# High-Resolution Displacement Sensor Using SQUID Array Amplifier

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## Abstract

We describe the design and operation principle of a new displacement sensor with a substantial improvement in resolution over what is currently possible. The potential for significant improvement in the measurement of displacement has profound implications for several high-impact gravitational physics and particle physics projects: the detection of gravitational waves, the test of the equivalence principle, the search for the postulated "axion" particle, and the test of the inverse square law of gravity, all of which rely on the detection of small displacements. If the new displacement sensor is sensitive enough to observe the quantum mechanical zero-point motion of a test mass, it will be possible to make a direct examination of the Heisenberg uncertainty principle. Based on our calculations, the achievable resolution ($\sim 10^{-20}$ to $10^{-22} m/\sqrt{Hz}$) is high enough to reach the quantum limit. Investigations of these world-class science issues will require the low vibration environment offered by free fall to achieve its ultimate sensitivity.

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## 1. Introduction

At the dawn of the last century, two outstanding mysteries motivated the birth of modern physics. The failure of classical physics to explain the black body radiation spectrum motivated the development of quantum mechanics, while the excessive precession of the perihelion point of the orbit of Mercury spurred the development of general relativity. Despite the astounding success of quantum mechanics and general relativity over the last century, several mysteries remain with answers that appear to lie outside the paradigm of modern physics. There are predictions within the realm of modern physics that have not yet been observed due to the lack of sensitivity of the best-known technologies. There are also fundamental assumptions upon which modern physics is based that are being tested at an ever-increasing level of sensitivity. Any observed deviations indicating fallacy of these assumptions would require a shift in our view of the Universe. In the following paragraphs, we review some of the most pressing science issues of our time and discuss how a new displacement sensor with substantial improvement in sensitivity may help resolve them.

### *1.1. The Missing Mass of the Universe and the The Strong CP Problem of the Standard Model*

Celestial mechanics of Newton and Keppler, refined by Einstein's general relativity, allowed predictions of the motion of bodies in our Solar system with triumphal precision. Yet, on the galactic scale, an exceptional mystery remains. From observations of the rotational velocity distribution of stars on the periphery of spiral galaxies [1], it appears that the majority of mass in the universe cannot be accounted for if our knowledge of the laws of gravity is correct. A hypothesis that has gained some acceptance is that the universe is composed largely of unknown weakly interacting particles that have mass. This "dark matter" would provide the needed gravitational pull on the stars in a galaxy to account for their rotational rates around the galaxy. One candidate for the missing mass is the weakly interacting particle called the "axion." Its existence is postulated by an attempt to explain the apparent absence of an electric dipole moment in a neutron down to a level of less than $11\times10^{-26} e-cm$ [2]. This absence is an outstanding problem in particle physics because the most general formalism of the gauge invariant quantum field theory allows the possibility of violations of the parity (*P*), the charge-parity (*CP*), and the time (*T*) symmetries in strong interaction; and the violation of CP should lead to a neutron electric dipole moment $\sim10^9$ larger than what is allowed by experimental observation. In 1977, Peccei and Quinn [3] proposed an elegant way to explain why the neutron electric dipole moment can be zero by hypothesizing that the strong interaction Lagrangian has a global chiral symmetry. Weinberg [4] and Wilczek [5] analyzed the consequence of the Peccei-Quinn symmetry and noticed that the spontaneous breaking of a global chiral symmetry leads to a light pseudoscalar, pseudo-Goldstone boson, called an axion. Axion is predicted to be a Yukawa particle that mediates a new short-range interaction. By precision measurements of the force acting between test masses, stringent limits have already been placed on the range and the strength of this new force. Substantial improvement in displacement sensitivity can help extend these limits and may lead to the observation of this new force at sensitivity levels that are inaccessible today.

