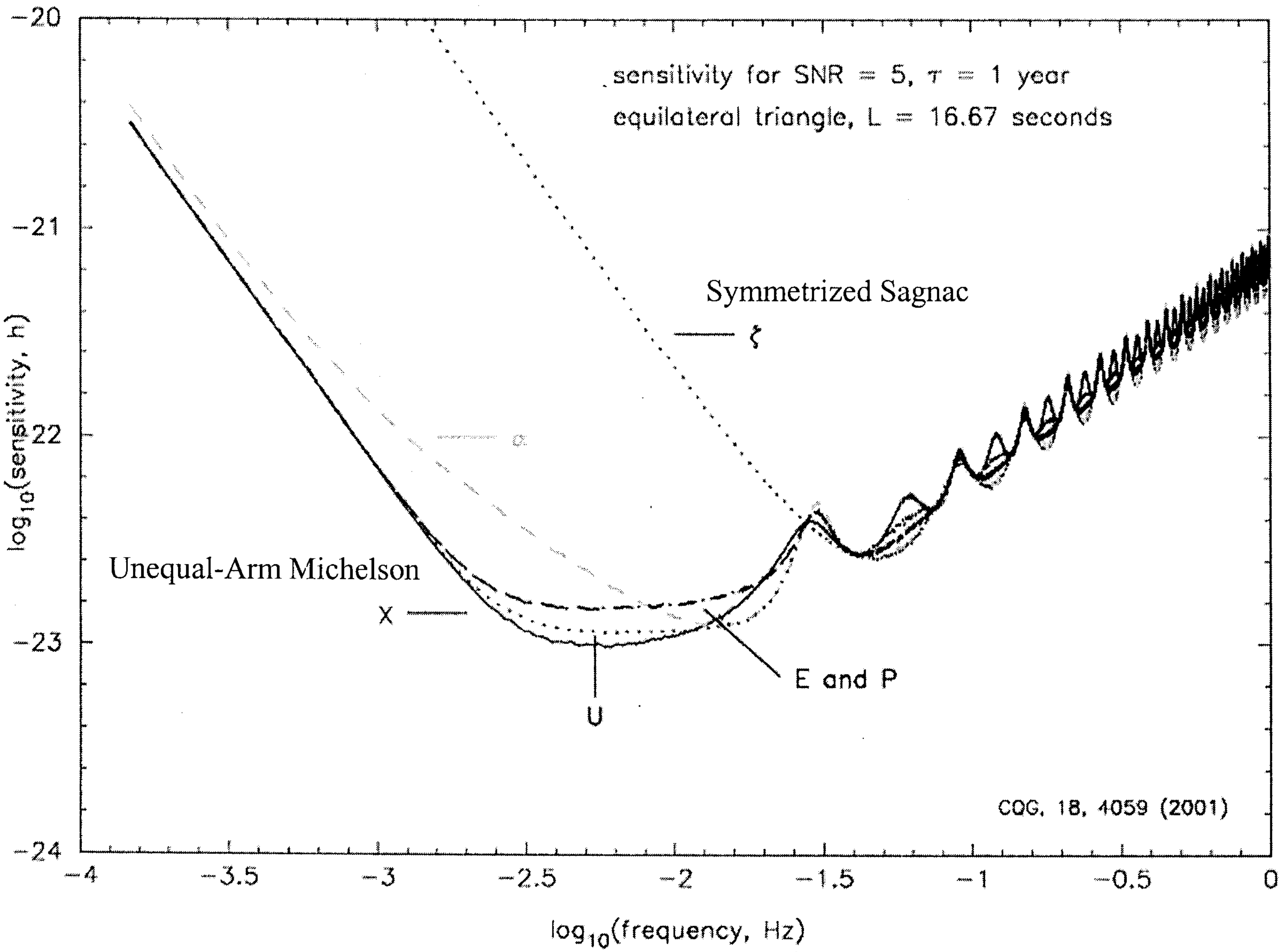


Data Combinations (Cont.)

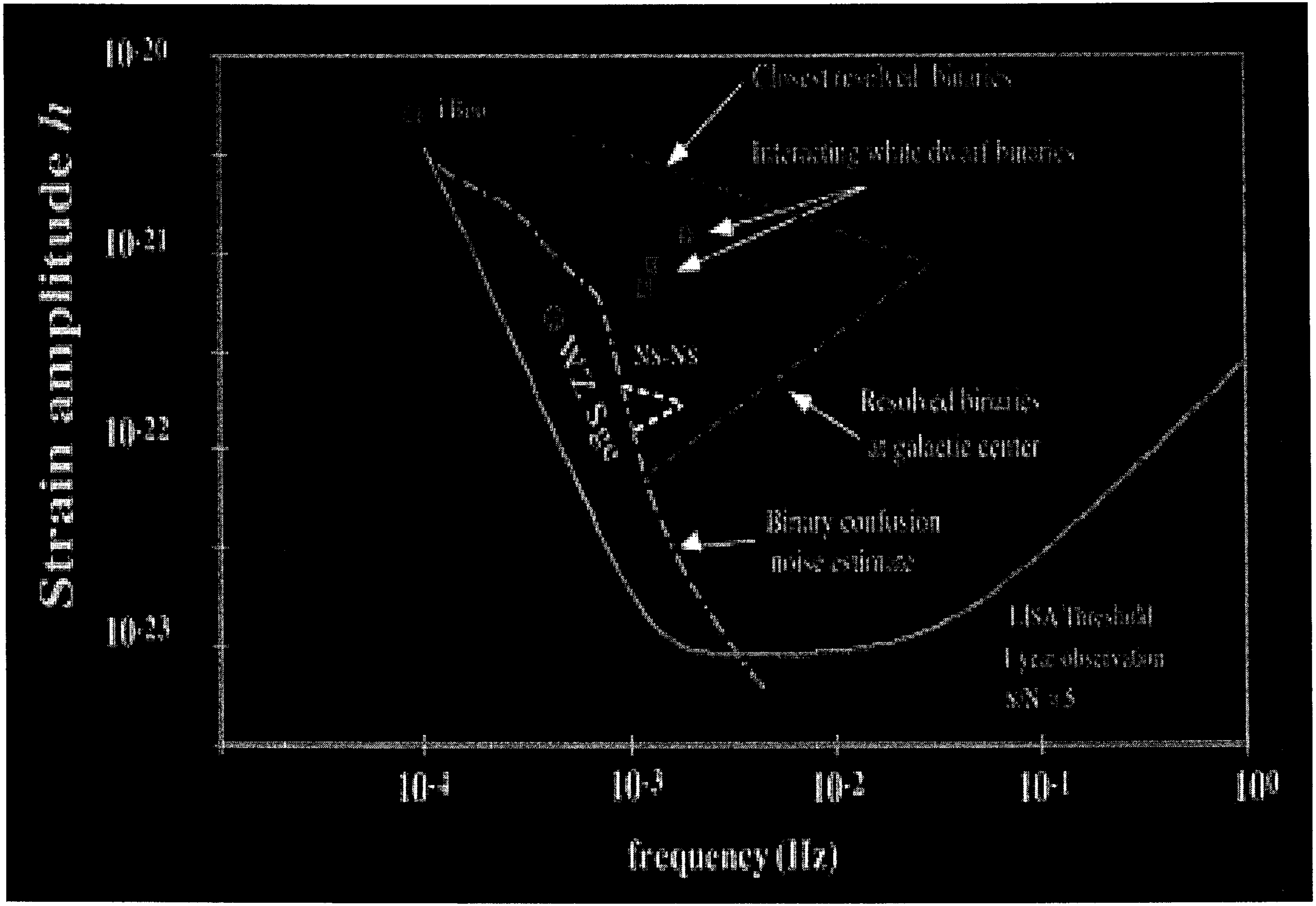
- There are 6 optical benches, 6 lasers, 3 Ultra Stable Oscillators (USO), and a total of 18 Doppler time series observed.
- The 6 beams exchanged between distant spacecraft contain the information about the GW signal; the other 12 signals are for comparison of the lasers, relative optical bench motions within the spacecraft, and calibration of the USO phase noises affecting the interferometric observables.
- The four combinations $(\alpha, \beta, \gamma, \varsigma)$ can be regarded as the generators of the functional space of all possible interferometric combinations.