### *1.2. The Unification of Quantum Mechanics and General Relativity*

Even though quantum mechanics successfully explains phenomena at small (atomic and sub-atomic) mass and length scales, and even though general relativity successfully explains phenomena at large mass and length scales, the two theories are based on incompatible formalisms. The apparent incompatibility has made the development of a common language to discuss phenomena over all mass and length scales difficult. Therefore, the unification of quantum mechanics and general relativity into a common mathematical framework is a quest of modern theoretical physics. It was clear early on that a common conceptual base must be developed and that the ultimate unification would involve a new mathematical formalism, invented to bridge the gap. The supersymmetric string theory provides such a conceptual and mathematical framework, and, so far, there is no other theory that can answer this challenge. This theory is now mature enough to be used to predict new quantitatively testable experimental results. For example, a paper by Damour and Polyakov [6] in 1994 predicted a violation of the equivalence principle in the range of $10^{-14}$ to $10^{-24}$ of the force of gravity. The most optimistic prediction is just beyond the reach of Earth-based measurements, which already have tested the validity of the equivalence principle to a level of $10^{-13}$. These Earth based measurements are limited by seismic disturbances. An on-orbit test with a high-resolution displacement sensor in a drag-free satellite would solve the seismic disturbance problem and allow the equivalence principle to be test to the fullest extend possible.

A relatively more recent paper by Dvali and Smirnov [7] suggested that string theory could be tested by sensitive measurements of the gravitational force between bodies at distances shorter than $1mm$. The new prediction suggests a deviation from the expected inverse square law of gravity at short distances. A satellite-based test of this prediction, the Inverse Square Law Experiment in Space (ISLES), has already been proposed by Paik [8]. Our technology will allow the limit of such a test to be extended.

## *1.3. The Gravitational Wave*

Gravitational waves are a remaining prediction of general relativity that have not yet been directly observed. However, evidence of their existence has been implied by the increasing rate of rotation of a binary pulsar as a result of energy loss by gravitational radiation [9]. With present technology, gravitational waves are expected to barely be detectable in the very rare supernova events in nearby stars. An improvement in displacement sensitivity by a factor of 100 can bring their observation within reach. A near simultaneous observation of a supernova event by gravitational waves and by optical observation would allow a direct comparison of the speed of light to that of gravitational wave. A basic assumption of general relativity is that they are equal.

## *1.4. Heisenberg's Uncertainty Principle*

One of the postulates of quantum mechanics is the Heisenberg uncertainty principle, which states that the measurement of the position and momentum of an object cannot be simultaneously made arbitrarily accurate. Many of the successes of quantum mechanics are implications of this postulate. Yet it has not been directly tested in a macroscopic mechanical system because measurement technology has not been sensitive enough. The technology proposed has the potential of reaching the quantum limit given by the zero point motion of a harmonic oscillator. If this limit is reached, it will allow the examination of this basic foundation of quantum mechanics.

One area of contemporary interest is the prospect of achieving a squeezed quantum mechanical state in a system consisting of a harmonic oscillator coupled with a displacement sensor. In such a state, the uncertainty in the position of an object can be made smaller than that given by the zero point motion. However, the Heisenberg uncertainty principle would require that the uncertainty in the momentum be increased so that their product remains larger than Planck's constant. Techniques for implementing this concept have been proposed. They are known as quantum non-demolition techniques [10]. A sensitive displacement sensor would open the door for testing these concepts of contemporary interest.

## 2. Operation Principle

The proposed displacement sensor is based on a combination of two of the world's most sensitive technologies. The high-Q superconducting resonator technology is used to amplify any off-null signal from a capacitance bridge. The amplified signal is detected using a sensitive SQUID array amplifier, which is a high bandwidth SQUID magnetometer available commercially [11]. As shown in Figure 1a, each of the four arms of the bridge consists of a capacitor fabricated with a superconductor. Depending on the implementation, one or more of these capacitors can be part of the displacement transducer. As shown by the Thevenin equivalent circuit in Figure 1b, the capacitors in the bridge together with the input inductor of the SQUID form a high-Q $LC$-resonator with a resonance of ~$10MHz$. An effective resistor with resistance value of $R_{eff}$ is included in the equivalent circuit to account for losses that contribute to the finite $Q$ value. The highest sensitivity is achieved when the bridge is driven at the resonant frequency. In the following we give an analysis of the thermal noise of the Thevenin equivalent circuit to show how a resolution of $7\times10^{-23} m/\sqrt{Hz}$ may be achieved.

### 2.1. *Thermal Noise of the Electronics*

This simple $LRC$ circuit follows the relation:

$$L\left(d^2q/dt^2\right)+R_{eff}\left(dq/dt\right)+q/C=V_{null}(t), \tag{1}$$

where $q$ is the charge in the capacitor, $L$ is the inductance of the SQUID input coil, and $C$ is the Thevenin equivalent capacitance of the bridge, which is equal to the capacitance on one arm, if the four arms have equal capacitance. Equation 1 is equivalent to that of a simple damped oscillator consisting of a mass attached to a spring. Since the thermal noise of the harmonic oscillator has already been derived [12], the power spectral density of $q$, *PSD_q,* obtained by a simple mapping is:

$$PSD\_q=q_\omega q_\omega^*=\left[4k_BTC/(\omega_eQ_e)\right]\cdot\left[\left(1-\omega^2/\omega_e^2\right)^2+\left(\omega/\omega_eQ_e\right)^2\right]^{-1}, \tag{2}$$

where $q_\omega$ is the Fourier transform of $q$, $Q_e=L\omega_e/R_{eff}$ and $\omega_e=\sqrt{1/(LC)}$ is the angular resonant frequency. Since $I_s=dq/dt$, in the frequency domain $I_{s\_\omega}I_{s\_\omega}^*=\omega^2q_\omega q_\omega^*$, which is the power spectral density of the current noise from the circuit. The total current noise seen by the SQUID consists of the noise from the circuit together with the SQUID current noise $I_n$. For the SQUID array amplifier [11], $I_n=2pA/\sqrt{Hz}$. Therefore, the voltage noise density associated with $V_{null}$ is $\Delta V_{null}=|Z|\sqrt{\omega^2q_\omega q_\omega^*+I_n^2}$, where $Z=1/(i\omega C)+i\omega L+R_{eff}$. The displacement noise density $\Delta d$ can be derived from the relation $\Delta d/d=2\Delta V_{null}/V$ for a capacitance transducer with a plate-separation of $d$ and a drive voltage of $V$.

Figure 2 shows the displacement noise density for two capacitance values of 700*pf* and 10*pf*, for $T = 1.5K$, $d = 0.1mm$, $V = 40V$, and $Q = 10^5$. These parameters are easily achievable on the ground and in space. The dotted straight line is the contribution from the Johnson noise of $R_{eff}$. As expected, the lowest noise is achieved at the resonance, where the resolution for the 700*pf* displacement sensor is $6.5\times10^{-19}\,m/\sqrt{Hz}$. This resolution is limited by the Johnson noise from $R_{eff}$. Away from the resonance, the sensitivity is limited by SQUID noise. By using $Q_e$ = 5x10$^6$, $d$ = 0.02*mm*, $V$ = 800*V*(near dielectric breakdown), $C$ = 3500 *pf* and $T$ = 20*mK*, we have shown that a noise density of $7\times10^{-23}\,m/\sqrt{Hz}$ is possible!!

2.2. *Thermal Noise Limit of Displacement Measurement*

Thermal noise of the proof mass being monitored by the displacement sensor can also degrade the performance of the overall system. Let us consider the behavior of a cylindrical proof mass that is floating in free fall, its length will stretch and shrink randomly due to thermal noise by an rms value $z_{thermal} = \sqrt{k_B T/k}$, where $k_B$ is the Boltzmann constant, and $k$ is the spring constant of the proof mass. For example, for a 0.1*m* bar and a 2.4*m* bar made of aluminum, the values of $z_{thermal}$ are $1.2\times10^{-16}$ and $5.7\times10^{-17}\,m$ respectively at 1.5*K*. At first glance, it may appear that the proposed displacement sensitivity may never be realized due to this intrinsic thermal noise in a proof mass. But the $z_{thermal}$ values given above assume that the sampling time for a series of *z's* is much longer than the internal thermal equilibration time inside the proof mass, so that each individual data is uncorrelated with its adjacent data. On the other hand, if the sampling time is much faster than the internal relaxation time, then the adjacent data are correlated and their relative values are not affected by thermal fluctuations. In a free falling test mass, the thermal equilibration time is given by $\tau_m = 2Q_m / \omega_m$, where $Q_m$ and $\omega_m$ are the quality factor and the angular resonant frequency of the mechanical resonance of the fundamental mode of the test mass. This equilibration time is the same as the ring-down time for the acoustic resonance generated in the bar when it is hit. This ring-down time can be as long as minutes.