$$\varsigma - \varsigma_{,123} = \alpha_{,1} - \alpha_{,23} + \beta_{,2} - \beta_{,31} + \gamma_{,3} - \gamma_{,12}$$
$$X_{,1} = \alpha_{,32} - \beta_{,2} - \gamma_{,3} + \varsigma$$
$$P = \varsigma - \alpha_{,1}$$
$$E = \alpha_{,1} - \varsigma_{,1}$$
$$U = \gamma_{,1} - \beta$$

S.V. Dhurandhar, K.R. Nayak, and J-Y. Vinet, *Phys. Rev. D*, **65**, 102002, 2002



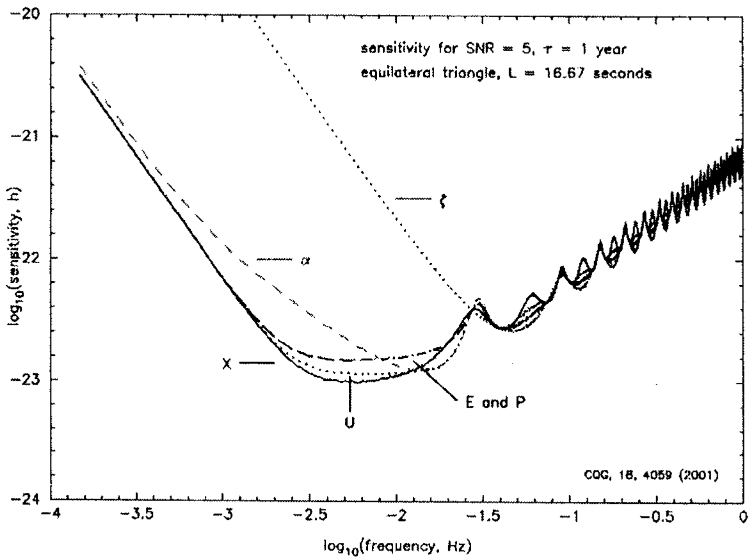
sensitivity for SNR = 5, τ = 1 year
equilateral triangle, L = 16.67 seconds
Symmetrized Sagnac
ζ
α
Unequal-Arm Michelson
X
U
E and P
CQG, 18, 4059 (2001)
log10(sensitivity, h)
log10(frequency, Hz)
−20
−21
−22
−23
−24
−4
−3.5
−3
−2.5
−2
−1.5
−1
−0.5
0


Strain amplitude h
10-20
10-21
10-22
10-23
10-4
10-3
10-2
10-1
10^0
frequency (Hz)
Closest resolved binaries
Interacting white dwarf binaries
NS-NS
Resolved binaries
at galactic center
Binary confusion
noise estimate
LISA Threshold
1 year observation
S/N = 5

NOISE CALIBRATION and DETECTION OF A STOCHASTIC BACKGROUND

- The gravitational wave background will be below the anticipated sensitivity curve of ζ by several orders of magnitude.
- The Sagnac combination provides a way for estimating the instrumental noise sources: Sagnac greatly attenuates the gravitational wave signal, but instrumental noise persists.
- This allows us to infer the actual on-orbit LISA instrumental noise in the Michelson interferometer mode X, and in turn to detect the stochastic background (**M. Tinto, J.W. Armstrong, & F.B. Estabrook,** *Phys. Rev. D,* **63**, 021101(R) (2001)).
- The ζ combination can of course be used also as a discriminator for sinusoidal signals and bursts.
- Hogan and Bender (*Phys. Rev. D*, **64**, 062002, 2001) have proposed a technique for improving the LISA sensitivity to backgrounds by properly combining the spectra of ζ, X, Y, and Z.