## 2.3. *Quantum Mechanical Limit of Displacement Measurement*

In quantum mechanics, the ground state of a harmonic oscillator as shown in Figure 3 has a zero-point energy $\hbar\omega_m / 2$ and a zero point motion $\langle z_{SQL}^2 \rangle = \hbar / (2\omega_m m_{eff})$. Here $m_{eff} = m / \pi^2$ for mapping a bar's fundamental mode resonant frequency into the frequency of a simple harmonic oscillator with the same spring constant, and $m$ is the mass of the bar. Such a limit, given by the zero point motion, is known as the Standard Quantum Limit (SQL). For the $0.1m$ aluminum bar weighing $0.675kg$ as discussed in the preceding paragraph, $z_{SQL} = 7 \times 10^{-20} m$. For gravity wave detection with the resonant bar technique, a typical bar is about $2.4m$ long, weighs $1620kg$ and has a resonant frequency of about $1kHz$. For these types of bars, the standard quantum limit on displacement is about $7 \times 10^{-21} m$. By taking data at a high sampling rate, one can avoid thermal fluctuations to reach the quantum mechanical limit. Our proposed displacement sensor will provide the needed resolution to achieve this goal.

## 2.4. *Effective Bandwidth*

Because our displacement sensor makes use of a resonance to amplify a signal, there is an effective bandwidth associated with the time it takes to build up the energy in a resonant circuit. This time is $\tau_e = 2Q_e / \omega_e$; and the effective bandwidth is $f_c = f_e / (2Q_e)$, where $f_e$ is the resonant frequency of the circuit. By making $f_e$ high, we can ensure that the effective bandwidth is reasonably high.

### 2.5. *Balancing the Bridge*

To balance the bridge, we plan to use a mechanically adjustable trim capacitor for rough adjustment. For fine adjustment, a magnetically adjustable capacitor using magnetostrictive material will be designed and fabricated for balancing the bridge. Graetz et al [13] reported that such a material could provide a stroke of 1% at cryogenic temperatures. The magnetostrictive adjustment can be set using a superconducting persistent current, which is free of noise.

### 2.6. *Back-Action Noise and Pump Noise*

Noise can be injected from the amplifier back into the proof mass. At present, the source of this back-action noise is not yet fully understood. However, in our scheme, the resonant amplification is used to amplify the signal ~$10^5$ times before being amplified by the SQUID array amplifier. Thus, the relative magnitude of back-action noise from the amplifier should be greatly reduced.

The AC drive of the bridge can also be a potential source of noise. This source of noise from the drive is also known as the pump noise. When the bridge is balanced, the pump noise does not appear at the input of the SQUID array amplifier to the first order. However, this noise can drive the proof mass into unpredictable motion. We plan to use low temperature filters to greatly attenuate the pump noise. The effect of the residual pump noise can be further mitigated by a simple implementation as shown in Fig. 3. The gap in one of the capacitors $C_{d1}$ or $C_{d2}$ can be adjusted to balance the force that the pump exerts on the prove mass. Then the capacitor $C_x$ can be used to balance the bridge.

## 3. Conclusion

We presented a concept for a very high-resolution displacement sensor. Our technique takes advantage of a recent advancement in DC SQUID technology that greatly extends its bandwidth from ~10*kHz* to ~10*MHz*. The new device based on this advancement is known as the SQUID array amplifier. It enables us to match the impedance of a SQUID to that of a capacitance bridge by operating at high frequencies. A resonance occurs when the impedances are matched. By driving the bridge very close to the resonant frequency, we should be able to combine the exceptional sensitivity offered by a high-Q superconducting resonator with the unmatched sensitivity offered by a SQUID. The combined sensitivity should allow us to surpass the sensitivities offered by competing technologies.

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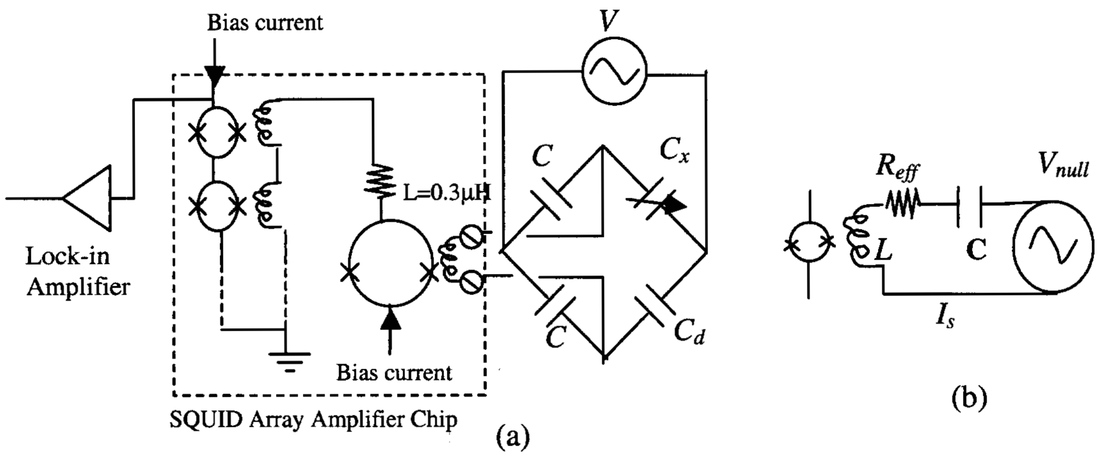


Figure 1: (a) The circuit for the displacement sensor. (b) The Thevenin equivalent circuit of the null detector.

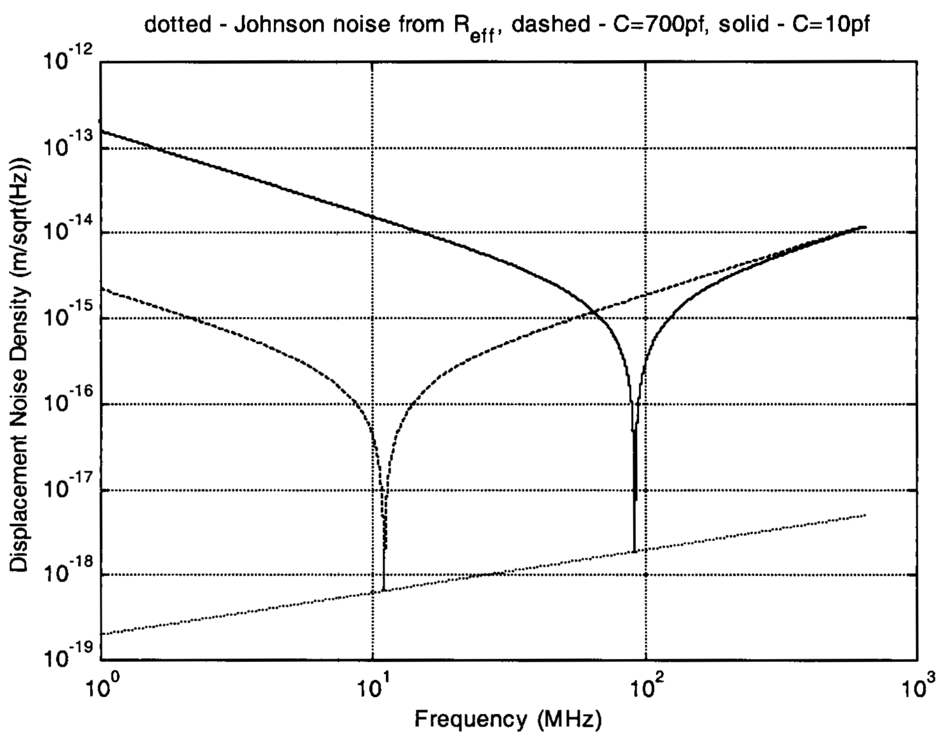


Figure 2: The displacement noise density as a function of the drive frequency. The dotted straight line is from the Johnson voltage noise across $R_{eff}$ alone. The curves include the noise from $R_{eff}$ as well as the SQUID noise. The dashed and solid lines are for $C = 700pf$ and $C = 10pf$ respectively. The temperature is $1.5K$. The drive voltage is $40V$, and $Q = 10^5$.

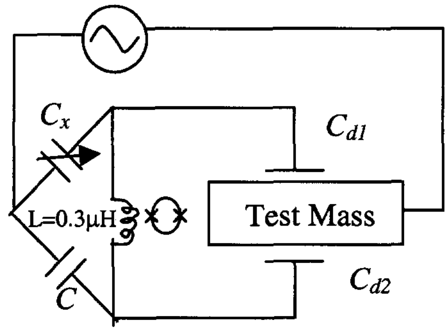


Figure 3: A way to reduce the back-action pump noise.

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