Optimal Interferometric Combinations



$$\eta = a_1(f)\tilde{\alpha} + a_2(f)\tilde{\beta} + a_3(f)\tilde{\gamma} + a_4(f)\tilde{\zeta}$$

$$\begin{aligned}[1-e^{2\pi i f(L_1+L_2+L_3)}]\tilde{\zeta} = {} & [e^{2\pi i f L_1} - e^{2\pi i f(L_2+L_3)}]\tilde{\alpha} \\ & + [e^{2\pi i f L_2} - e^{2\pi i f(L_1+L_3)}]\tilde{\beta} \\ & + [e^{2\pi i f L_3} - e^{2\pi i f(L_1+L_2)}]\tilde{\gamma}\end{aligned}$$

$$SNR_{\eta}^{2} = \int \frac{|a_1(f)\tilde{\alpha}_s + a_2(f)\tilde{\beta}_s + a_3(f)\tilde{\gamma}_s|^2}{<|a_1(f)\tilde{\alpha}_n + a_2(f)\tilde{\beta}_n + a_3(f)\tilde{\gamma}_n|^2>} df$$

- One should regard the SNR as a functional of the functions a_i (f), and extremize it with respect of them.

T. Prince, M. Tinto, S. Larson, and J.W. Armstrong, *Phys. Rev. D*, **to appear** (2002)

Optimal Interferometric…(Cont.)

$$SNR_{\eta}^{\;2} = \int \frac{a_i \, A_{ij} \, a_j^*}{a_r \, C_{rt} \, a_t^*} df$$

$$A_{ij} = X_i^{(s)} X_j^{(s)^*} \; ; \; C_{ij} = \langle X_i^{(n)} X_j^{(n)^*} \rangle$$

$$(X_1, X_2, X_3) \equiv (\alpha, \beta, \gamma)$$

- The stationary values of the SNR_{η} correspond to the stationary points of the integrand.
- Since the quadratic form **C** is non-singular, we can identify the stationary points of the integrand by using the **Rayleigh's Principle for Quadratic Forms**: *The stationary values of the integrand are attained at the eigenvalues of the matrix (*$\mathbf{C}^{-1}A$*)*

Optimal Interferometric...(Cont.)

- Since **A** has rank 1, it follows that also $\mathbf{C}^{-1}\mathbf{A}$ has rank 1.

- The only non-zero eigenvalue of the matrix $\mathbf{C}^{-1}\mathbf{A}$ is therefore equal to Tr ($\mathbf{C}^{-1}\mathbf{A}$). This implies the following expression for the optimal signal-to-noise ratio:

$$SNR^2{}_{\eta_{opt.}} = \int X_i^{(s)^*} C^{-1}{}_{ij} X_j^{(s)} df$$

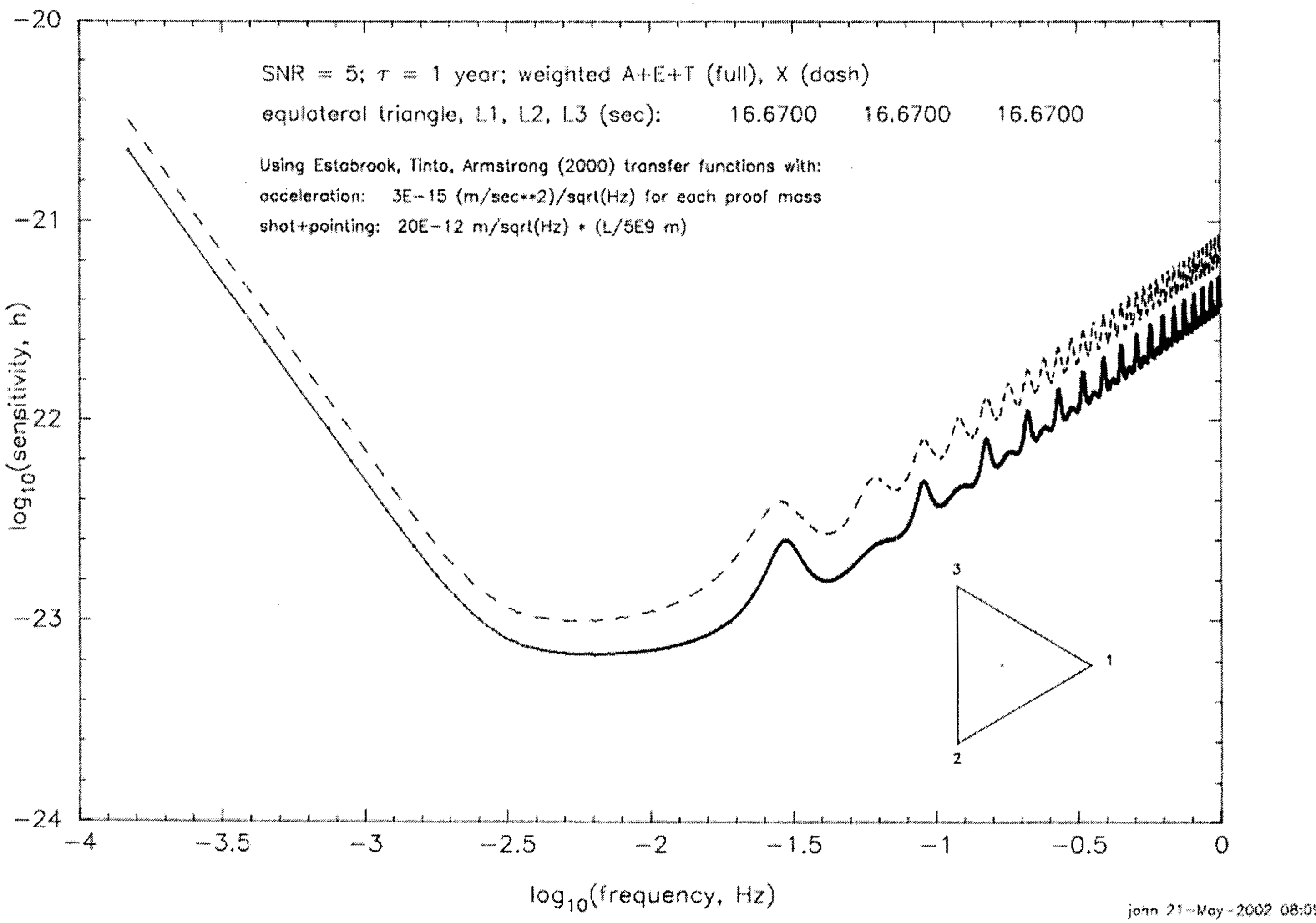
- We can change basis, and work with a triad of interferometric observables that have diagonal covariance matrix.

Optimal Interferometric....(Cont.)

$$\begin{bmatrix} A \\ E \\ T \end{bmatrix} = \begin{bmatrix} \frac{1}{\sqrt{2}}(\tilde{\gamma} - \tilde{\alpha}) \\ \frac{1}{\sqrt{6}}(2\tilde{\beta} - \tilde{\alpha} - \tilde{\gamma}) \\ \frac{1}{\sqrt{3}}(\tilde{\alpha} + \tilde{\beta} + \tilde{\gamma}) \end{bmatrix}$$

$$SNR_{\eta}^{2} = \int \left[\frac{A_s A_s^*}{<A_n A_n^*>} + \frac{E_s E_s^*}{<E_n E_n^*>} + \frac{T_s T_s^*}{<T_n T_n^*>} \right] df$$

Optimal Interferometric….(Cont.)



Conclusions

- T.D.I. provides an exact method for canceling the leading noise source – laser phase fluctuations – in an interferometer with unequal, time-variable arms.
- It allows analysis of signals, noises, achievable sensitivities, and architectural design (including system-level tradeoffs between, e.g. Laser stability, arm length accuracy, stability of optical bench, Doppler shifts due to chosen orbits, USO stability).
- It provides robustness of the mission with respect to failures of subsystems.
- It shows existence of alternate LISA configurations offering potential design, implementation, or cost advantages.
- It gives a data combination (ζ) for assessing the LISA on-orbit instrumental noise performance.
- The LISA optimal sensitivity can be obtained by regarding LISA as a network of three Interferometers.